

Homework 0

Purpose: Self Assessment

1. Let U be a $U(0, 1)$ random variable, and let $X = -\ln(U)$. Show that X is an $\text{exp}(1)$ random variable.
2. Let $X_i \sim P(\lambda_i)$, $i = 1, 2$, be two independent random variables. Show that $X = X_1 + X_2$ is a $P(\lambda_1 + \lambda_2)$ random variable.
3. A stick of unit length is cut at two points at distances U_1 and U_2 from the left end. Suppose U_1, U_2 are iid $U(0, 1)$ random variables. Compute the probability that the three pieces can form three sides of a triangle.
4. Suppose X_1, X_2, \dots, X_n are iid $\text{exp}(\lambda)$ random variables. Show that $X = X_1 + X_2 + \dots + X_n$ is an $\text{Erl}(n, \lambda)$ random variable.
5. Number votes cast in a county election is a $P(\lambda)$ random variable. Each vote is Democratic with probability p and Republican with probability $1-p$. All votes are independent of each other. Let D and R be the Democratic and Republican votes counted in the final tally, respectively. Compute the joint pmf of (D, R) .
6. Consider Example 1 on page 575. Let N denote the number of outcomes that never occur in the n trials. Compute $E(N)$.