HW 9 Solutions

Due: Thursday, Nov 8, 2007

Ch. 6, Mod. Ex. 3. Let the state of the system be the queue of machines in the repair shop. Thus, state 0 implies that the queue is empty, i.e., both machines are working, state i, $(i = 1, 2)$ indicates that machine is is in the workshop (under repair) and the other machine is up, state ij, $i, j = 1, 2, i \neq j$ indicates that machine i is under repair, while machine j is waiting in the workshop. Thus the state space is $\{0, 1, 2, 12, 21\}$. Doing the usual triggering event analysis we get the generating matrix as

$$Q = \begin{bmatrix} -(\mu_1 + \mu_2) & \mu_1 & \mu_2 & 0 & 0 \\ \lambda_1 & -(\lambda_1 + \mu 2) & 0 & \mu_2 & 0 \\ \lambda_2 & -(\lambda_2 + \mu 1) & 0 & 0 & \mu_1 \\ 0 & 0 & \lambda_1 & -\lambda_1 & 0 \\ 0 & \lambda_2 & 0 & 0 & -\lambda_2 \end{bmatrix}.$$

Ch. 6, Mod. Ex. 6. Since the arrival process is Poisson, service times are exponential, and the number of active servers depends only on the number of customers in the system, $\{X(t), t \geq 0\}$is a birth and death process with the following parameters:

$$\lambda_i = \lambda, \quad i \geq 0,$$

$$\mu_i = \begin{cases} \mu & \text{for } 1 \leq i \leq 5 \\ 2\mu & \text{for } 6 \leq i \leq 8 \\ 3\mu & \text{for } 9 \leq i \leq 12 \\ 4\mu & \text{for } 13 \leq i \leq 15 \\ 5\mu & \text{for } 16 \leq i \end{cases}$$

Ch. 6, Mod. Ex. 7. The state-space of the system is $\{0, 1A, 1B, 2, 3, 4, ...\}$. The state 1A (1B) indicates that there is one customer in the system and he is being served by server A (B). Otherwise, the state i indicates that there are i customers in the system. The triggering event analysis shows that $\{X(t), t \geq 0\}$ is CTMC with the following transition rates (we show only the positive rates.)

$$q_{0,1a} = \lambda\alpha, \;\; q_{0,1B} = \lambda(1 - \alpha),$$

$$q_{1A,0} = \mu_1, \;\; q_{1A,2} = \lambda,$$

$$q_{1B,0} = \mu_2, \;\; q_{1B,2} = \lambda,$$

$$q_{2,1A} = \mu_2, \quad q_{2,1B} = \mu_1, \quad q_{2,3} = \lambda,$$

$$q_{i,i+1} = \lambda, \quad q_{i,i-1} = \mu_1, \mu_2, \quad i \geq 3.$$

Ch. 6, Mod. Ex. 11. The arrival process is Poisson, service times are exponential, and the admission policy depends only on the number of customers in the system. This makes $\{X(t), t \geq 0\}$a birth and death process with birth parameters

$$\lambda_i = \begin{cases} \lambda_1 + \lambda_2 & \text{if } 0 \leq i < s \\ \lambda_1 & \text{if } i \geq s, \end{cases}$$

and death parameters $\mu_i = \min(i, s)\mu, \quad i \geq 0\}$.

Ch. 6, Mod. Ex. 22. Note that at most one order can be outstanding at any time. The state space of $\{X(t), t \geq 0\}$is $\{0, 1, ..., K + R\}$. Consider state i, $R < i \leq K + R$. In this state no orders are outstanding, and a new demand takes the system to state $i - 1$. Next consider state i, $0 \leq i \leq R$. In this state one order is outstanding. If the order is delivered, the system state moves to $i + K > R$, and if a new demand occurs the state moves to $i - 1$. If $i = 0$, the demand is lost. $\{X(t), t \geq 0\}$is a CTMC with transition rates given below:

$$q_{i,i+K} = \lambda, \quad 0 \leq i \leq R,$$

$$q_{i,i-1} = \mu, \quad 1 \leq i \leq K + R.$$