SOLUTION HW 1.

P1. The state space of $\{X(t), t \geq 0\}$ is $\{0, 1, ..., k\}$. Note that when $X(t) = i$, i machines are working and hence can fail at rate $i\mu$, and $(k-i)$ machines are under repair at a total rate of $(k-i)\lambda$. Following the development in Example 6.5, we see that $\{X(t), t \geq 0\}$is a CTMC with the following transition rates:

$$q_{i,i+1} = (k-i)\lambda, \;\; 0 \leq i \leq k-1,$$
$$q_{i,i-1} = i\mu, \;\; 1 \leq i \leq k,$$
$$q_{i,i} = -i\mu - (k-i)\lambda, \;\; 0 \leq i \leq k.$$

All other elements of the rate matrix are zero.

P2. Same as above, except that in state i, $\min(r, k-i)$ machines are under repair. Hence we have

$$q_{i,i+1} = \min(k-i, r)\lambda, \;\; 0 \leq i \leq k-1,$$
$$q_{i,i-1} = i\mu, \;\; 1 \leq i \leq k,$$
$$q_{i,i} = -i\mu - \min(k-i, r)\lambda, \;\; 0 \leq i \leq k.$$

All other elements of the rate matrix are zero.

P5. The state space of $\{X(t), t \geq 0\}$is $\{0, 1, ..., k\}$. In state i, i wires share a load of M kilograms. Thus each breaks at rate $\mu M/i$. Hence the next break occurs ate rate μM.. This yields the following rates:

$$q_{0,0} = 0, \;\; q_{i,i-1} = \mu M, \quad 1 \leq i \leq k.$$

P6. Since the arrival process is Poisson, service times are exponential, and the number of active servers depends only on the number of customers in the system, $\{X(t), t \geq 0\}$is a birth and death process with the following parameters:

$$\lambda_i = \lambda, \quad i \geq 0,$$
$$\mu_i = \begin{cases} \mu & \text{for } 1 \leq i \leq 5 \\ 2\mu & \text{for } 6 \leq i \leq 8 \\ 3\mu & \text{for } 9 \leq i \leq 12 \\ 4\mu & \text{for } 13 \leq i \leq 15 \\ 5\mu & \text{for } 16 \leq i \end{cases}$$

P7. The state-space of the system is $\{0, 1A, 1B, 2, 3, 4, ...\}$. The state 1A (1B) indicates that there is one customer in the system and he is being served by server A (B). Otherwise, the state i indicates that there are i customers in the system. The triggering event analysis shows that $\{X(t), t \geq 0\}$ is CTMC with the following transition rates (we show only the positive rates.)

$$q_{0,1a} = \lambda\alpha, \;\; q_{0,1B} = \lambda(1-\alpha),$$
$$q_{1A,0} = \mu_1, \;\; q_{1A,2} = \lambda,$$
$$q_{1B,0} = \mu_2, \;\; q_{1B,2} = \lambda,$$
$$q_{2,1A} = \mu_2, \;\; q_{2,1B} = \mu_1, \;\; q_{2,3} = \lambda,$$
$$q_{i,i+1} = \lambda, \;\; q_{i,i-1} = \mu_1, \mu_2, \;\; i \geq 3.$$