Continuous EOQ Model

Assumptions and Notation

- λ = deterministic demand rate per unit time.
- h = holding cost per item per unit time.
- K ordering/setup cost.
- No shortages permitted.
- Zero lead time.

Objective

Decide how much to order and when to order in order to minimize the long run cost per unit time.

ANALYSIS

- Q = order size (decision variable $Q > 0$).
- T = Cycle time = Q/λ.
- Cost in one cycle = $K + \frac{1}{2}hTQ$.
- $C(Q)$ = Cost per unit time = $\frac{K\lambda}{Q} + \frac{1}{2}hQ$.
- $C(Q)$ is minimized at $Q = Q^*$, where
$$Q^* = \sqrt{\frac{2K\lambda}{h}}.$$
- Q^* is called the Economic Order Quantity, EOQ.
- $C(Q^*) = \sqrt{2K\lambda h}$.
- $\frac{C(Q)}{C(Q^*)} = \frac{1}{2}[\frac{Q^*}{Q} + \frac{Q}{Q^*}]$.
- Insensitivity:
$.5Q^* \le Q \le 2Q^* \Rightarrow C(Q) \le 1.25C(Q^*)$.
- $T^* = \sqrt{\frac{2K}{h\lambda}}$.
- Order when the inventory hits zero, every T^* time units.

Extension I
Non-zero Lead Time.

Assumptions and Notation

- The lead time is a fixed constant $\tau > 0$.
- All other assumptions same as before.

Analysis

- If $\tau \leq T^*$, order when the inventory reaches $\lambda\tau$. At most one order is outstanding in this case.
- If $\tau > T^*$, order when the inventory reaches $\lambda(\tau \bmod(T^*))$. The number of outstanding orders is k or $k+1$, where $k = \text{floor}(\tau/T^*)$.
- What happens when τ is random?

Extension II
Shortages Allowed.

Assumptions and Notation

- Cost of shortage: s per item per unit time.
- Zero lead time.
- All other assumptions same as before.

ANALYSIS

- Place an order for Q items when the shortage reaches $Q - M$.
- The inventory is M after an order is placed, and reduces linearly to $-(Q - M)$, at which point another order is placed.
- T = Cycle time = Q/λ.
- Cost in one cycle =

  $$K + \frac{1}{2}h(M/\lambda)M + \frac{1}{2}s((Q - M)/\lambda)(Q - M).$$

- $C(Q, M)$ = Cost per unit time =

  $$\frac{K\lambda}{Q} + \frac{1}{2}\frac{hM^2}{Q} + \frac{1}{2}\frac{s(Q - M)^2}{Q}.$$

- $C(Q, M)$ is minimized at $Q = Q^*$ and $M = M^*$, where

  $$Q^* = \sqrt{\frac{2K\lambda}{h}\frac{h + s}{s}} = EOQ\sqrt{\frac{h + s}{s}}.$$

  $$M^* = EOQ\sqrt{\frac{s}{h + s}}.$$

Other Extensions.

- Quantity Discounts.
- Uncertainty in orders received.
- Random lead times.
- Pricing and perishability.
- Discounted costs.
- Continuous production.
- Price increases.

Power of Two Solution.

- Back to the basic EOQ model.
- Let $T_L \leq T^*$ be a base planning period: (a shift, a day, a week, a month, etc.)
- Restrict the reorder interval T to lie in the set
$$\{2^k T_L : k = 0, 1, 2, ...\}.$$
- The optimal k^* satisfies
$$\frac{T^*}{\sqrt{2}} \leq 2^{k^*} T_L < \sqrt{2} T^*.$$
- Let $C(T)$ be the cost per unit time when the reorder interval is T. Then
$$C(T^*) \leq C(2^{k^*} T_L) \leq 1.06 C(T^*).$$
- The worst power of two solution will produce costs that are within 6% of the optimal cost.
- The average power of two solution will produce costs that are within 2% of the optimal cost.

Discrete EOQ Model

Assumptions and Notation

- Demand D occurs at times nA, $n = 0, 1, 2, ...$
- $h =$ holding cost per item per unit time.
- K ordering/setup cost.
- No shortages permitted.
- Zero lead time.

ANALYSIS

- The optimal order quantity is an integer multiple of D.
- The optimal cycle time is an integer multiple of A.
- $q * D$ = order size (decision variable) ($q = 1, 2, ...$).
- T = Cycle time = $q * A$.
- Cost in one cycle = $K + \frac{hAq(q-1)D}{2}$.
- $C(q)$ = Cost per cycle = $\frac{K}{Aq} + \frac{h(q-1)D}{2}$.
- $C(q)$ is minimized at the smallest integer q^* satisfying
$$q^*(q^*+1) \geq \frac{2K}{hAD}.$$
- q^*D is called the (Discrete) Economic Order Quantity, DEOQ.
- Insensitivity as in the continuous case.
- Order when the inventory is zero and just before a new order is expected.