ASSEMBLY SYSTEM.

- The supply chain consists of N nodes, numbered 1 thru N, arranged as shown in Figure 6.
- Node N receives items from nodes 1 thru $N-1$ and assembles them into a final product.
- External demands only occur at node N at times 1,2,3,...
- Back orders are allowed at node N alone.
- l_i = shipping time from node i to node N, $1 \leq i \leq N-1$ (non-negative integers).
- No ordering/setup cost at any node.
- Lead times at node i are zero, $1 \leq i \leq N-1$.
- h_i = holding cost at node i.
- c_i = cost of an item from node i.
- p = cost of a backordered assembly at the end of a period.
- Push System. Decision variables: how much to order and how much to ship at node i, $1 \leq i \leq N-1$.

OBJECTIVE.

Determine the ordering and shipping policies to minimize the long run rate at which shipping, holding and backordering costs are incurred.

ANALYSIS.

- Renumber the nodes 1 thru $N-1$ so that $l_1 \geq l_2 \geq \cdots \geq l_{N-1}$.
- Since the lead times and ordering costs at each of these nodes are zero, it makes sense not to carry any inventory of item i at node i. As soon as a shipping order arrives, the required amount can be produced and shipped off.
- Imagine a sequential assembly process where node 1 ships an item to node 2, where it is combined with an item from node 2 and shipped to node 3, ..., all the way to node N.
- This creates a series system that we have already studied. The nodes in the series system are numbered from 1 to N, node 1 being the top, and N being the bottom.
- The hoding and procurement costs at node i in this equivalent system are the same as those at node i in the original system.
- The lead time at node 1 is zero, while that at node i $(2 \leq i \leq N)$ is $L_i = l_{i-1} - l_i$. $(l_N = 0.)$ See Figure 7.

- X_i^m = all units of item i in the system plus plus all units ordered in the last m periods.
- Assume that the initial inventory satisfies:

$$X_i^m \leq X_{i-1}^m \quad i = 2, 3, .., N; \quad m = 0, 1, ..., l_i$$

MAIN RESULT.

If the initial inventory levels satisfy the bove condition, the optimal shipping policy for the original system is the same as the optimal policy for the series system (which is a modified base-stock policy) and can be computed as described earlier.

References

1. Rosling, K. (1987). Optimal inventory policies for assembly systems under random demands, Working paper, Dept. of Management and Economics, Linkoping Institute of Technology, Linkoping, Sweden.

2. Schmidt, C. P. and S. Nahmias (1985). Optimal policy for a two stage assembly system under random demand. Oper. Res. 33, 1130-1145.

DISTRIBUTION SYSTEM.

- The supply chain consists of $J+1$ nodes, numbered 0 thru J, arranged as shown in Figure 8.
- Node 0 receives items from outside, and ships them to nodes 1 thru J.
- External demands only occur at nodes 1 thru J at times 1,2,3,...
- Back orders are allowed at nodes 1 thru J.
- l_j = shipping time from node 0 to node j, $1 \leq i \leq J$ (non-negative integers).
- L = lead time for orders at node 0.
- K= ordering/setup cost at node 0.
- Shipping costs are linear.
- h_j = holding cost per unit time per item at node j.
- c_0 = procurement cost per item at node 0.
- c_j = cost of shipping an item to node j from node 0, $1 \leq j \leq J$.
- p_j = cost of a backordered demand at the end of a period at node j, $1 \leq j \leq J$.
- Push System. Decision variables: how much to order at node 0 and how much to ship to node j, $1 \leq j \leq J$.

OBJECTIVE.

Determine the ordering and shipping policies to minimize the long run rate at which ordering, shipping, holding and backordering costs are incurred.

ANALYSIS: NO CENTRALIZED INVENTORY.

- Node zero acts as a centralized ordering and allocation station.
- T = finite horizon length.
- y_t = size of the order placed at node 0 at the beginning of period t, $0 \leq t \leq T$.
- $y^t = [y_{t-L} \ \ldots \ y_{t-1}]$.
- x_{jt} = inventory position at node j at the beginning of period t, $0 \leq t \leq T$.
- $x^t = [x_{1t} \ x_{2t} \ \ldots \ x_{Jt}]$.
- z_{jt} = allocation to location j in period t.
- $z^t = [z_{1t} \ z_{2t} \ \ldots \ z_{Jt}]$.
- u_{jt} = demand at node j at time t.
- $u^t = [u_{1t} \ u_{2t} \ \ldots \ u_{Jt}]$.
- $U_{jt} = \sum_{s=t}^{t+l_j} u_{js}$.

SYSTEM DYNAMICS.

- The state of the system at the beginning of period t is given by (x^t, y^t). This is a $J + L$ dimensional vector.
- System constraints:

$$y_t \geq 0, \quad z_{jt} \geq 0, \quad j = 1, 2, ..., J,$$

$$\sum_{j=1}^{J} z_{jt} = y_{t-L}.$$

- System dynamics:

$$x^{t+1} = x^t + z^t - u^t$$

$$y^{t+1} = [y_{t-L+1} \ \ ... \ \ y_t].$$

- The holding and shortage costs incurred in period t are charged in period $t - l_j$.

$$q_{jt}(x) = h_j E[(x - U_{jt})^+] + p_j E[(U_{jt} - x)^+] \quad \text{if } t \leq T - l_j.$$

$$q_{jt}(x) = 0 \quad \text{if } t > T - l_j.$$

DYNAMIC PROGRAM.

- $f_t(x, y)$ = minimum total expected cost from periods t thru T starting in state $x^t = x = (x_1, x_2, ..., x_J)$ and $y^t = y = (y_1, y_2, ..., y_L)$.
- Decision variables: a = amount ordered at node 0, $z = (z_1, ..., z_J)$, z_j = amount allocated to node j.
- DP recursion:

$$f_T(x, y) = 0,$$

$$f_t(x, y) = \min_{a,z}\{K\delta(a) + c_0 a + \sum_{j=1}^{J} q_{jt}(x_j + z_j)$$
$$+E(f_{t+1}[(x + z - u), (y_2, y_3, ..., y_L, a)])$$

  The minimum is taken over all (a, z) satisfying the system constraints:

$$a \geq 0, \quad z_j \geq 0, \quad \sum_{j=1}^{J} z_j = y_1.$$

- The state space is $J + L$ dimensional. No decomposition possible!
- Exact solution is impractical.

APPROXIMATION BY RELAXATION.

- Ignore the non-negativity part of the system constraints. The resulting value function provides an lower bound on the real value function.
- Assume that the parameters h_j, p_j and c_j are independent of j.
- Consequences (can be proved by induction):
  1. Value function depends on x only via its sum, i.e., the system wide inventory position.
  2. Myopic policy is optimal in each period, i.e., it is optimal to choose z to minimize the one step cost.

- **References:**

  1. Federgruen and Zipkin(1984c). Allocation policies and cost approximation for multilocation inventory inventory systems. Naval research Logistics Quarterly, 31, 97-131.
  2. Zipkin, P. (1982) Exact and approximate cost functions for product aggregates. Management Science, 28, 1002-1012.

APPROXIMATION BY RESTRICTION.

- Consider policies that base their decisions only on systemwide inventory position.
- Further consider either base-stock or (s, S) policies based on systemwide position.

ANALYSIS: CENTRALIZED INVENTORY ALLOWED.

- Inventories may be kept at node zero.
- Consider infinite horizon stationary problem.
- Three decision variables:
  1. Size of the order to be placed at node 0,
  2. amount to be withdrawn from the central inventory for shipping,
  3. allocation of this amount among the retail nodes.
- One can develop similar DPs, and find approximate solutions by relaxation or restriction approach.