GAME THEORY MODELS: SUPPLY CONTRACTS.

- A Supply chain consists of multiple parties linked by flow of goods, information, and funds.
- Centralized management: all decisions made by a single party.
- Decentralized management: Local decisions are made by local parties based on local information.
- Rational local behavior can lead to inefficient global performance.
- Need contractual arrangements modify the local behavior to reduce inefficiency.
- Contracts can specify: reallocation of decision rights, rules for sharing inventory costs, pricing policies, delivery policies, return policies, rules for information sharing, etc. A legal document that includes this specification is called a supply contract.

A SIMPLE MODEL.

- Two node supply chain: one manufacturer, one retailer.
- c = cost per item incurred by the manufacturer.
- Q = amount ordered by the retailer, and shipped by the manufacturer.
- $W(Q)$ = amount charged by the supplier to the retailer to ship Q units.
- p = retail price charged by the retailer to its customers.
- $D(p)$ = demand seen by the retailer if the retail price is p.
- Π^c = total profits under centralized control.
- Π^d = total profits under decentralized control.
- Typically $\Pi^d < \Pi^c$.
- The objective is to write a contract that will benefit both parties and minimize the difference.

MOTIVATING FACTORS.

- **Risk Sharing.** The retailer shares demand forecast with the manufacturer, and guarantees minimum purchase quantity, or accepts penalties for returns, so that the manufacturer does not assume all the risk of demand uncertainty.
- **Channel Coordination.** Alleviate inefficiencies caused by double marginalization. This arises because there are two cost (or profit) centers, and each perceives costs differently. For example the profit is $W(Q)-cQ$ for the manufacturer, while it is $pD(p) - W(Q)$ for the retailer. This leads to inefficient decisions.
- **Long-term Relationships.** Longer term planning vs. short term planning. Creates discounts for recurrent high-volume customers, etc.
- **Legality.** Make the relationship legal with recourse for non-compliance.

COMPONENTS OF A SUPPLY CONTRACT.

- **Decision Rights.** Common model: manufacturer chooses the wholesale price function $W(Q)$, retailer chooses retail price p, and the order quantity Q. Under Retail price management (RPM), manufacturer stipulates p. Under Quantity Fixing, manufacturer controls Q.
- **Pricing.** Specify $W(Q)$. Liner pricing, quantity discounts, etc.
- **Minimum Purchase Commitments.** Specifies minimum order quantity (for a single order or cumulative over the contract period.)
- **Quantity Flexibility.** Ability to change previously placed orders, or forecasts.
- **Return policies.** Also called buyback policies. Specifies how much can be returned, and for what price.
- **Allocation Rules.** Specifies how the manufacturer distributes available stock among its retailers.
- **Lead Times.** Quantifies financial consequences of deviating from agreed upon lead times.
- **Quality.** Quantifies the notion of quality and the consequences of deviating from the agreed upon levels.

TWO-PERSON GAME THEORY:

- x = decision vector of player 1. $x \in X$, the feasible decision space of player 1.
- y = decision vector of player 2. $y \in Y$, the feasible decision space of player 2.
- $f(x, y)$ = profit to player 1 if player 1 chooses decision x, and player 2 chooses decision y.
- $g(x, y)$ = profit to player 2 if player 1 chooses decision x, and player 2 chooses decision y.
- Game is called zero sum if $f(x, y) + g(x, y) = 0$ for all $x \in X$ and $y \in Y$. Else, it is called non-zero sum game.
- Game is called co-operative if f and g are known to both players.
- A pair $(x^*, y^*) \in X \times Y$ is called a Saddle Point solution to the game if

  $$x^* = \text{argmax}_{x \in X} f(x, y^*),$$

  $$y^* = \text{argmax}_{y \in Y} f(x^*, y).$$

  Effectively, if a saddle point solution exists, each rational player will gravitate to it, and once both players reach it, they have no incentive to move.

- What happens if a saddle point solution does not exist?

  1. Randomized solutions. Each player chooses a decision randomly from the feasible decisions according to a distribution. A saddle point exists in the distribution space under mild restrictions.
  2. Nash Equilibrium. Maximize $f(x, y)g(x, y)$ over all $(x, y) \in X \times Y$. Satisfies certain intuitive properties like consistency under linear transformation, Pareto optimality, etc.