Commitments and Stochastic Demand

Supply Contracts with Quantity

R. Anupindi and V. Bassok

7.2 Supply Contracts: A General Framework

- Information Sharing
- Quality (defect rates, specifications, etc.)
- Delivery Commitment (lead time, shipment policy)
- Flexibility
- Quantity Commitment
- Periodicity of Ordering (fixed, random)
- Pricing
- Horizon Length

Supply contract parameters

Material, information, and financial
buyer usually reserves fraction of supplier’s capacity

Suppliers

buyer commits to source fraction of all his demand from specific

Demand Commitment

- Periodic Commitment
  - Total Dollar Volume Commitment (multiple products)
  - Total Minimum Quantity Commitment (single products)

- Order Commitment

Quantity Commitment
Flexibility may come at extra cost to buyer. • Frequency of adjustments – magnitude of adjustments — these quantities to be purchased, supplier often provides some flexibility to adjust when buyer is required to make some commitments on quantities.
Two Types of Research Questions

- **Analysis of Contracts:**
  Given that several types of contracts are observed in practice, what is a decision maker’s (buyer or supplier) optimal policy under these different contracts?

- **Design of Contracts:**
  What types of contracts are written and why?
Supply Contracts

7.3 Classical Inventory Theory as Analysis of
7.3.1 Commitment Type Contracts

- Buyer wants to ensure supply
- Supplier wants to ensure markets

volume (commitment contracts)

- Reasons for existence of total minimum (quantity or dollar

- Supplier and buyer share burden of demand uncertainty

- Quantity commitments with flexibility

- Buyer burdens demand uncertainty

- Stationary quantity commitments with no flexibility

- Supplier burdens uncertainty in demand process

- Newsvendor supply contract
Types of Contracts using Quantity Commitments

- 7.8 Periodical Commitments with Options
- 7.7 Rolling Horizon Flexibility (RHF)
- Periodical Commitment with Flexibility
- 7.6 Total Minimum Dollar Volume Commitment
- 7.5 Total Minimum Quantity Commitment
- 7.4 Total Minimum Quantity Commitment with Flexibility
per unit price decreases as total minimum commitment increases

Supplier offers buyer menu of (per unit price, total minimum commitment) pairs

In return for buyer’s commitment, supplier provides price discount

Long as cumulative quantity restriction is met at end of horizon, buyer has flexibility to order any amount in specific periods as horizon will exceed specified minimum quantity

Guarantees cumulative orders across all periods in planning

Contracts

7.4 Total Minimum Quantity Commitment
available stock $I^O + D^O$ demanded and satisfied as much as possible from demand observed and delivered instantaneously

(order of size $I^O$ placed)

$N$, $Y^T = \min$ minimum remaining commitment in periods $1, \ldots, T$

commitment quantity $Y^T$ observed at beginning of each period $t$, starting inventory and remaining

minimum quantity $Y^T$ for entire horizon at beginning of horizon, buyer makes commitment to buy

demand inventory problem

classical periodic review finite horizon $(N)$ stochastic

7.4.1 Model and Analysis
\( a \) with pdf and cdf \( q_a \sim D \)
- Marginal discounted backlog penalty costs
- Marginal discounted holding costs
- Marginal discounted purchasing costs
- Excess inventory carried to next period
- Excess demand backlogged
- Demands iid across periods
- \( a \) f
- \( a \sim q_a \)

\( \nu \)

\( \eta \)

\( c \)

\( \gamma \)
$$\left(\mathcal{O}, K, I\right) \min_{C} \left(\mathcal{O}, K, I\right) = (\mathcal{O}, K, I)$$

Period \( t \) value function

$$\bigwedge_{N} \text{Periodical purchase quantities} \quad \mathcal{O} = (\mathcal{O}, K, I)$$

Total expected cost for periods \( t \) through \( N \)

Decision variables:

- Remaining commitment \( K \)
- On-hand inventory \( I \)

Two state variables:

Stochastic dynamic program

Optimization Problem
\[
\left\{ ([I+\gamma X I+\gamma I] I+\gamma \gamma)_{\gamma} \mathcal{E} + (\gamma \mathcal{O} + \gamma I) T + \gamma \mathcal{O} \right\}_{\text{min}} = (\gamma X, \gamma I)_{\gamma} T
\]

Dynamic programming recursion •

\[
([I+\gamma X I+\gamma I] I+\gamma \gamma)_{\gamma} \mathcal{E} + (\gamma \mathcal{O} + \gamma I) T + \gamma \mathcal{O} = (\gamma \mathcal{O}, \gamma X, \gamma I)_{\gamma} \mathcal{O}
\]

\[
[+((\mathcal{O} + I) - A)]_{\gamma} \mathcal{E} + [+((A - \mathcal{O} + I)]_{\gamma} \mathcal{E} = (\mathcal{O} + I) T
\]

Single period expected holding and penalty costs •

\[
N \cdot \cdot \cdot \cdot I = \gamma \quad [+((\mathcal{O} - \gamma X)] = I+\gamma \gamma X
\]

\[
N \cdot \cdot \cdot \cdot I = \gamma \quad \gamma A - \gamma \mathcal{O} + \gamma I = I+\gamma I
\]

\[
0 \equiv (0, I+\gamma N I)_{I+\gamma N} I
\]

define •
\[ N \cdot \prod_{i=1}^{\mathcal{O}} 0 \preceq \mathcal{O} \]

\[ \prod_{i=1}^{\mathcal{O}} \bigcap_{\mathcal{O}} \min_{\mathcal{O}} \]

optimization problem
3. \( \mathbb{I}^{(3)}(I, \mathcal{K}, \mathcal{O}) \) is convex in \( I, \mathcal{O}, \mathcal{K} \), and \( \mathcal{K} \).

\[ \frac{\gamma + \eta}{\eta} = (\infty S)(\mathcal{H}) \]

The newsvendor problem without commitment and period standard

where \( S = I + t - N \), and

\[ \begin{cases} \gamma \mathcal{K} - \gamma S > \gamma I \geq 0 \\ \gamma \mathcal{K} - \infty S > \gamma I \geq \gamma \mathcal{K} - \gamma S \\ \infty S > \gamma I \geq \gamma \mathcal{K} - \infty S \\ \infty S \leq \gamma I \end{cases} \]

\( \mathcal{O} \)

2. The optimal order for period \( t \) is:

\[ \mathcal{O} \]

I. Proposition 1.

Proposition 1.
Figure 7.1
stock levels $S_1, \ldots, S_\infty$ and $S$ to implement optimal order policy, need only calculate base
offered

(purchase price, total minimum commitment) pair from menu
of commitment and purchase price, and thus to choose best
solution allows buyer to evaluate optimal costs as function


unit regular price = $1.00

mean demand 100.0 units, with CV 0.25

per cent age savings defined as savings in total costs of contract over that of base case

regular cost
discounted contract cost price specified as per cent age costs over

with no restriction on total quantities purchased

base case: no contract signed, buyer purchases at regular price

minimum commitment contract

numerical example to illustrate benefits to buyer of signing total

7.4.2 Example
be advantageous, depending on discount offered and horizon length. Commitment for mean demand over horizon (1000 units) may not

(loss) commitment below (above) 920 units results in expected savings • buyer should not commit to purchase more than about 900 units

price discount 5%: 

demand distributions normal • horizon length = 10 periods • unit holding cost = 25% of purchase price • unit penalty cost = $2.50
minimum quantity \( K \) for entire horizon

at beginning of horizon, buyer makes commitment to buy

supplier at higher price (supplier may offer to supply quantitites larger than a certain amount at higher price (or buyer may source these from another discounted price (e.g., when production capacity restricted)

but supplier may impose restrictions on total purchases at

in previous section, assumed buyer could purchase any additional quantity above minimum commitment at same discounted price

Contracts with Flexibility

7.5 Total Minimum Quantity Commitment
units purchased at discounted price \( c \) \( \neq \emptyset \) ●

any further purchases above available at regular price \( c < c \) ●

purchases up to discounted price available \( \sum_{t=1}^{T} Y_t (a + 1) = \sum_{t=1}^{T} Y_t \) ●

total purchases over horizon of contract

fraction \( a \) flexibility that supplier offers to buyer to adjust

fixed fraction \( a \) above minimum

marginial discounted purchasing costs for purchases up to

minimum remaining commitment in periods \( 1, \ldots, T \) \( c = c \) ●

remaining \( \sum_{t=1}^{T} Y_t \) quantity observed

commitment quantity observed \( \sum_{t=1}^{T} I_t \) at beginning of each period, \( t \), starting inventory and remaining
$\mathcal{F}$ demands iid across periods with pdf and cdf $f$ (\(D \sim \mathcal{D}\))

\begin{align*}
\nu &= \text{marginal discounted backlog penalty costs} \\
\eta &= \text{marginal discounted holding costs} \\
\text{excess inventory carried to next period} &\text{ excess demand backlogged} \\
\text{available stock observed and satisfied as much as possible from demand of size } \mathcal{D} &\text{ placed and delivered instantly (wlog)} \\
n \# &= \mathcal{W}$
Proposition 2. The optimal order policy for period \( t \) is defined by three critical levels \( S_0, S^*, \) and \( S_m \), as follows:

- The function is convex in the arguments and can be shown that expected cost and corresponding value function are convex in their arguments.
- Total minimum commitment flexibility (supplier offers buyer menu of regular price, discounted price, ...)
- The optimal order policy for period \( t \) is defined by three critical levels \( S_0, S^*, \) and \( S_m \), as follows:
$\int (1 + t - N) (I + t - N)$

order cost, and $\int S$ base stock level for standard

$\int (1 + t - N) (I + t - N)$ standard

where

$\int X - S > I > \int X - wS \notin$

$\int X - S > I > \int X - S \notin$

$\int X - \infty S > I > \int X - S \notin$

$\infty S > I > \int X - \infty S \notin$

$\infty S < I \notin$

$\int I - \int X - wS$ \( \left\{ \begin{array}{ll}
(0, \int X) \\
(0, \int I - S) \\
(0, \int X) \\
(0, \int I - \infty S) \\
(0, 0)
\end{array} \right. 
= (\int *I, \int *O)$
stock levels $S^*, \ldots, S^0, S^N$, and $N^*$, \ldots, $N^0$.

• To implement optimal order policy, need only calculate base

offered

price, total minimum commitment flexibility, 4-tuple from menu

flexibility, and thus to choose best (regular price, discounted

purchase prices (regular and discounted), and

solution allows buyer to evaluate optimal costs as function of
price (may be from different supplier)

purchases in excess of upper bound charged regular (market)

( flexibility)

purchases up to a certain fraction above minimum commitment

supplier extends discounted price for cumulative dollar volume

during specified time horizon

buyer commits to minimum cumulative dollar value of purchases

multiple products: contracts on total dollar volume of purchases

single products: total minimum quantity contracts

Contracts

7.6 Total Minimum Dollar Volume Commitment
Advantages to Buyer and Supplier

- by offering discounts, supplier increases total business volume
- by pooling purchases, buyer can negotiate higher discount rates
- 
- and increases market presence for higher priced products
7.6.1 Model

- Fraction \( a = \text{flexibility supplier offers to buyer to adjust total purchases over horizon of contract} \)
- \( c_p = \text{unit discounted price for product} p \)
- \( P = \text{total # products in family, indexed by} p \)
- \( K' = \text{minimum dollar volume across all products} \) for entire horizon

- at beginning of horizon, buyer makes commitment to buy total purchases up to \( K' = (1 + a)K' \) available at discounted price
\[ dW + dO + dI \]

available stock observed and satisfied as much as possible from demand observed (\( \log \))

for each product placed and delivered order of size of \( d \) units of product purchased at regular price

\[ \frac{d}{d} \text{ unit of product purchased at discounted price} \quad \# = \frac{d}{d} W \]

\[ \frac{d}{d} \text{ unit of product purchased at discounted price} \quad \# = \frac{d}{d} O \]

remaining commitment quantity observed

at beginning of each period \( t \), starting inventories and

price

\[ \frac{d}{d} o < \frac{d}{d} O \]

any further purchases of product above \( d \) available at regular price
(total minimum commitment flexibility)
supplier offers buyer menu of (regular price, discounted price, ...

\[
\mathbb{P}(d\mathcal{I} \mid \mathcal{I}) d\mathcal{I} \sim \frac{1}{d\mathcal{I}} \]

with

demands for each product iid across periods $d$ with each other

demands for products independent of each other

\[
\begin{align*}
&d \text{ marginal discounted backlog penalty costs for product } d = d\nu \\
&d \text{ marginal discounted holding costs for product } d = d\eta \\
&\text{salvage value of excess inventory (in last period)} = 0 \\
&\text{excess inventory carried to next period} = 0 \\
&\text{excess demands backlogged} = 0
\end{align*}
\]
\[ 0 \equiv (0, 0, 0, 0, 0, 0, 0, 0, 0, N, 1)_{t + N} \]

**Define**

\[
(\Omega Y, \frac{\hat{Y}}{I}, \frac{\hat{W}, \hat{O}, \hat{I}}{I})_{\min} = (\Omega Y, \frac{\hat{Y}}{I}, \frac{\hat{W}, \hat{O}, \hat{I}}{I})_{t}
\]

Problem starting in period \( t \)

\[
(\Omega Y, \frac{\hat{Y}}{I}, \frac{\hat{W}, \hat{O}, \hat{I}}{I})_{t + N}
\]

optimal expected costs for period \( t + N \)

\[
(\Omega Y, \frac{\hat{Y}}{I}, \frac{\hat{W}, \hat{O}, \hat{I}}{I})_{t}
\]

Total expected costs for period \( t \)

\[
(\Omega Y, \frac{\hat{Y}}{I}, \frac{\hat{W}, \hat{O}, \hat{I}}{I})_{t}
\]

Single period expected holding and penalty cost

\[
(\Omega Y, \frac{\hat{Y}}{I}, \frac{\hat{W}, \hat{O}, \hat{I}}{I})_{t}
\]

Specified: e.g., \( I \) denote vectors of size unless otherwise denoted
\[ \left\{ \left( \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C} \right) \right\} \frac{1}{\tau^t} \mathcal{E} + \left( \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C} \right) \right\} \frac{1}{\tau^t} \mathcal{E} = \left( \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C} \right) \frac{1}{\tau^t} \mathcal{E} \]

dynamic programming recursion

\[ \left\{ \left( \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C} \right) \right\} \frac{1}{\tau^t} \mathcal{E} + \left( \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C} \right) \right\} \frac{1}{\tau^t} \mathcal{E} = \left( \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C} \right) \frac{1}{\tau^t} \mathcal{E} \]

\[ \left( \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C} \right) \right\} \frac{1}{\tau^t} \mathcal{E} = \left( \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C}, \frac{1}{\tau^t} \mathcal{C} \right) \frac{1}{\tau^t} \mathcal{E} \]

for each \( t = 1 \), \( \cdots \)
\[
N^{\ldots} I = \tau \quad d^{\ldots} I = d \\
N^{\ldots} I = \tau \quad d^{\ldots} I = d \\
N^{\ldots} I = \tau \quad d^{\ldots} I = d
\]

\[
0 \leq \frac{d}{d} I \quad 0 \leq \frac{d}{d} O
\]

\[
\left( \bigcap_{d} \frac{d}{d} O \bigcap_{d} \bigcap_{d} I = \bigcap_{d} I + \bigcap_{d} I \right)
\]

\[
\bigcap_{d} O d \bigcap_{d} Y = \bigcap_{d} I + \bigcap_{d} I
\]

\[
\text{s.t. (\bigcap_{d} I, \bigcap_{d} O, \bigcap_{d} I, \bigcap_{d} I) } \text{ minimization problem}
\]
\[ (d^\omega m = \frac{\partial}{\partial t} \xi) \]

such that \( m > 0 \) in contract indentical (for all \( \xi \), there exists an \( m < 0 \))

uniformly discounted policy property: price discount across all

\[
\left\{ \left[ \left( (dW + d\mathcal{O} + dI) - dQ \right) \right]^{dQ} \mathcal{E} d\mu + \sum_{i=1}^{d} \left[ \left( dQ - dW + d\mathcal{O} + dI \right) \right]^{dQ} \mathcal{E} d\eta \right\} = \left( W', \mathcal{O}, I \right) \mathcal{T}
\]

assuming proportional holding and penalty costs.
at regular price only when $Y_T$ is exhausted

• buyer will first purchase goods at discounted price, then purchase

flexibility is not optimal

• Generalization of policy structure for single product case with

will also likely be complex

multi-product constrained dynamic program, optimal solution

since optimization problem for buyer is complex multi-period,

7.6.2 Analyses
last period problem (dollar volume across products) need be solved only for under order policy assumption, can be shown that allocation
not optimal, but gives upper bound on total expected costs
and independent of purchase price (either discounted or regular)
base stock levels for product identical for periods 1, 2, ..., \( N \) – 1

\[
\frac{d\mu + d\nu}{d\nu} = \left( \int_{dS}^\infty \right) d\nu\ \text{where} \quad \int_{dS}^\infty \text{up to period} \ d \text{order each product until}
\]

Order Policy Assumption •

Upper Bound
allocation algorithms

lower and upper bounds on budget (use standard resource

last period problem is multi-product newvendor problem with
expected costs

can be shown that LB policy gives lower bound on optimal

units to be salvaged in any period at their purchase price

LB policy: same as order policy assumption above, but allow

Lower Bound
(By itself offers enough flexibility)

Pooling of demands over multiple periods and multiple products small even when number of products small (intuitively, risk little loss in committing to mean dollar volume, value of flexibility)

demand equal to 1.0, smaller gap for lower CV's gap between bounds usually very small (≥ 3.03%) for CV of

Results of Computational Studies
costs affected

optimal for entire supply chain, especially if supplier’s production demand uncertainty “pass-through” to supplier may not be

demand due to finite horizon nature or remaining problem

commitment exhausted (subsequent orders may differ from orders exactly demand in previous period until total minimum

for single product problems, policy structure implied that buyer

quantities purchased each period
discussed previously, no specific restrictions imposed on exact

in total quantity (dollar volume) commitment contracts

Flexibility

7.7 Periodical Commitment Contracts with
unit price could increase with band-width (greater flexibility)

specification lower and limits that are stationary over time

— buyer required to restrict all order quantities to be within order bands

— more than discount price, may not be delivered immediately

— additional units available at extra cost (unit price possibility)

— discounts based on commitment level

— buyer required to purchase minimum amount in each period

stationary commitments

Two Examples
- buyer updates period 3 commitment to 130
  \[150 \div 30 = 120, 180\]
- commitment for period 3 can be adjusted by 20%:

  \[\text{in period 1:}\]

  \[\text{in period 0:}\] commitment for period 3 is 150

  20% flexibility in commitment two periods from now

  10% flexibility in commitment one period from now

  5% flexibility in current period’s order

  \text{Example: Rolling Horizon Flexibility (RF)}
Low level of short-term flexibility

High level of long-term flexibility

\[ 120 \div 6 = 114 \text{, } 126 \]

- Buyer is allowed to purchase within 5% of 120

In period 3:

- Buyer updates period 3 commitment to 120

\[ 130 \div 13 = 117 \text{, } 143 \]

- Commitment for period 3 can be adjusted by 10%

In period 2:
0 = \bar{\gamma}_t \equiv \gamma_t - \gamma_t^* \\
\text{for } t \geq 0 \\
\text{for period } t \\
\text{commitment (or actual order if } t = \ell) \\
\text{fraction by which buyer may adjust period } t \\
\text{commitment (or actual order if } t = \ell) \\
\text{fraction by which buyer may adjust period } t \\
\text{for all periods } t = 0, 1, \ldots, N \\
\text{initial commitment made by buyer at beginning of horizon} \\
\text{finite horizon RHE contract} \\
7.7.1 Model and Measurements
\[
\begin{align*}
\begin{bmatrix} \zeta \end{bmatrix} = \begin{bmatrix} x \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \zeta \end{bmatrix} = \begin{bmatrix} x \end{bmatrix}
\end{align*}
\]

upward and downward flexibilities specified as two matrices \( \begin{bmatrix} \zeta \end{bmatrix} \) and \( \begin{bmatrix} \zeta \end{bmatrix} \) for \( x \in \mathbb{R} \) and \( x \in \mathbb{R} \) for \( \zeta \) and \( \zeta \) in period \( t \) and period \( t \) for \( x \).
excess inventory carried to next period • excess demands backlogged • available stock

\( t_0 + t_1 \) observed and satisfied as much as possible from demand updates commitment for future periods • buyer updates commitment for future periods

order of size \( t_0 + t_1 \) placed and delivered instantaneously (WLOG) •

at beginning of each period \( t, t_1 \) starting inventory observed •

\[
\mathcal{O} = \mathcal{O}_{t-1} + \mathcal{O}_{t} + \mathcal{O}_{t+1} \quad \text{and} \quad \mathcal{O} = \mathcal{O}_{t} + \mathcal{O}_{t+1} \]

future period depends only on number of periods into future.

usually, RHF contracts have property that flexibility offered for
commitments made in period \( t - 1 \) for periods \( t \) through \( N \). Given starting inventory and total expected cost \( C(t) \), the supplier offers menu of (discounted price, flexibility) with demands independent across periods with \( p(t) \) and \( c(t) \).

\[
C(t) = \sum_{t=1}^{N} c(t) p(t) + I(t) f(t) + \frac{1}{2} D(t) b(t) + s(t) \]

where:

- \( C(t) \) = marginal discounted backlog penalty costs
- \( s(t) \) = salvage value of excess inventory (in last period)
\[
[+((\bar{\mathcal{O}} + \bar{I}) - \bar{A})]\mathcal{E}v + [+\left(\bar{A} - \bar{\mathcal{O}} + \bar{I}\right)]\mathcal{E}u = (\bar{\mathcal{O}} + \bar{I})\bar{I}
\]

single period expected holding and penalty costs

\[
N \cdot \ldots \cdot I = \bar{\tau} \quad \bar{A} - \bar{\mathcal{O}} + \bar{I} = I + \bar{I}
\]

define:

\[
I + NIs - \equiv (I + NI)^{1 + N}I
\]

\[
(N^{\bar{\mathcal{O}}} - I^{\bar{\mathcal{O}}}, \ldots, N^{\bar{\mathcal{O}}} - I^{\bar{\mathcal{O}}}, I)\mathcal{O}
\]

\[
= (N^{\bar{\mathcal{O}}} - I^{\bar{\mathcal{O}}}, \ldots, N^{\bar{\mathcal{O}}} - I^{\bar{\mathcal{O}}}, I)\mathcal{I}
\]

period \(t\) value function \( = (N^{\bar{\mathcal{O}}} - I^{\bar{\mathcal{O}}}, \ldots, N^{\bar{\mathcal{O}}} - I^{\bar{\mathcal{O}}}, I)\mathcal{I}\)
\[
\left\{ \left[ \left( \sum_{i=1}^{N-1} \mathcal{O} \cdot \mathcal{I} \right)^t \right] \right\}_{m \in \mathbb{N}} + \left( i \mathcal{O} + \mathcal{I} \right)^t \mathcal{I} + t \mathcal{O} \mathcal{O}^2 \\
= \left( \sum_{i=1}^{N-1} \mathcal{O} \cdot \mathcal{I} \right)^t \mathcal{I}
\]

Dynamic Programming-recursion

\[
\left[ \left( \sum_{i=1}^{N-1} \mathcal{O} \cdot \mathcal{I} \right)^t \right] \mathcal{A} + \left( i \mathcal{O} + \mathcal{I} \right)^t \mathcal{I} + t \mathcal{O} \mathcal{O}^2 \\
= \left( \sum_{i=1}^{N-1} \mathcal{O} \cdot \mathcal{I} \right)^t \mathcal{O}^t
\]

For each \( t = 1, \ldots, N \).
\[ N^{\cdot \cdot \cdot, t} = \gamma \quad t_{t-1} \quad \delta (\gamma \cdot \gamma + 1) \geq t \quad \delta \geq t_{t-1} \quad \delta (\gamma \cdot \gamma - 1) \]

subject to

\[ (N^{t-1} \quad \delta, \ldots, t_{t-1} \quad \delta, I \mid N^{t} \quad \delta, \ldots, t_{t} \quad \delta, \) \]

\[ \min \quad N^{t} \quad \delta, \ldots, t_{t} \quad \delta, \]

optimization problem in period \( t = 2, \ldots \)
of demand

- Converted value of CV of orders significantly lower than CV

- Order converges as horizon length increases

- In computational studies with stationary demands, CV of

\[
\frac{\left[\text{CV}\right] \mu}{\left(\left[\text{CV}\right] \mu - \left[\text{CV}\right] \sigma^2\right)^{2}} = \text{ORCV}
\]

- Coefficient of variation (CV) of period t order

• Order process variability

Order Process Measurements
\[
[\mid |\epsilon | + |\epsilon | + |\epsilon | \mid ] E = \theta \theta \theta \theta \theta \theta D \theta \theta \theta \theta \theta \theta \theta \theta \theta \\
\text{In period } t \text{ for period } t + 1 \text{ and actual order placed in period } t \text{ and committed made} \text{ – mean absolute deviation (MAD) between commitment made}\n\text{ advance information}
\[
+ \cdots + \left( \frac{1}{2} \right)
= \left( \frac{1}{2} \right)
\]

OLFC policies not optimal, but close to optimal

OFCC policies will not be changed in future

periodic commitments at beginning of period, assuming

Open Loop Feedback Control (OLFC): determine optimal

perhaps unattainable for implementation

optimal policy for RHF problem will be extremely complex and

7.7.2 Analysis
\[
((\varphi_1^2 \mathcal{O})^2 \mathcal{W} \mathcal{L}^2 \chi) + (N)^2 \mathcal{O} = (\mathcal{O})^2 \mathcal{W} \mathcal{L}^2 \chi
\]

\[
\left[ + ((N)^2 \mathcal{O} - (N)^2 \mathcal{A}) \right] \mathcal{H} \nu + \left[ + ((N)^2 \mathcal{A} - (N)^2 \mathcal{O}) \right] \mathcal{H} (s - \eta) = ((N)^2 \mathcal{O})^N \mathcal{W}
\]

For \( N = \) starting in period \( \mathcal{O} \)

\[
\left[ + ((\varphi_1^2 \mathcal{O} - (\varphi_1^2 \mathcal{A})) \right] \mathcal{H} \nu + \left[ + ((\varphi_1^2 \mathcal{A} - (\varphi_1^2 \mathcal{O})) \right] \mathcal{H} \eta = ((\varphi_1^2 \mathcal{O})^2 \mathcal{W}
\]

Since \( N = \) starting in period \( \mathcal{A} \)

Expected holding and penalty costs in period \( N \)

Expected costs for periods \( t \) through \( N \)

\[
\text{through } N = \text{total expected costs for periods } t = (\mathcal{O})^2 \mathcal{W} \mathcal{L}^2 \chi
\]
\[ \begin{align*} \bar{t} - N & \geq 0 \\
 & \geq (\bar{t} + \bar{t} t x + I) \geq (\bar{t} + \bar{t} t - \bar{t}) \geq (\bar{t} + \bar{t} t \bar{O} - I) \\
 \bar{t} - N & \geq 0 \quad \Rightarrow (I - \bar{t} + \bar{t}) \bar{O} \lesssim (\bar{t} + \bar{t}) \bar{O} \quad \text{s.t.} \\
 (N) \bar{O} + \bar{t} x & \cdots (\bar{t}) \bar{O} + \bar{t} x \bar{O} \\
 \text{min} \\
 \text{for period } \bar{t} = 2 \end{align*} \]

\[ \begin{align*} N & \geq I \geq 1 \quad \Rightarrow (I - \bar{t}) \bar{O} \lesssim (\bar{t}) \bar{O} \quad \text{s.t.} \\
 (N) \bar{O} & \cdots (I) \bar{O} \\
 \text{min} \\
 \text{for period } I \end{align*} \]
- Zero Lead time Flexibility (ZLF) contract

- buyer makes commitment in first period for quantities to be purchased in each period

- allow actual order quantity to be adjusted, but not future commitments

- special case of RHF with \( \alpha_{t,i} = \overline{\alpha}_{t,i} = 0 \) for \( i > t \)
\[ \mathcal{C} = ? \quad \mathcal{C} 
eq ? \]
\[ \left\{ I - \left( \mathcal{C}, \mathcal{C} \right) \mathcal{C}^{\dagger} + I \right\}^{1=\gamma} = \mathcal{C} \]}
\[ \left\{ I - \left( \mathcal{C}, \mathcal{C} \right) \mathcal{C}^{\dagger} - I \right\}^{1=\gamma} = \mathcal{C} \]}

\[ \mathcal{C} \] and \[ \mathcal{C} \] define matrices and \[ \mathcal{C} \] and \[ \mathcal{C} \] define matrices.
between mean demand and order

less variability between information and order than variability
information within 1-3 periods of actual order more "reliable"
creates information regarding future orders
process of generating and updating commitments for future
contract
variability in order process significance reduced under RHFC
price
OLFC heuristic performs well when salvage value equals purchase
computational studies with stationary demand

7.7.3 Performance Evaluation
two-period model with correlated normal demands •

quantity of goods at exercise price (up to number of options purchased) enable buyer to procure, in future period, additional options for each unit of option purchased •

option price for each unit of option purchased •

flexibility: buyer may purchase ahead of time some options commitments by buyer similar to supply contracts in previous sections: quantity

Contracts with Options 7.8 Coordination and Flexibility using Supply

Contracts with Options
value of options increases with correlation in demand

usually gains more

both buyer and supplier gain from use of options, though supplier

under certain conditions, channel coordination can be achieved

options to buyer at a price

supplier mitigates risk of raw material purchase by offering

expected

but there might be opportunities to sell more products than

raw material (all at once at beginning of season)

changed eliminates risk of supplier buying too much or too little

requiring buyer to commit to order quantities which cannot be
7.9 Summary and Conclusion

- Coordination commitments together with options can achieve channel
- Between supplier and buyer (contractual bull-whip effect) and share risks, due to uncertainty,
- Periodic commitments reduce variability in order process
- Buyer's requirements (ensuring market for supplier and supply for supplier) to prepare enough capacity and raw materials to meet
- Wrt buyer's overall demand and specific future orders allowing
- Quantity commitments provide supplier with reliable information

-
single period) to incorporate more realistic settings
– relax assumptions (single supplier, single buyer, risk neutrality,
– include information asymmetries

• design of contracts

• complex settings: multiple players, multiple products
• consider supplier’s perspective
decision makers’ behaviors and implications on other player
• incorporate other parameters of supply contract to study

• analysis of contracts

Future Research