Continuous Review: Stochastic Demand.

- Inventory level is monitored continuously.
- Demand process needs to be specified.
- K = order cost.
- h = holding cost.
- p = shortage cost per item per unit time.

OBJECTIVE

Find an ordering policy that minimizes the long run cost per unit time.

Deterministic Demand: Stochastic Lead Time.

- Demand occurs continuously at a rate λ per unit time.
- L_n = nth lead time, $\{L_n, n \geq 0\}$ a sequence of iid random variables.
- $\zeta_n = \lambda L_n$ = demand during the nth lead time.
- Backlogging is allowed.
- At most one outstanding order allowed.

ANALYSIS

- When an order is received, place another order immediately if inventory on hand is s or less. Else, wait for the inventory to dwidle to s. Always order upto S.
- A cycle starts whenever an order is received.
- $X_n =$ inventory on hand at the beginning of the nth cycle.
- If $X_n > s$, an order of size $S - s$ is placed when the inventory reaches s.
- If $X_n \leq s$, an order of size $S - X_n$ is placed immediately.
- The order is received after a lead time of L_n.
- Thus

$$X_{n+1} = \begin{cases} s - \zeta_n + S - s & = S - \zeta_n \quad \text{if } X_n > s \\ X_n - \zeta_n + S - X_n & = S - \zeta_n \quad \text{if } X_n < s \end{cases}$$

- $T_n =$ length of the nth cycle.

$$T_n = \begin{cases} (X_n - s)/\lambda + L_n & \text{if } X_n > s \\ L_n & \text{if } X_n < s \end{cases} = \frac{(X_n - s)^+}{\lambda} + L_n.$$

- The expected ordering cost during a cycle is K.
- Considering the ranges $(-\infty, 0], (0, s]$, and $(s, S]$ for X_n and $(0, \min(s, X_n)]$, and $[\min(s, X_n), \infty)$ for ζ_n, we get

$$E(\text{ Holding Cost in a cycle}) = \frac{h}{2\lambda}[X_n^2 - ((\min(s, X_n) - \zeta_n)^+)^2]^+$$

$$= \frac{h}{2\lambda}[(S - \zeta_{n-1})^2 - ((\min(s, S - \zeta_{n-1}) - \zeta_n)^+)^2]^+.$$

- Similarly,

$$E(\text{ Shortage Cost in a cycle}) = \frac{p}{2\lambda}[((\zeta_n - \min(s, X_n))^+)^2]$$

$$= \frac{p}{2\lambda}[((\zeta_n - \min(s, S - \zeta_{n-1}))^+)^2].$$

- Using regenerative process theory, we get

$$E(\text{Long run cost rate}) = \frac{E(\text{ord + hol + shor cost in a cycle})}{E(\text{Length of a cycle})}.$$

- Find (s, S) to minimize the above cost rate. No analytical solution. Hence, we look for approximations.

APPROXIMATION: BACKORDER CASE

- p= shortage cost per unit. (Not per unit time)
- $$E(T_n) = \frac{S-s}{\lambda}$$

  $$E(\text{ Holding Cost in a cycle}) = h\frac{S-s}{\lambda}[\frac{S-s}{2}+E(s-\zeta)].$$

  $$E(\text{ Shortage Cost in a cycle}) = pE((\zeta - s)^+).$$

- Long run expected cost rate

  $$\frac{K\lambda}{S-s} + \frac{p\lambda}{S-s}E((\zeta-s)^+) + h[\frac{S-s}{2} + E(s-\zeta)].$$

- Optimal solution:

  $$Q = S - s = \sqrt{2\lambda[K + pE((\zeta - s)^+)]/h},$$

  $$P(\zeta \geq s) = \frac{Qh}{p\lambda}.$$

- Solve these two simultaneous equations iteratively. Start with Q, and compute s using the second equation. Then update Q using the first equation. Continue until convergence occurs.

APPROXIMATION: LOST SALES CASE

- p= cost of a lost sale.
- $$E(T_n) = \frac{S-s}{\lambda}$$

  $$E(\text{ Holding Cost in a cycle}) = h\frac{S-s}{\lambda}[\frac{S-s}{2}+E((s-\zeta)^+)].$$

  $$E(\text{ Shortage Cost in a cycle}) = pE((\zeta - s)^+).$$

- Long run expected cost rate

  $$\frac{K\lambda}{S-s} + \frac{p\lambda}{S-s}E((\zeta - s)^+) + h[\frac{S-s}{2} + E((s-\zeta)^+)].$$

- Optimal solution:

  $$Q = S - s = \sqrt{2\lambda[K + pE((\zeta - s)^+)]/h},$$

  $$P(\zeta \geq s) = \frac{Qh}{p\lambda + Qh}.$$

- Solve these two simultaneous equations iteratively. Start with Q, and compute s using the second equation. Then update Q using the first equation. Continue until convergence occurs.

- Thus the s value under lost sales case is smaller than that in the backordering case, while the Q value is larger.