Pergamon
www.elsevier.com/locate/jaerosci
PII: S0021-8502(99)00025-7
J. Aerosol Sci. Vol. 30, No. 10, pp. 1317–1320, 1999

Printed in Great Britain
0021-8502/99/$ - see front matter

A SIMPLE DERIVATION OF THE CUTPOINT OF AN IMPACTOR

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(*First received* 14 *September* 1998; *and in final form* 16 *February* 1999)

Abstract—Impactors are designed according to numerical computations, which have been shown to give accurate predictions for the 50% cutpoint. However, such computer calculations obscure the basic physics of the impactor. Textbook derivations commonly assume circular streamlines and do not yield accurate results for the cutpoint. In this paper, a simple but accurate derivation is given for the cutpoint of an impactor with a rectangular nozzle, based on only very general assumptions as to the flow. For an impactor with a circular nozzle, the derivation requires an additional input datum and is less accurate. The derivation makes clear the essential physics of the impactor. © 1999 Elsevier Science Ltd. All rights reserved

INTRODUCTION

Impactors are among the most widely used particle size-selective samplers. They are commonly designed by reference to published numerical computations such as those by Marple and Lui (1974) or Rader and Marple (1985). These computations give the impaction efficiency as a function of the dimensionless particle size expressed as the square root of the Stokes number, St. St is defined (Hinds, 1982) as the ratio of the stop distance, L, to the half-width of the nozzle aperture:

$$\mathrm{St} = \frac{2L}{W}, \tag{1}$$

$$\sqrt{\mathrm{St}} = \sqrt{\frac{\rho_\mathrm{p} D_\mathrm{p}^2 V_0 C}{9\mu W}}, \tag{2}$$

where ρ_p is the particle density, D_p the particle diameter, V_0 the average air velocity in the nozzle throat, C the slip correction, μ the fluid viscosity, and W the nozzle width (rectangular nozzle) or nozzle diameter (round nozzle).

Of particular interest is the value of $\sqrt{\mathrm{St}}$ corresponding to an impaction efficiency of 50%, designated as $\sqrt{\mathrm{St}_{50}}$, and referred to as the cutpoint. An ideal impactor would have a collection efficiency of 100% for sizes larger than the cutpoint and zero efficiency for smaller sizes; i.e. the efficiency curve would be a step function. A plot of $\sqrt{\mathrm{St}_{50}}$ vs the Reynolds number shows that $\sqrt{\mathrm{St}_{50}}$ varies with Re, but there is a midrange near Re = 500 where $\sqrt{\mathrm{St}_{50}}$ is essentially constant (Marple and Liu, 1974). Re is given by

$$\mathrm{Re} = \frac{\rho 2 W V_0}{\mu} \text{ (rectangular)} \quad \text{and} \quad \mathrm{Re} = \frac{\rho W V_0}{\mu} \text{ (round)}, \tag{3}$$

where ρ is the air density. The results of the numerical methods have been verified by extensive experimental measurements and are essential for the design of practical impactors. However, the basic physics of the impactor are obscured. Attempts to give simplified analytical derivations of the impactor cutpoint in text books usually involve assumptions as to the flow field, for example, that the streamlines are arcs of circles, and do not yield accurate results (Fuchs, 1964; Hinds, 1982). In this note, it is shown that it is possible to give a simple but accurate derivation of the cutpoint of an impactor with a rectangular slit, making only very general assumptions. In the case of a circular nozzle, the derivation is semiquantitative, but requires an additional assumption.

THEORY

Rectangular nozzle

Figure 1 shows an ideal impactor having a rectangular nozzle. The air flow inside the nozzle is uniform (plug flow) with velocity V_0 parallel to the axis of the nozzle. The particle concentration is uniform and the particle velocity is also V_0 and parallel to the axis.

By symmetry, it is only necessary to consider the flow on one side of the axis. At the exit of the nozzle, the streamline enclosing 50% (when both sides are considered) of the particles between it and the axis is at distance a from the axis, where

$$a = \frac{W}{4}. \tag{4}$$

After the jet is deflected by 90° at the impaction plate, the 50% streamline will be parallel to the impaction plate at distance a by the conservation of flux, assuming that the flow is incompressible and the air speed remains unchanged. A particle with no inertia would remain on the 50% streamline while a particle with inertia will be closer to the impaction plate. The condition for the cutpoint is that the particle just reaches the impaction plate, i.e. the vertical component of the particle velocity is zero. Thus the vertical stop distance is identified as equal to a. (A detailed justification is given in Appendix A.) It follows from equation (1) that

$$\mathrm{St} = \frac{2a}{W} = \frac{1}{2} \tag{5}$$

and

$$\sqrt{\mathrm{St}_{50}} = \sqrt{\tfrac{1}{2}} = 0.707. \tag{6}$$

For Re = 500, Marple and Liu (1974) obtained a value of 0.70 while Rader and Marple (1985) obtained 0.73 (from their "quadruple" calculation with the highest grid resolution). The present value is intermediate between the two and probably within the uncertainty of the numerical calculations. Besides questions involved with computer computations of fluid flow and particle transport, there are other factors which could influence the numerical results. The nozzle of the Marple impactor used in the calculations has a tapered inlet.

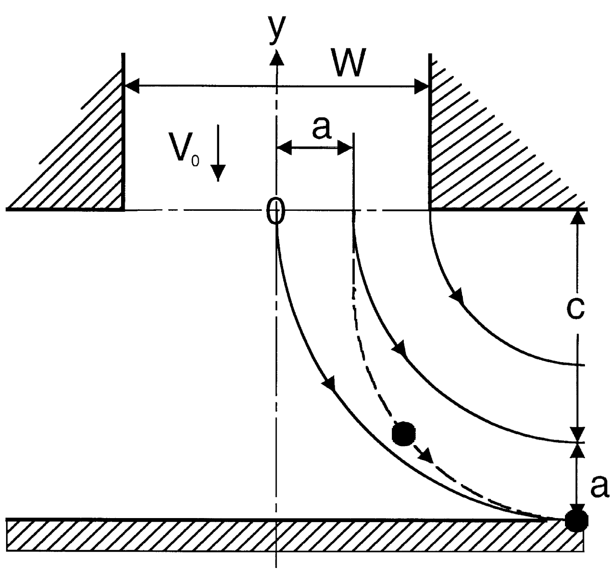

Fig. 1. Schematic diagram of the air flow and particle trajectory for the calculation of the cutpoint of an impactor with a rectangular nozzle.

Jurcik and Wang (1995) showed by numerical calculations that the presence of a taper at the inlet affects the shape of the cutoff curve; however, the effect on the 50% cutpoint was insignificant. Another difference from ideal in the numerical work is the flow at the edge of the nozzle where the velocity is zero. This evidently did not produce a significant change in the cutoff.

Circular nozzle

A complication arises for a circular nozzle because the jet spreads azimuthally at the impaction plate. Even without an inertial force, the particles would be transported closer to the impaction plate than their original distance from the axis; i.e. the vertical stop distance, L, is less than a. It is necessary to specify a value for the distance b, the radial distance from the axis where the particle originally on the 50% streamline deposits on the impaction plate. Measurements of impactor deposit profiles by Sethi and John (1993) show that for $\sqrt{\mathrm{St}_{50}} = 0.48$, the profile peaks near the nozzle edge. Therefore, the distance b will be taken to be $W/2$. From the 50% condition, it follows that

$$\pi a^2 = \frac{1}{2}\pi\left(\frac{W}{2}\right)^2, \tag{7}$$

$$a^2 = \frac{W^2}{8}. \tag{8}$$

In Fig. 2, the stop distance, L, is shown. Because the air flux is conserved, the wall area of the cylinder with radius b and height L must equal the cross section inside the 50% streamline at the nozzle exit:

$$2\pi b L = \pi a^2, \tag{9}$$

$$L = \frac{W}{8}, \tag{10}$$

$$\mathrm{St} = \frac{2L}{W} = \frac{1}{4}, \tag{11}$$

$$\sqrt{\mathrm{St}_{50}} = 0.50. \tag{12}$$

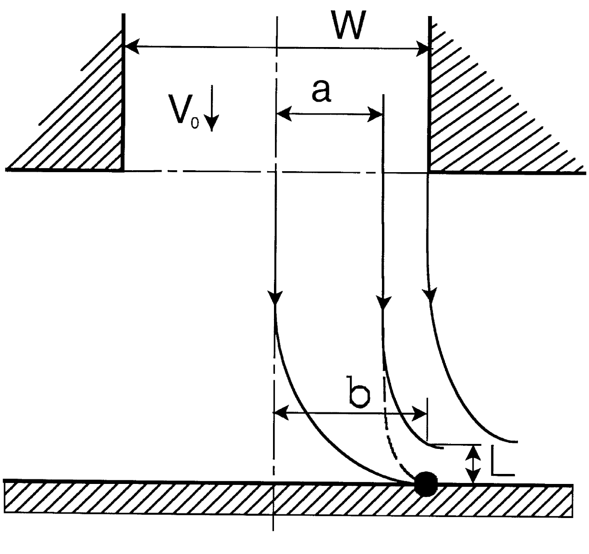

Fig. 2. Schematic diagram of the air flow and particle trajectory for the calculation of the cutpoint of an impactor with a circular nozzle.

The result is only 2% larger than the value of 0.49 obtained by Rader and Marple (1985). The result is moderately insensitive to the magnitude of b. For example, if b is decreased or increased by 25%, $\sqrt{\mathrm{St}_{50}}$ increases or decreases, respectively, by 12%.

CONCLUSIONS

For an ideal impactor having a rectangular nozzle, it is possible to give a simple but accurate derivation of the cutpoint. The result is in excellent agreement with numerical calculations. The same approach can be used for an impactor with a circular nozzle, but it is necessary to specify the radius of the impact circle. If the latter is taken to be the nozzle radius, a value for which there is experimental evidence, the calculated cutpoint is in good agreement with numerical computations.

In this theoretical approach, the basic physics is clear. The cutpoint is dependent only on the vertical stop distance and, hence, the details of the flow field are not needed. $\sqrt{\mathrm{St}_{50}}$ is smaller for the circular nozzle than for the rectangular nozzle because the spread of the jet on the impaction plate for the former reduces the vertical stop distance.

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APPENDIX

The stop distance can be calculated by integrating the vertical component of the equation of motion of the particle having the cutpoint size. In Fig. 1, the y co-ordinate is shown with the origin on the axis at the nozzle exit. The equation of motion can be written in dimensionless form (Marple and Liu, 1974):

$$\frac{\mathrm{St}}{2}\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = V_y - \frac{\mathrm{d}y}{\mathrm{d}t}, \tag{A1}$$

where the time $t = t'/W/V_0$, the fluid velocity $V_y = V_y'/V_0$ and $y = y'/W$, the primed values being actual values.

Equation (A1) is integrated over the total time, T for the particle to travel from the starting point at the nozzle exit at distance a from the axis to the impaction plate:

$$\frac{\mathrm{St}}{2}\int_0^{\mathrm{T}} \frac{\mathrm{d}^2 y}{\mathrm{d}t^2}\mathrm{d}t = \int_0^{\mathrm{T}} V_y \,\mathrm{d}t - \int_0^{\mathrm{T}} \frac{\mathrm{d}y}{\mathrm{d}t}\mathrm{d}t, \tag{A2}$$

$$\frac{\mathrm{St}}{2}\left[\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)_{\mathrm{T}} - \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)_0\right] = \frac{1}{W}[-c-(-c-a)], \tag{A3}$$

$$\frac{\mathrm{St}}{2}[0-(-1)] = \frac{a}{W}, \tag{A4}$$

$$\mathrm{St} = \frac{2a}{W}. \tag{A5}$$

From the definition of St (equation (1)), a is identified to be the stop distance. The derivation simplifies because the terms on the right-hand side of the equations are additive, and the integrated motion depends only on the endpoint values.

The cancellation of c from the equations is consistent with the numerical computations (Marple and Liu, 1974) which show that $\sqrt{\mathrm{St}_{50}}$ is independent of the nozzle-to-impaction plate distance if that distance is greater than W.