Practice Problems on Sigma Algebras

1. Practice Problems on sigma algebras are available at:
   - 2010, Final Exam, #4
   - 2009, Final Exam, #2
   - 2009, HW 10, #1
   - 2008, HW 10, #8
   - 2008, Final Exam, #1, 2
   - 2009, HW 10, #2 & 3 (Each of these questions describes a set of subsets of the real numbers. Identify whether the set of subsets is a sigma algebra.)

2. Some of the problems in the above list mention the concept of one sigma algebra containing another. Given below are definitions and examples related to this concept.
   a. Example: Let $X$ be a set and let $S$ be a nonempty set of subsets of $X$. For example, $X$ might equal $\mathbb{N}$, the set of natural numbers, and $S$ might consist of all sets of the form $\{n, n + 2, n + 8\}$ which includes sets such as $\{2,4,10\}$ and $\{50,52,58\}$.

   b. **Definition:** Let $X$ be a set and let $S$ be a set of subsets of $X$. Note that $S$ is not necessarily a sigma algebra. The sigma algebra $S'$ contains $S$ if every element of $S$ is an element of $S'$.

   c. Example: Let $X=\{1,2,3,4,5,6\}$, $S=\{\emptyset, X, \{1,2,3\}, \{4,5,6\}\}$
   and $S'=\{\emptyset, X, \{1,2,3\}, \{4,5,6\}, \{1\}, \{2,3,4,5,6\}, \{1,4,5,6\}, \{2,3\}\}$.
   In this example, The sigma algebra $S'$ contains the sigma algebra $S$.

4. **Definition:** Let $X$ be a set and let $S$ be a set of subsets of $X$. (Note that $S$ is not necessarily a sigma algebra.) The smallest sigma algebra that contains $S$ denoted $\sigma(S)$ has three properties:
   - $\sigma(S)$ is a sigma algebra.
   - $\sigma(S)$ contains $S$, and
   - any other sigma algebra that contains $S$ also contains $\sigma(S)$.

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1Keep in mind that the elements of a sigma algebra such as $S'$ are themselves sets. Given the notation used in this problem, the elements of $S'$ are subsets of $X$. 