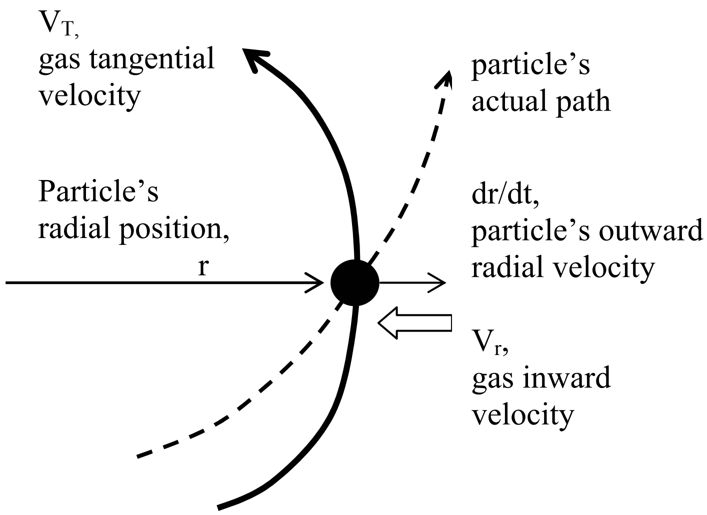
CYCLONES

Consider a force balance on a particle that moves in rotating flow as illustrated in the drawing below. The outward, centrifugal force on the particle, F_c, will be balanced by the inward, drag force on the particle, F_D, as it moves through the fluid. The relative velocity between the particle and the fluid that determines drag depends on both the particle's radial velocity outward, dr/dt, and on the inward velocity of the gas, V_r, as it moves to the cyclone axis.



Next, we will make some assumptions about the behavior of the particle and the fluid:

1. The particle moves with a tangential velocity that is the same as the gas tangential velocity at radial position "r"; that is, the particle does not "slip" tangentially,

2. The "positive" direction is radially outward.

Now we can construct a balance for the forces acting on the particle in the radial direction using Newton's second law, which says that the sum of these forces will equal mass times acceleration.

$$\sum F = m\ a \tag{1}$$

$$F_C - F_D = m\ a \tag{2}$$

$$\left(\frac{\pi d^3 \rho_p}{6}\right)\left(\frac{V_T^2}{r}\right) - \frac{3\pi\mu d}{C_C}\left(\frac{dr}{dt} + V_r\right) = \left(\frac{\pi d^3 \rho_p}{6}\right)\frac{d^2 r}{dt^2} \tag{3}$$

which, using the definition that $\tau = \frac{d^2 \rho_p C_C}{18 \mu}$ reduces to

$$\frac{d^2 r}{dt^2} + \frac{1}{\tau}\frac{dr}{dt} + \left(\frac{V_r}{\tau} - \frac{V_T^2}{r} \right) = 0 \quad . \tag{4}$$

This is a second order, non-linear differential equation. Note that gas tangential velocity, V_T, varies with radial position, r. If

$$V_T \; r^n \; = \text{constant} \; = \; V_{T,wall} \; r_{wall}^n \quad , \tag{5}$$

where the subscript "wall" corresponds to conditions at the wall of the cyclone as has been shown from experiments, then Eq (4) becomes

$$\frac{d^2 r}{dt^2} + \frac{1}{\tau}\frac{dr}{dt} + \left(\frac{V_r}{\tau} - \frac{V_{T,wall}^2 \; r_{wall}^{2n}}{r^{2n+1}} \right) = 0 \quad . \tag{6}$$

Collection Efficiency: Barth Solution

Equation (6) can be solved in different ways, depending on the assumptions made. Barth's solution, the "static particle approach", considers a particle for which the outward centrifugal force just balances the inward drag force. This particle is "static", because it moves neither outward nor inward. In this case,

$$\frac{d^2 r}{dt^2} = \frac{dr}{dt} = 0 \tag{7}$$

so that Eq (6) reduces to

$$d \; = \sqrt{\frac{18 \; V_r \; \; \mu \; r}{V_T^2 \; \; \rho_p \; \; C_C}} \tag{8}$$

where the values of V_r, r, and V_T are taken at the radial position of maximum tangential velocity. Iozia and colleagues used this approach along with measurements that determined the location of maximum tangential velocity to find d_c, the particle size collected with 50% efficiency.

$$d_c = \sqrt{\frac{9 \mu Q}{\pi \rho_p \; Z_c \; v_{t,max}{}^2}} \tag{9}$$

where

d_c is particle size collected with 50% efficiency,
μ is gas viscosity,
Q is gas flow
ρ_p is particle density,
Z_c is height of the control surface,
$v_{t,max}$ is the gas tangential velocity on the control surface

The height of the control surface depends on the radius of the control surface, r_c:

$$r_c = 0.52\left(\frac{D}{2}\right)\left(\frac{a\,b}{D^2}\right)^{-0.25}\left(\frac{De}{D}\right)^{1.53}, \qquad (10)$$

an empirical equation for which $r^2 = 0.889$

After r_c is known, the proper equation for Z_c can be used

$$Z_c = H - S \qquad \text{if } 2\,r_c < B \qquad (11a)$$

$$Z_c = (H-S) - \frac{(H-h)}{\left(D/B - 1\right)}\left(\frac{2\,r_c}{B} - 1\right) \qquad \text{if } 2\,r_c > B \qquad (11b)$$

The value for maximum tangential velocity on the control surface can be determined from:

$$v_{t,max} = 6.1\,v_{inlet}\left(\frac{a\,b}{D^2}\right)^{0.61}\left(\frac{De}{D}\right)^{-0.74}\left(\frac{H}{D}\right)^{-0.33}, \qquad (12)$$

an empirical equation for which $r^2 = 0.983$, where v_{inlet} is the gas velocity through the inlet to the cyclone,

$$v_{inlet} = \frac{Q}{a\,b} \quad . \qquad (13)$$

Once the particle size that is collected with 50% efficiency, d_c, has been calculated, the efficiency for that cyclone on particles of any size can then be determined from:

$$\eta = \frac{1}{1 + \left(\frac{d_c}{d}\right)^{\beta}} \qquad (14)$$

where the slope parameter, β, is given by:

$$\ln\beta = 0.62 - 0.87\ln d_c + 5.21\ln\left(\frac{a\,b}{D^2}\right) + 1.05\left[\ln\left(\frac{a\,b}{D^2}\right)\right]^2 \qquad (15)$$

for which $r^2 = 0.833$

In Eq. (15) above, d_c must be given in units of **centimeters**.

Pressure Drop in a Cyclone

Recall that:

$$\Delta P = \Delta H \left(\frac{1}{2} \rho_g \, v^2 \right) \left(\frac{1}{\rho_L \, g} \right) \tag{16}$$

where

ΔP is pressure drop in cm of water, or whatever fluid is used in the manometer

ΔH is the number of velocity pressures of pressure drop for the cyclone (see below)

ρ_g is gas density

v is gas inlet velocity

ρ_L is the density of the fluid in the manometer, and

g is the acceleration of gravity.

If no value for ΔH is available from experiments, then theory can be used to calculate it. The Dirgo approach is probably best.

$$\Delta H = 20 \frac{a\, b}{D_e^{\,2}} \left[\frac{S/D}{\left(H/D\right)\left(h/D\right)\left(B/D\right)} \right]^{1/3} \tag{17}$$

References:

Iozia, Donna Lee and David Leith, "Effect of Cyclone Dimensions on Gas Flow Pattern and Collection Efficiency", *Aerosol Science and Technology*, **10**: 491 (1989).

Iozia, Donna Lee and David Leith, "The Logistic Function and Cyclone Fractional Efficiency", *Aerosol Science and Technology*, **12**: 598-606 (1990).

Ramachandran, G., David Leith, John Dirgo, and Henry Feldman, "Cyclone Optimization Based on a New Empirical Model for Pressure Drop", *Aerosol Science and Technology*, **15**: 135-148 (1991).