Cyclone Optimization Based on a New Empirical Model for Pressure Drop

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An empirical model for predicting pressure drop across a cyclone, developed by Dirgo (1988), is presented. The model was developed through a statistical analysis of pressure drop data for 98 cyclone designs. The model is shown to perform better than the pressure drop models of Shepherd and Lapple (1940), Alexander (1949), First (1949), Stairmand (1949), and Barth (1956). This model is used with the efficiency model of Iozia and Leith (1990) to develop an optimization curve which predicts the minimum pressure drop and the dimension ratios of the optimized cyclone for a given aerodynamic cut diameter, d_{50}. The effect of variation in cyclone height, cyclone diameter, and flow on the optimization is determined. The optimization results are used to develop a design procedure for optimized cyclones.

INTRODUCTION

Pressure drop and collection efficiency are the two major criteria used to evaluate cyclone performance. Both criteria are functions of cyclone dimensions: inlet height, a, inlet width, b, gas outlet diameter, D_e, outlet duct length, S, cylinder height, h, cyclone height, H, and dust outlet diameter, B; see Figure 1. The goal of cyclone design is to obtain the greatest efficiency for a given operating cost (pressure drop) by adjusting these dimensions.

Any design method based on theory depends on the accurate prediction of efficiency and pressure drop. Dirgo and Leith (1985) developed an optimization program to investigate design changes that would improve performance. Owing to the poor prediction by the efficiency and pressure drop theories used (Stairmand, 1949; Barth, 1956; Leith and Licht, 1972), substantial improvement in performance was not shown by pilot scale cyclones designed according to the program. Thus, efficiency and pressure drop theories with better predictive capabilities are needed.

Subsequently, Iozia and Leith (1990) developed an improved method to predict cyclone aerodynamic cut diameter, d_{50}. Aerodynamic cut diameter is the particle size collected with 50% efficiency. Iozia and Leith claimed that their method works better than the theories of Lapple (1950), Leith and Licht (1972), and Dietz (1981). The equations necessary to use this model are in the Appendix.

Pressure drop models currently used in-

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0278-6826/91/$3.50

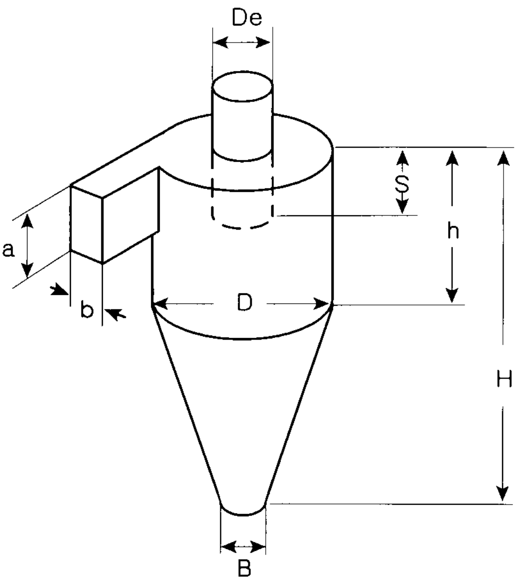

FIGURE 1. Reverse flow cyclone.

clude those by Shepherd and Lapple (1940), Alexander (1949), First (1949), Stairmand (1949), and Barth (1956). None of these predicts pressure drop accurately for a wide range of cyclone designs; predictions can differ from measured values by more than a factor of 2 (Dirgo, 1988). Further, evaluation of cyclone pressure drop models by different investigators has produced conflicting conclusions as to which models work best. Stern et al. (1955) found that the model of Shepherd and Lapple provided the best fit to experimental data, although for some cyclones the models of Alexander and First worked equally well. Leith and Mehta (1973) found that the models of Barth, Stairmand, and Shepherd and Lapple predicted results most accurately. They recommended the model of Shepherd and Lapple because of its simplicity. In a later review based on a larger data set, Leith et al. (1986) found that the models of Stairmand and Shepherd and Lapple were the most accurate. Caplan (1977) recommended the model of Alexander. This paper presents a new empirical model for predicting pressure drop which was developed by Dirgo (1988). The model was developed through statistical analysis of pressure drop data for 98 cyclone designs, a substantially larger data set for evaluation of pressure drop models than has been used previously.

The present study uses the efficiency model of Iozia and Leith (1990) and the pressure drop model developed by Dirgo (1988) to determine design changes that should optimize cyclone performance. Our goal was to design cyclones that provide the greatest efficiency (minimum d_{50}) for a given pressure drop.

PRESSURE DROP MODEL OF DIRGO (1988)

Pressure drop, ΔP, will be expressed as the number of gas inlet velocity heads, ΔH. This dimensionless term is related to pressure drop in height of a column of a liquid, ΔP, by

$$\Delta P = \Delta H(v_i^2 \rho_G)/(2\rho_L g), \quad (1)$$

where ΔP is cyclone pressure drop (meters of liquid), v_i is gas inlet velocity (meters per second), ρ_G is gas density (kilograms per cubic meter), ρ_L is liquid density (kilograms per cubic meter), and g is the acceleration of gravity (m/s^2) ΔH is a function of the cyclone dimension ratios only and is not affected by operating conditions such as gas flow. ΔH should remain constant for any cyclone configuration, regardless of size, as long as the relative proportions of the dimensions stay the same.

A literature review was conducted to identify data that could be used to evaluate existing cyclone pressure drop models and to develop a better model. Decisions to include specific studies for this review were based on four criteria. Data were included only if all four criteria were met:

1. ΔH was presented or could be calculated from reported values of cyclone pressure drop, ΔP, and operating conditions.
2. All cyclone dimensions were presented

or could be determined from drawings of known scale. For nine cyclones, all dimension ratios except the dust outlet diameter, B, could be determined by these methods. A value of $0.25D$ was assigned to B and these cyclones were included in this study.

3. Sufficient information was presented so that ΔH could be calculated from the pressure drop models of Shepherd and Lapple (1940), Alexander (1949), First (1949), Stairmand (1949), and Barth (1956).
4. The cyclone was similar in configuration to the reverse flow cyclone in Figure 1. Cyclones that were significantly different were excluded; e.g., cyclones with scroll-type inlets.

Data were found for 98 cyclones that met these criteria. The seven dimension ratios, the reported ΔH for each cyclone, and the literature sources from which the data were obtained are in Table 1.

A correlation matrix was developed for these data to investigate the relationship between cyclone dimension ratios and ΔH. The matrix included three dimension ratios in addition to the seven basic dimension ratios:

1. $(H-h)/D$, the ratio of cyclone cone length to cyclone diameter,
2. $(H-S)/D$, the ratio of cyclone core length to diameter; the core is defined as the distance from the bottom of the gas outlet duct to the dust outlet,
3. (ab/D_e^2), the ratio of the gas inlet area to the square of the gas outlet duct diameter; this term is proportional to the ratio of gas inlet to outlet area.

Table 2 is the correlation matrix for these dimension ratios and ΔH. SAS (1982) programs produced the correlation matrix and all subsequent statistical analyses. The correlation matrix shows that cyclone inlet (a/D, b/D) and outlet (D_e/D) dimension ratios are most highly correlated with ΔH. When these ratios are combined as (ab/D_e^2), the correlation coefficient with ΔH is 0.976. The inlet and outlet dimension ratios are not strongly correlated with other cyclone dimension ratios, and none of the other dimension ratios is strongly correlated with ΔH. However, strong correlations exist among these other ratios, particularly for ratios that include cyclone height, H.

Stepwise and backward regression was used to suggest possible models for ΔH based on cyclone dimension ratios (Draper and Smith, 1966). Both regressions used natural logarithms of ΔH and the dimension ratios to produce models of the form

$$\Delta H = KA^X B^Y C^Z, \tag{2}$$

where K is a constant, and A, B, and C are cyclone dimension ratios.

Two sets of stepwise and backward regressions were run; each set investigated three possible models. The first set included the cyclone gas inlet and outlet dimension ratios, a/D, b/D, and D_e/D, independently. Each of the three models also included cylinder height, h/D, outlet duct length, S/D, and dust outlet diameter, B/D. The first model included cyclone height, H/D, the second included cone height, $(H-h)/D$, and the third core height $(H-S)/D$. These last three dimension ratios were not included in the same model because they were highly correlated and as general measures of cyclone height, they are likely to explain the same variability in ΔH. The second set of three models was identical to the first set, except that cyclone gas inlet and outlet dimension ratios were grouped as (ab/D_e^2).

The stepwise and backward regression procedures produced identical results for all potential models investigated. In each case, the stepwise procedure included all dimension ratios in the model [seven dimension ratios for models using a, b, and D_e; five dimension ratios for models using (ab/D_e^2).] The backward procedure left all dimension ratios in the model. After review-

TABLE 1. Reported Values for Cyclone Dimension Ratios and Pressure Drop, ΔH

Source[a]	D_c	a	b	S	H	h	B	ΔH
A	0.500	0.533	0.233	1.600	4.267	2.133	0.267	7.2
A	0.500	0.533	0.156	1.600	4.267	2.133	0.267	4.7
A	0.500	0.533	0.111	1.600	4.267	2.133	0.267	3.7
B	0.313	0.500	0.148	0.500	2.375	1.625	0.313	11.3
B	0.500	0.500	0.148	0.500	2.375	1.625	0.313	5.1
B	0.313	1.000	0.250	0.500	1.688	1.125	0.313	40.2
B	0.500	1.000	0.250	0.500	1.688	1.125	0.313	14.5
B	0.500	0.250	0.125	1.500	4.000	2.000	0.250[b]	3.5
B	0.500	0.500	0.250	1.500	4.000	2.000	0.250[b]	5.7
B	0.500	0.600	0.250	1.500	4.000	2.000	0.250[b]	10.7
B	0.500	0.500	0.250	1.500	4.000	2.000	0.250[b]	6.5
B	0.333	0.667	0.133	0.400	1.967	1.133	0.250[b]	11.5
B	0.250	0.367	0.117	0.733	2.200	0.950	0.250[b]	12.7
C	0.500	0.283	0.150	0.600	1.450	0.700	0.200	4.9
C	0.500	0.288	0.151	0.613	1.463	0.700	0.200	5.0
C	0.500	0.293	0.150	0.600	1.475	0.700	0.200	5.0
C	0.500	0.283	0.067	0.600	1.450	0.700	0.200	2.5
C	0.500	0.142	0.150	0.600	1.450	0.700	0.200	2.8
C	0.667	0.283	0.150	0.600	1.450	0.700	0.200	3.1
C	0.333	0.283	0.067	0.600	1.450	0.700	0.200	6.0
C	0.500	0.283	0.150	0.600	1.158	0.700	0.200	5.5
D	0.500	0.283	0.094	0.600	1.158	0.700	0.200	3.8
D	0.667	0.283	0.094	0.600	1.158	0.700	0.200	2.3
D	0.333	0.283	0.150	0.600	1.158	0.700	0.200	11.8
D	0.667	0.283	0.150	0.600	1.158	0.700	0.200	3.3
D	0.333	0.283	0.150	0.600	1.450	0.700	0.200	10.5
D	0.500	0.113	0.150	0.600	1.450	0.700	0.200	2.3
D	0.500	0.208	0.150	0.600	1.450	0.700	0.200	3.9
D	0.500	0.283	0.094	0.600	1.450	0.700	0.200	3.5
D	0.333	0.283	0.067	0.400	1.450	0.700	0.200	6.2
D	0.417	0.292	0.208	0.556	2.056	0.667	0.140	9.5
D	0.476	0.393	0.119	0.667	1.607	0.655	0.141	5.3
E	0.500	0.500	0.200	0.500	4.000	1.500	0.375	5.3
F	0.500	0.620	0.230	1.170	3.180	1.330	0.250[b]	3.7
F	0.573	0.667	0.333	0.893	3.280	1.307	0.250[b]	6.2
F	0.564	0.609	0.318	0.909	2.727	1.364	0.250[b]	7.3
G	0.513	0.561	0.211	0.763	2.666	0.561	0.531	2.3
G	0.434	0.526	0.156	0.632	3.579	0.632	0.316	4.7
G	0.435	0.538	0.162	0.673	3.373	0.681	0.404	4.6
G	0.400	0.527	0.149	0.636	2.909	0.636	0.345	4.9
H	0.405	0.486	0.268	0.568	2.335	0.649	0.405	12.0
H	0.300	0.267	0.267	0.390	2.486	0.501	0.300	25.0
I	0.500	0.900	0.100	0.967	2.217	1.035	0.500	9.6
I	0.500	0.900	0.100	0.967	3.467	1.035	0.500	8.5
I	0.500	0.900	0.100	0.967	5.967	1.035	0.500	6.9
I	0.500	0.900	0.100	0.967	10.970	1.035	0.500	5.2
I	0.500	0.900	0.200	0.967	2.217	1.035	0.500	14.4
I	0.500	0.900	0.200	0.967	3.467	1.035	0.500	13.2
I	0.500	0.900	0.200	0.967	5.967	1.035	0.500	11.4
I	0.500	0.900	0.200	0.967	10.970	1.035	0.500	9.1
I	0.500	0.900	0.300	0.967	2.217	1.035	0.500	19.3
I	0.500	0.900	0.300	0.967	3.467	1.035	0.500	17.5
I	0.500	0.900	0.300	0.967	5.967	1.035	0.500	15.5
I	0.500	0.900	0.300	0.967	10.970	1.035	0.500	13.1

TABLE 1. *Continued*

Source[a]	D_e	a	b	S	H	h	B	ΔH
I	0.500	0.900	0.400	0.967	2.217	1.035	0.500	25.2
I	0.500	0.900	0.400	0.967	3.467	1.035	0.500	21.9
I	0.500	0.900	0.400	0.967	5.967	1.035	0.500	18.9
I	0.500	0.900	0.400	0.967	10.970	1.035	0.500	16.6
I	0.333	0.900	0.100	0.967	1.801	1.035	0.333	24.1
I	0.333	0.900	0.100	0.967	2.634	1.035	0.333	19.0
I	0.333	0.900	0.100	0.967	4.301	1.035	0.333	16.0
I	0.333	0.900	0.200	0.967	1.801	1.035	0.333	40.0
I	0.333	0.900	0.200	0.967	2.634	1.035	0.333	34.4
I	0.333	0.900	0.200	0.967	4.301	1.035	0.333	29.7
I	0.333	0.900	0.300	0.967	1.801	1.035	0.333	56.7
I	0.333	0.900	0.300	0.967	2.634	1.035	0.333	49.9
I	0.333	0.900	0.300	0.967	4.301	1.035	0.333	43.5
I	0.333	0.900	0.400	0.967	1.801	1.035	0.333	73.5
I	0.333	0.900	0.400	0.967	2.634	1.035	0.333	65.3
I	0.333	0.900	0.400	0.967	4.301	1.035	0.333	58.4
I	0.250	0.900	0.100	0.967	1.592	1.035	0.250	49.1
I	0.250	0.900	0.100	0.967	2.217	1.035	0.250	41.4
I	0.250	0.900	0.100	0.967	3.467	1.035	0.250	33.9
I	0.250	0.900	0.200	0.967	1.592	1.035	0.250	80.8
I	0.250	0.900	0.200	0.967	2.217	1.035	0.250	77.2
I	0.250	0.900	0.200	0.967	3.467	1.035	0.250	66.9
I	0.250	0.900	0.300	0.967	1.592	1.035	0.250	121.4
I	0.250	0.900	0.300	0.967	2.217	1.035	0.250	110.6
I	0.250	0.900	0.300	0.967	3.467	1.035	0.250	98.0
I	0.250	0.900	0.400	0.967	1.592	1.035	0.250	155.3
I	0.250	0.900	0.400	0.967	2.217	1.035	0.250	148.3
I	0.250	0.900	0.400	0.967	3.467	1.035	0.250	136.8
J	0.433	0.555	0.162	0.543	3.263	0.684	0.384	6.4
J	0.431	0.553	0.161	0.552	3.245	0.681	0.383	7.3
J	0.432	0.553	0.161	0.561	3.255	0.682	0.382	7.8
K	0.400	0.440	0.210	0.500	3.900	1.400	0.400	9.2
K	0.500	0.500	0.250	0.600	3.750	1.750	0.400	7.6
L	0.541	0.557	0.331	0.962	5.939	3.350	0.287	8.8
M	0.575	0.575	0.230	0.584	3.510	0.750	0.480	10.0
M	0.575	0.573	0.223	0.580	3.460	0.750	0.477	7.0
N	0.514	0.372	0.186	0.541	2.095	0.743	0.253	2.8
O	0.407	0.494	0.247	0.740	3.961	2.662	0.586	2.3
P	0.313	0.375	0.188	1.125	4.313	1.813	0.375	9.1
Q	0.500	0.500	0.200	0.500	4.000	1.500	0.375	5.8
Q	0.333	0.500	0.300	0.558	6.000	3.500	0.375	11.2
Q	0.583	0.375	0.200	3.052	6.000	3.500	1.000	2.9
Q	0.583	0.375	0.200	2.865	6.000	3.500	0.688	2.9
Q	0.333	0.500	0.300	2.073	6.000	3.500	1.000	12.2

[a]Sources are abbreviated according to the code below:

A Shepherd and Lapple (1940)
B Stairmand (1949)
C First (1949)
D First (1950)
E Stairmand (1951)
F Stern et al. (1955)
G Petroll and Langhammer (1962)
H van Ebbenhorst Tengbergen (1965)
I Leineweber (1967)
J Petroll et al. (1967)
K Swift (1969)
L Hejma (1971)
M Kalmykov et al. (1976)
N Yuu et al. (1978)
O Ernst et al. (1980)
P Parker et al. (1981)
Q Dirgo and Leith (1985)

[b]Estimated value (see text).

TABLE 2. Correlation Matrix for Cyclone Dimension Ratios and ΔH

	D_c/D	a/D	b/D	S/D	H/D	h/D	B/D	$(H-h)/D$	$(H-S)/D$	(ab/D_c^2)	ΔH
D_c/D	—										
a/D	−0.378	—									
b/D	−0.121	0.442	—								
S/D	0.092	0.148	0.171	—							
H/D	0.190	0.289	0.240	0.378	—						
h/D	0.096	−0.052	0.195	0.685	0.393	—					
B/D	0.199	0.271	0.244	0.528	0.557	0.473	—				
$(H-h)/D$	0.172	0.331	0.192	0.171	0.947	0.078	0.439	—			
$(H-S)/D$	0.181	0.274	0.217	0.184	0.980	0.269	0.476	0.969	—		
ab/D_c^2	−0.661	0.607	0.632	0.085	−0.051	−0.046	−0.073	−0.040	−0.073	—	
ΔH	−0.668	0.540	0.511	0.063	−0.156	−0.113	−0.165	−0.129	−0.179	0.976	—

ing these models, the following model was selected for further evaluation:

$$\Delta H = 19.7\,(ab/D_c^2)^{0.99}(S/D)^{0.35} \times (H/D)^{-0.34}(h/D)^{-0.35} \times (B/D)^{-0.33}. \qquad (3)$$

The first three exponents in Eq. (3) are statistically significant at the $p < 0.003$ level based on partial F-test values; the exponent for B/D is significant at the $p = 0.006$ level. The coefficient of determination, R^2, for this model, which uses natural logarithms of ΔH and the dimension ratios, is 0.917. A major consideration in selecting Eq. (3) is that this model can be expressed in simpler form. Rounding off the constant and exponents, and combining secondary dimension ratios produces

$$\Delta H = 20\{ab/D_c^2\}\{[S/D]/[(H/D)(h/D)(B/D)]\}^{1/3}. \qquad (4)$$

Partial F-tests show that Eqs. (3) and (4) are not significantly different. Equation (4) was used to predict ΔH values for the 98 cyclones in the data set. Figure 2 plots observed ΔH vs. the values predicted by Eq. (4).

Comparison of Equation (4) with Other Pressure Drop Models

For each of the cyclones in the data set, the pressure drop values predicted by the models of Shepherd and Lapple (1940), Alexander (1949), First (1949), Stairmand (1949), and Barth (1956) and also by Eq. (4) were calculated. For each model, a geometric mean difference between the observed and

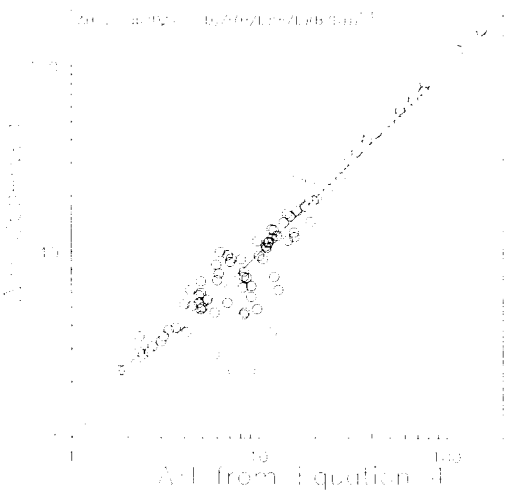

FIGURE 2. Observed ΔH vs. predicted ΔH for Eq. (4). ΔH is the number of gas inlet velocity heads and is related to pressure drop, by $\Delta P = \Delta H(v_i^2\rho_G)/(2\rho_L g)$.

predicted ΔH values was calculated by

$$\ln d_g = \sum_{i=1}^{n} (\ln \Delta H_{obs} - \ln \Delta H_{pred})/n. \quad (5)$$

The geometric mean difference can be used to determine whether, on average, a model overestimates or underestimates ΔH. A least squares performance index, I, was also calculated for each model:

$$I = \sum_{i=1}^{n} (\ln \Delta H_{obs} - \ln \Delta H_{pred})^2/n. \quad (6)$$

I is derived from the quantity minimized when a regression line is fitted to data by the method of least squares. The model with minimum value of I best fits the data.

Results of these analyses presented in Table 3 show that Eq. (4) performed better than the other five models; d_g was closest to 1 and I was lowest. These results were expected because Eq. (4) was developed from a least squares regression for the same ΔH values used for the inter-model comparison. Table 3 shows that Eq. (4) and the models of First and Barth tended to predict ΔH values higher than those observed; d_g was < 1. The models of Shepherd and Lapple, Stairmand, and Alexander tended to underestimate ΔH. The models of Alexander and Shepherd and Lapple were the poorest predictors of ΔH based on values of I.

The accuracy of cyclone pressure drop models was examined in a second way. Table 3 shows the fraction of predictions for each model within 10%, 20% and 30% of the observed ΔH. The results in Table 3 generally agree with the evaluation of models based on I; Eq. (4) performs best. For Eq. (4), over 40% of the predictions were within 10% of the observed ΔH; nearly 70% were within 20% of ΔH; over 80% were within 30% of ΔH. Table 3 also shows that the First model was more accurate than the Barth model for most cyclones. Since the First model is also much simpler, it should be preferred over the Barth model. These comparisons show that the number of cyclone dimension ratios included in a model is an important factor in predicting ΔH. The two models that performed much worse than the others, those of Alexander and Shepherd and Lapple, base their predictions on only the gas inlet and outlet dimension ratios, a/D, b/D, and D_e/D. The four models that predict ΔH much better include additional dimension ratios.

Although Eq. (4) is more accurate than existing pressure drop models, Figure 2 shows that this equation cannot predict ΔH accurately for all cyclone designs. Two factors may have contributed to differences between predicted and observed ΔH values. First, most literature sources did not report how pressure drop was measured. Some measurements may not have been corrected for differences in velocity pressure at the gas inlet and outlet. This would have affected reported ΔH values, particularly for cyclones with large differences in inlet and outlet cross-sectional areas. Second, most literature sources did not provide data on dust loading. Cyclone pressure drop de-

TABLE 3. Performance Parameters for Cyclone Pressure Drop Models

Model	Geometric mean difference, d_g	Least squares performance index, I	Fraction of predictions		
			Within 10%	Within 20%	Within 30%
Shepherd and Lapple (1940)	1.165	0.229	0.17	0.34	0.46
Alexander (1949)	1.386	0.321	0.15	0.28	0.38
First (1949)	0.922	0.129	0.39	0.64	0.80
Stairmand (1949)	1.180	0.149	0.26	0.39	0.56
Barth (1956)	0.957	0.121	0.15	0.53	0.71
Equation (4)	0.977	0.102	0.42	0.67	0.83

creases as dust concentrations increase (Stern et al., 1955; Tengbergen, 1965; Yuu et al., 1978); however, cyclone pressure drop models do not consider this effect. Model predictions could have overestimated ΔH for dusty gas streams.

Cross-Validation

Testing Eq. (4) on the data from which it was developed will overestimate its performance. Cross-validation is one method of evaluating a model's predictive ability for new data (Mosteller and Tukey, 1977). This statistical procedure also helps to evaluate the form of a model, in this case, the dimension ratios used to predict ΔH, and the numerical values of the model's coefficients; i.e., the exponents for the dimension ratios. A simple cross-validation was carried out on the data set in Table 1. The data were randomly divided in half. The modeling procedure described above was repeated for half the data. The model was then tested on the other half.

The model that resulted from this analysis was very similar to the model developed from the full data set. All terms in the cross-validation model were also in Eq. (4); however, the values for the constants and the exponents were slightly different. The major difference between the cross-validation model and the full data set model was that the dust outlet diameter, B, was not included in the cross-validation model.

When tested on the second half of the data set, the cross-validation model performed better than three existing pressure drop models and nearly as well as the other two. The least squares performance index for the cross-validation model, when tested on the second half of the data set, was 0.162, somewhat worse (higher) than the indices for the Barth model ($I = 0.149$) and the Stairmand model ($I = 0.160$), but considerably better (lower) than the index values for the models of Shepherd and Lapple, Alexander, and First. When the entire data set was considered, the cross-validation model performed better than any of the existing models. Full details of the cross-validation procedure are given in Dirgo (1988). Thus, Eq. (4), which is very similar to the cross-validation model, might also be expected to predict ΔH accurately for new cyclones.

OPTIMIZATION

Our goal was to predict dimension changes that would improve cyclone performance compared to a baseline defined by the Stairmand high efficiency cyclone ($D = 0.254$ m, $Q = 0.094$ m^3/s). To do this, we used Eqs. (1) and (4) for pressure drop and the efficiency model of Iozia and Leith (Appendix). Since the equation for efficiency, Eq. (A6), shows that efficiency can be maximized by minimizing d_{50}, the optimization procedure minimized d_{50} for a given pressure drop. The procedure (Dirgo and Leith, 1985; Iozia and Leith, 1989) used in this study is a variation of the single factor method (Cochran and Cox, 1957). Two cyclone dimensions were varied simultaneously. Gas outlet diameter, D_e, selected as the primary dimension, was varied first. This changed the pressure drop. Three dimensions, inlet height, a, inlet width, b, and gas outlet duct length, S, were then varied, one at a time, to bring pressure drop back to the original value. Pressure drop was held constant because improvements in performance are difficult to evaluate if pressure drop changes. The d_{50} for each new design was then predicted from theory. The new design with lowest d_{50} became the baseline for the next iteration. In the next iteration, the gas outlet diameter, D_e, was again varied in a pairwise fashion with each of the three other dimensions, a, b, and S, to find the second dimension change that most reduced d_{50} while keeping pressure drop constant. Iterations were continued until the predicted reduction in d_{50} from iteration to iteration was less than one nanometer.

The other cyclone dimensions, dust outlet diameter, B, cylinder height, h, and cyclone height, H, were subject to constraints. The optimization program would increase dust outlet diameter, B, until it equaled the cyclone diameter. However, in this work, B was set equal to 0.375 D to bring the collected dust to a central point. The cylinder height, h, was set equal to 1.5 D so that cyclone height varied by changing cone height, $(H - h)$.

Optimization was done for 40 pressure drops from 0.1 to 4.0 kPa in steps of 0.1 kPa. Thus, for operation over this ΔP range, we determined cyclone designs that would give the lowest d_{50} possible according to the models used for efficiency and pressure drop. The plots of ΔP vs. d_{50} and cyclone dimension ratios vs. d_{50} are the "optimization curves." This was done for a cyclone diameter of $D = 0.254$ m, a flow of $Q = 0.094$ m^3/s and cyclone height of $H = 5D$.

Next we studied the effect of changing cyclone height, H, flow, Q, and cyclone diameter, D, on the optimization curves. The effect of H was studied by plotting optimization curves for $H = 4D$, $5D$, and $6D$. The effect of Q was studied by plotting optimization curves at three flows: the baseline value of 0.094 m^3/s and at $\pm 25\%$ of the baseline value. The effect of D was determined by plotting optimization curves at the baseline value of 0.254 m and at $\pm 25\%$ of the baseline value.

RESULTS AND DISCUSSION

The straight line in Figure 3 shows the relationship between predicted pressure drop and predicted aerodynamic d_{50} for optimized designs at baseline conditions of $D = 0.254$ m, $Q = 0.094$ m^3/s, and $H = 5D$. Pressure drop was predicted using Eqs. (1) and (4) and aerodynamic d_{50} was predicted by the Iozia-Leith theory (Appendix). It indicates the predicted optimum performance for cyclones. The area above the line indicates current practice. To show this, predicted performance for selected cyclone designs from the experimental work of Lapple (1950), Peterson and Whitby (1965), Dirgo and Leith (1985), and Iozia and Leith (1990) are shown. The relationship shows a tradeoff between efficiency and operating cost. Greater efficiency (lower d_{50}) requires greater operating cost (pressure drop).

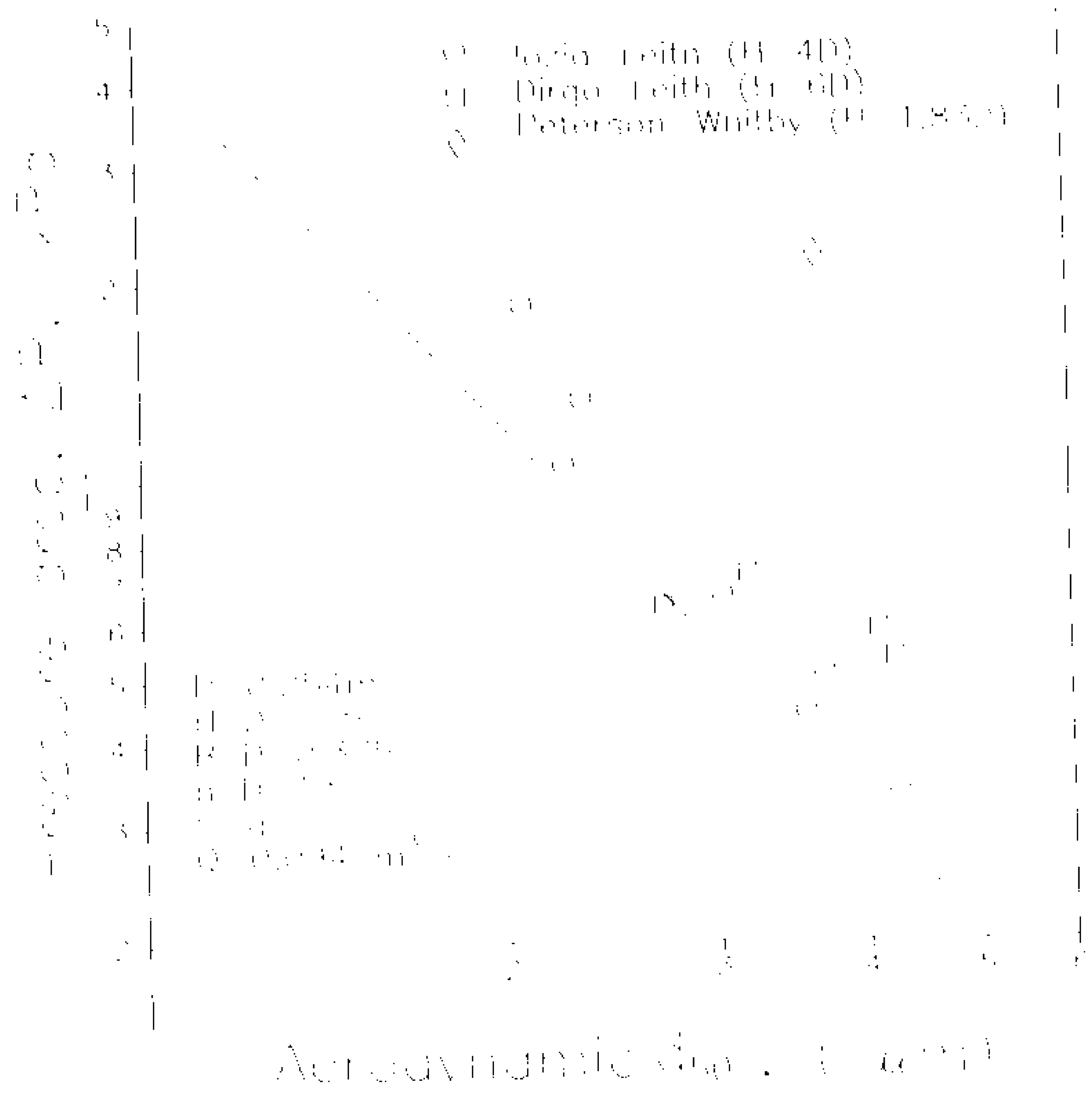

FIGURE 3. Optimum pressure drop, ΔP, vs. d_{50} for $H = 5D$ with experimental data.

Figure 4 shows how inlet height, a, inlet width, b, and outlet diameter, D_e, varied with aerodynamic d_{50}. A program constraint was that $b < (D - D_e)/2$, to prevent "crowding" of the entering gas. At $b \geq 0.2$ D, this limit was achieved, for $D_e = 0.6$ D, resulting in a flat curve for b at $d_{50} \geq 3$ μm. In this region, inlet height was increased to achieve higher d_{50} cuts. Inlet width and outlet diameter were relatively constant regardless of d_{50}, and so were largely independent of pressure drop. Gas outlet duct length, S, was always found equal to inlet height.

For optimized designs at all pressure drops, the outlet area, $\pi D_e^2/4$, was larger than inlet area, ab. Thus the inlet velocity head is greater than the outlet velocity head.

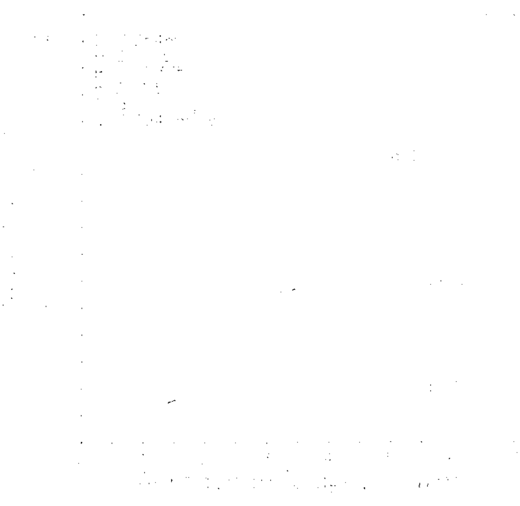

FIGURE 4. Optimum dimension ratios vs. d_{50} for $H = 5D$.

This leads to a lower total pressure drop across the cyclone than if the inlet and outlet areas were equal.

Figure 5 shows pressure drop vs. aerodynamic d_{50} for $H = 4D$, $5D$, and $6D$. Q and D are kept at baseline values. For any pressure drop, increases in cyclone height, H, decrease d_{50}. An explanation for this can be obtained from the "static particle" theory. According to this theory, the particle with diameter d_{50}, called the critical particle, remains suspended within the cyclone owing to the balance of centrifugal and radial drag forces. The radial drag force depends on the average radial velocity of gas, V_r, flowing inward past the critical particle. V_r is given by

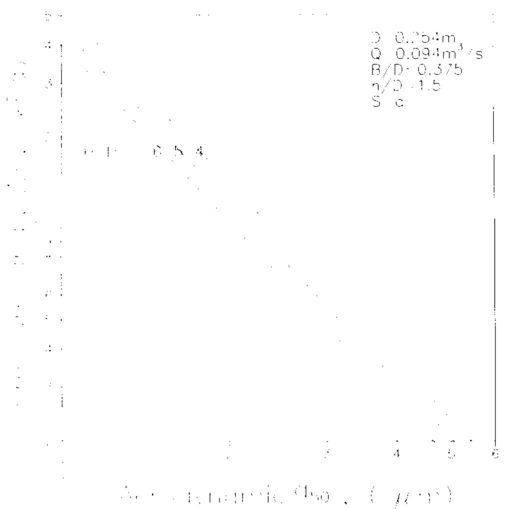

FIGURE 5. Effect of variation in H on optimum pressure drop.

$$V_r = Q/(2\pi r_c z_c), \tag{7}$$

where Q is the gas flow, r_c is the core radius, and z_c is the core length (Barth, 1956). Core length, z_c, is directly proportional to cyclone height, H. An increase in H will increase z_c, thereby reducing V_r and hence the drag force. Thus, as H increases, a smaller d_{50} particle can be balanced on the edge of the core.

Another way to view Figure 5 is that for any d_{50}, increases in cyclone height reduce pressure drop. First (1949) suggested that a shorter cone causes the gas outlet opening to function as a valve. Gas is diverted from the downward flowing outer vortex to the upward flowing inner vortex at a faster rate than it can enter the exit duct. This causes a secondary downward flow within the inner vortex, increasing pressure drop. A longer cyclone would reduce this effect.

Figure 6 shows how inlet height, a, inlet width, b, and outlet diameter, D_e, varied with aerodynamic d_{50} for cyclones of each height. This figure shows that D_e and b do not change with changes in H but inlet height, a, increases with H. This is consistent with the observation that, for the optimizations presented here, b and D_e are independent of pressure drop and hence are also independent of H.

Figure 7 shows pressure drop vs. aerodynamic d_{50} for different flows through the cyclone. H and D are kept at baseline conditions. It illustrates a trade-off between flow and d_{50}. For a particular operating cost (pressure drop) as flow increases, efficiency decreases. Figure 8 shows that an increase in flow is accompanied by an increase in inlet height, a. Inlet width and outlet diameter do not change significantly with flow.

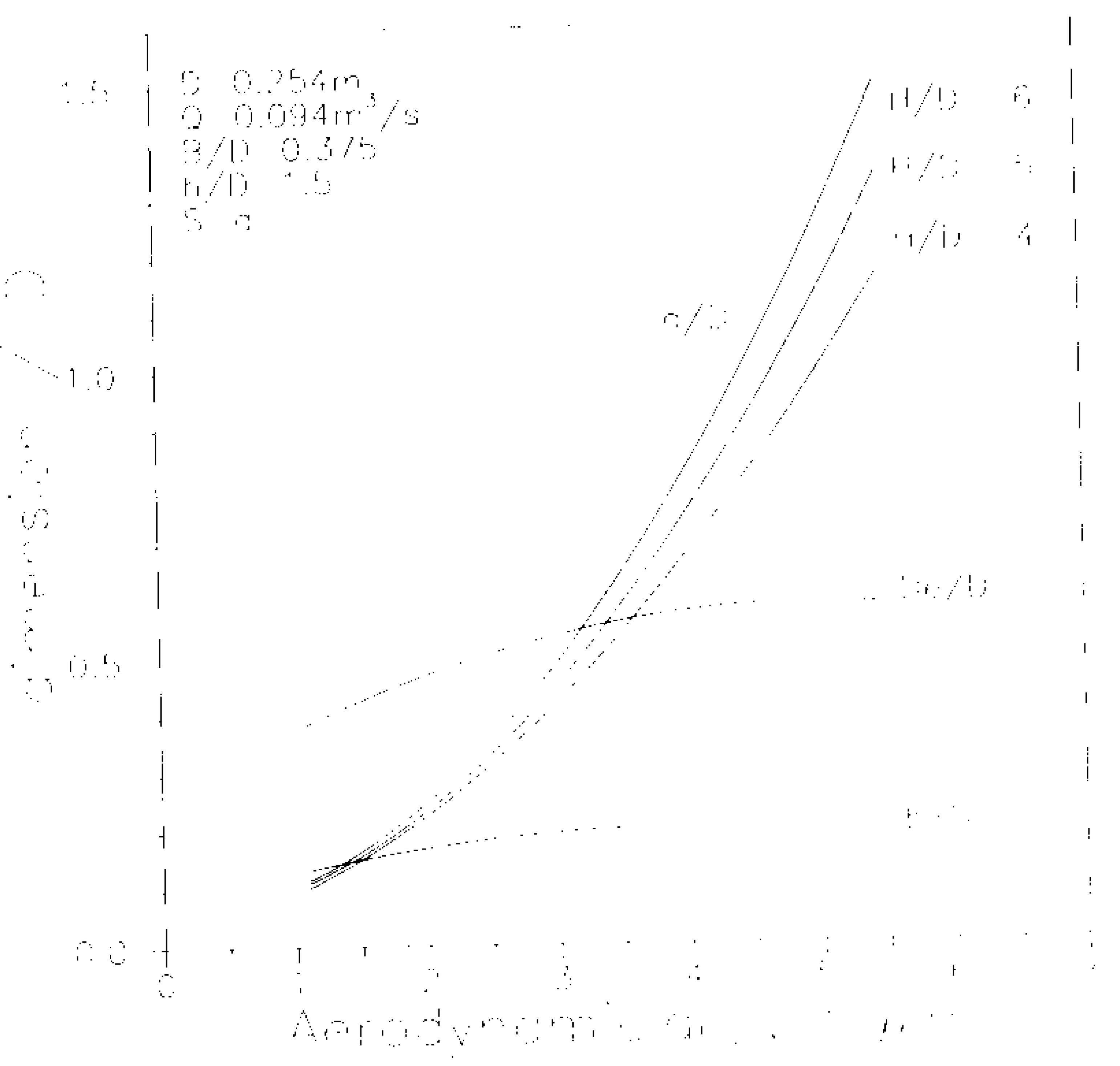

FIGURE 6. Effect of variation in H on optimum dimension ratios.

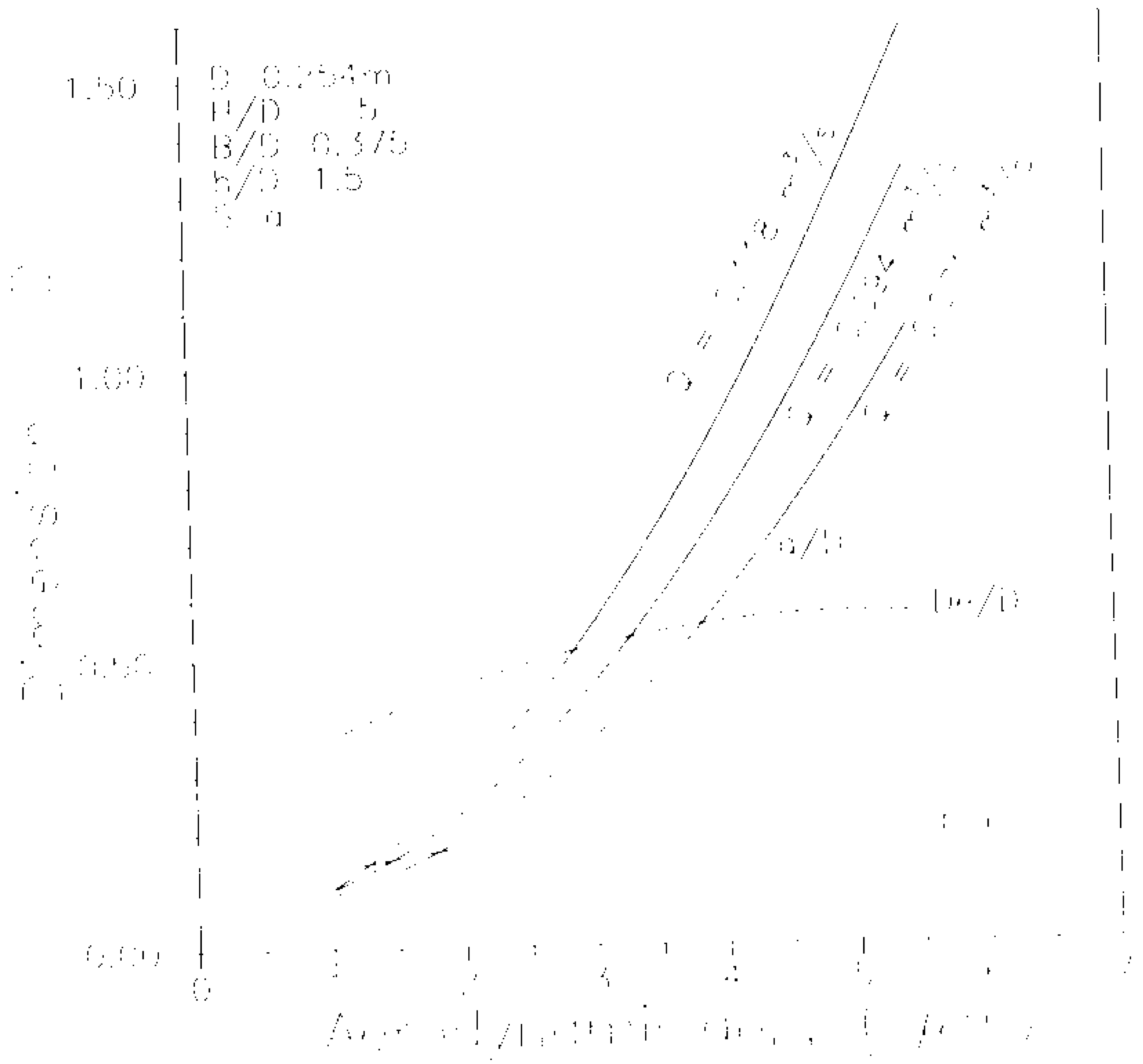

FIGURE 8. Effect of variation in Q on optimum dimension ratios.

This can be understood by substituting Eqs. (A2), (A3), and (A5) from the Appendix into Eq. (A1) to obtain

$$d_{50} = \left(9\mu/(\pi\rho_p 6.1^2)\right)^{1/2}\left(1/(H-S)\right)^{1/2} \times (1/Q)^{1/2}\left(ab/D^2\right)^{0.39}\left(D_e/D\right)^{0.74} \times (H/D)^{0.33} D^2. \quad (8)$$

D_e and b are roughly constant. H and D are kept constant. If Q increases, then a must increase to keep d_{50} the same.

Figure 9 shows pressure drop vs. aerodynamic d_{50} for different cyclone diameters. It shows that as cyclone diameter, D, increases, pressure drop (operating cost) decreases. Cyclone capital costs are propor-

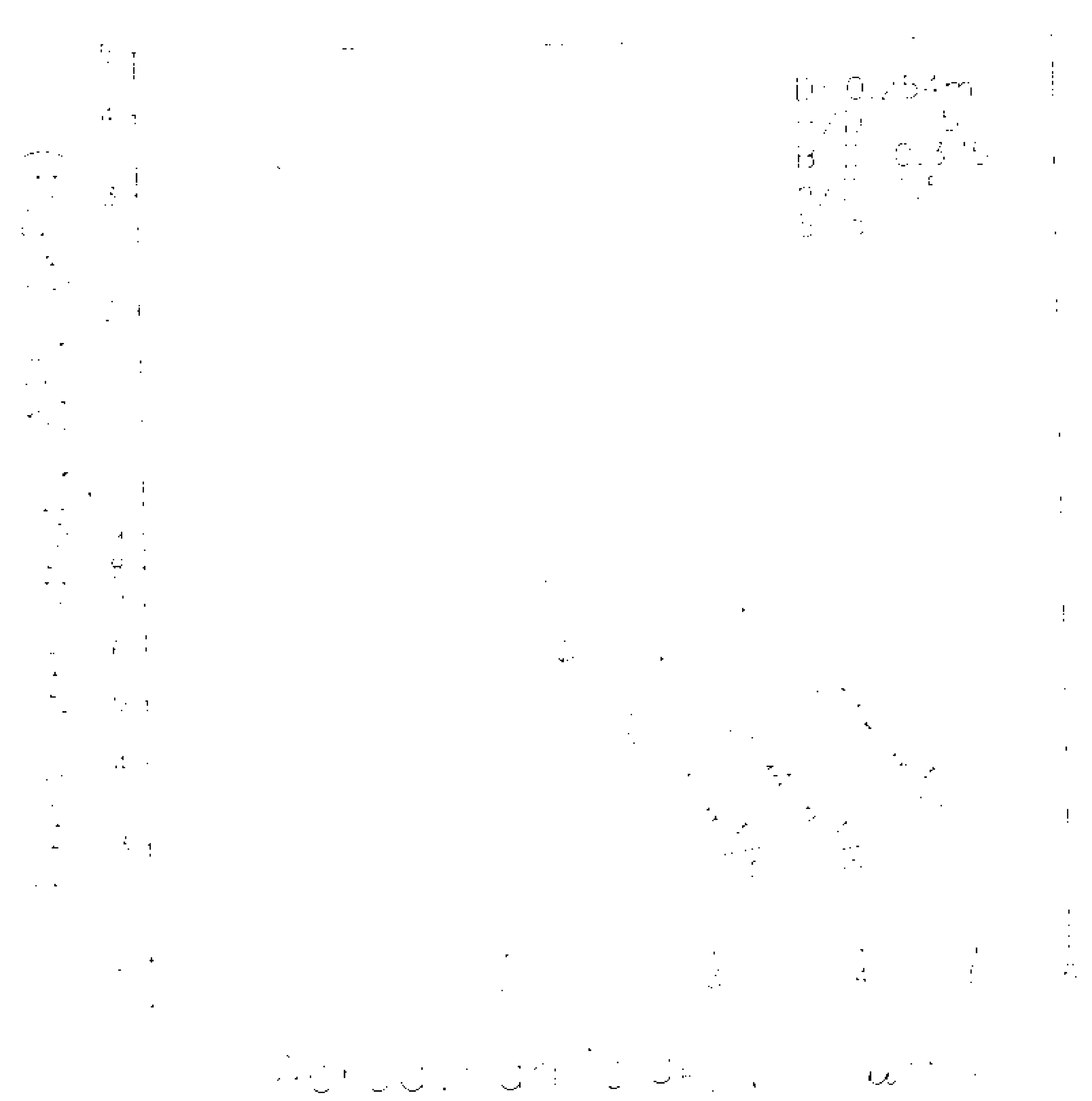

FIGURE 7. Effect of variation in Q on optimum pressure drop.

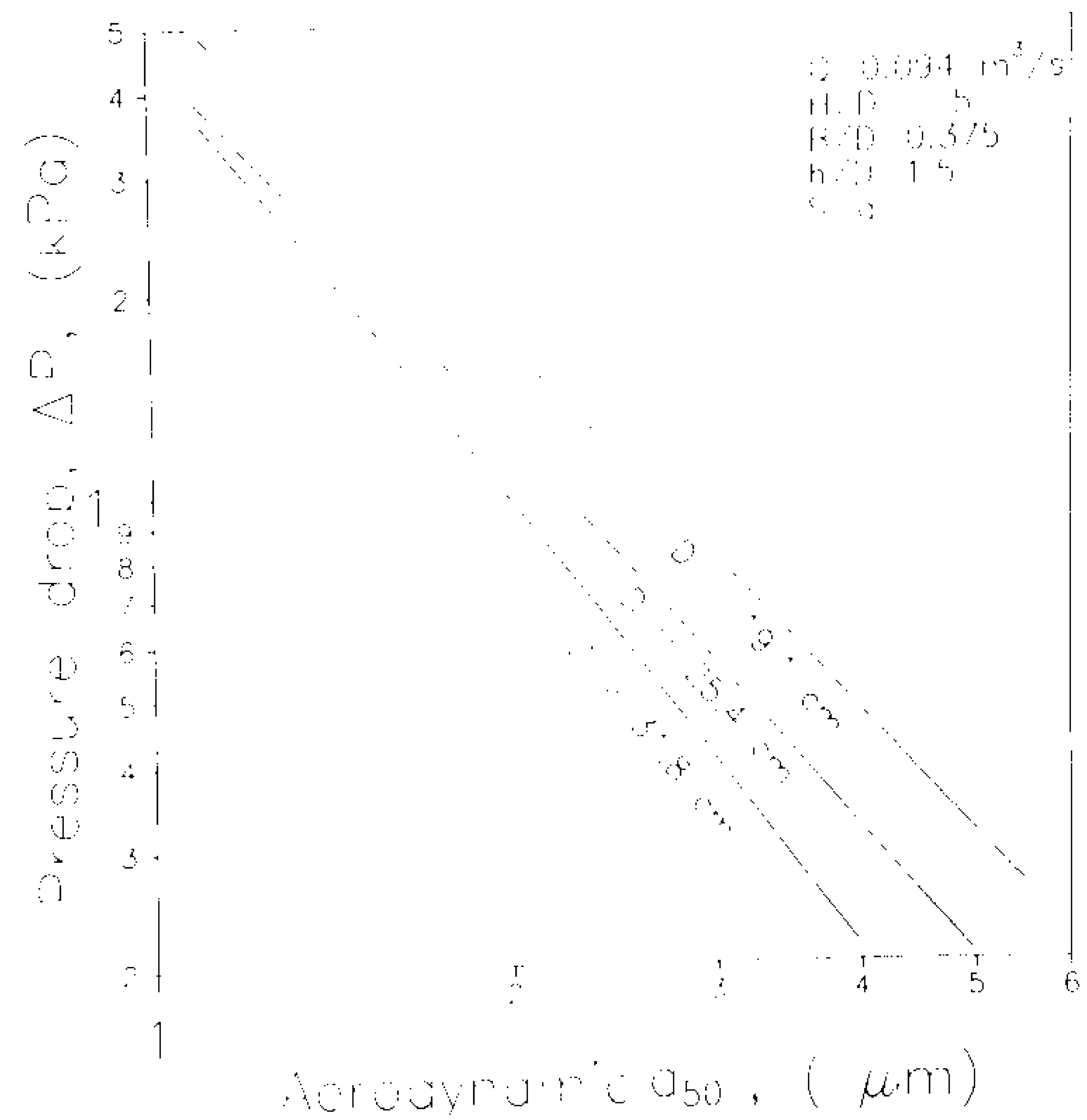

FIGURE 9. Effect of variation in D on optimum pressure drop.

tional to surface area and are therefore proportional to D^2. Thus, if capital costs increase, operating costs decrease. The designer must strike a balance between the higher capital cost that a bigger cyclone entails and the lower operating cost it brings.

Figure 10 shows that, for a given d_{50}, an increase in cyclone diameter is accompanied by a decrease in the ratio a/D. This again can be understood by referring to Eq. (8). If D increases, then since b and D_e are almost constant and H and Q are constant, a must decrease to keep d_{50} constant.

ENGINEERING DESIGN

For any application, the required overall collection efficiency is first specified. A d_{50} is then selected in the following way. The particle size distribution of the dust must be known. Let $c_1, c_2, \ldots, c_N$ be the fraction of particles in each size range. A value of d_{50} is chosen arbitrarily. For each size range, the collection efficiency is calculated from Eq. (A6) as $\eta_1, \eta_2, \ldots, \eta_N$. The overall efficiency is then calculated as

$$\eta_{\text{overall}} = c_1\eta_1 + c_2\eta_2 + \cdots + c_N\eta_N. \quad (9)$$

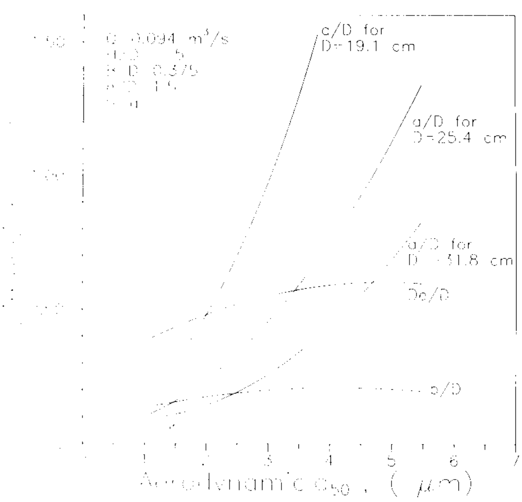

FIGURE 10. Effect of variation in D on optimum dimension ratios.

If the required overall efficiency is greater than the calculated overall efficiency, then d_{50} is decreased, and this process is repeated. By trial and error a value of d_{50} is found such that the required overall efficiency is obtained. This d_{50} is located on the optimization curve (for $H = 5D$) of Figure 5. The pressure drop, ΔP, corresponding to this d_{50} is found. Cyclone inlet height, a, width, b, and outlet diameter, D_e, are found from the dimension ratios of Figure 6 for the specified d_{50}; other cyclone dimensions are fixed at the values listed on that figure. Cyclone diameter, D_2, is found by

$$D_2 = D_1\left(\rho_{p2}Q_2/\rho_{p1}Q_1\right)^{1/3} \quad (10)$$

after Stairmand (1951), where D_1, ρ_{p1}, and Q_1 are the cyclone diameter (0.254 m), particle density (1000 kg/m^3), and flow (0.094 m^3/s) of the cyclone optimized in Figure 6, and ρ_{p2} and Q_2 are the corresponding values for the system being designed. Design pressure drop, ΔP_2, is found (Iozia and Leith, 1989) according to

$$\Delta P_2 = \Delta P_1\left(Q_2 D_1^2/Q_1 D_2^2\right)^2, \quad (11)$$

where ΔP_1 is the pressure drop from Figure 5 corresponding to the constant d_{50}; Q_1 and D_1 are the flow and cyclone diameter of the optimization curve. Q_2 and D_2 are the flow and cyclone diameter of the system to be designed. D_2 is obtained from Eq. (10).

If the pressure drop, ΔP_2, is too high, then the designer should explore other alternatives. A taller cyclone can be chosen as the starting point and the design procedure described above can be repeated. However, a tall cyclone may not always be feasible, especially if it is to be installed at an indoor location with space constraints. A larger diameter can be chosen as the starting point of the design. This would lower pressure drop but increase capital costs. Another way to lower pressure drop would be to reduce flow through the cyclone. This would mean installing additional cyclones in parallel, leading to higher capital costs. Thus every choice presents a trade-off to the designer.

CONCLUSIONS

In this work, a new empirical model for predicting pressure drop across a cyclone developed by Dirgo (1988) is presented. This model is based on statistical analysis of cyclone data and does not add to fundamental understanding of cyclone mechanics. The model also does not take into account the effect of inlet dust loading on cyclone performance. Despite these shortcomings, this model seems capable of better predictions than other models.

Using this model and the efficiency model of Iozia and Leith (1989, 1990), an optimization curve was developed which predicted the minimum d_{50} and the dimension ratios of the optimized cyclone for a given pressure drop. The effect of variation in cyclone height, H, diameter, D, and flow, Q, on the optimization curves was determined.

The optimization curves were used to develop a design procedure for optimized cyclones. The procedure shows that for any set of design criteria, several optimized cyclones may be used. Each of these presents a trade-off that must be made. The designer must choose the alternative most economically feasible.

APPENDIX

Equations for Predicting d_{50} from Iozia and Leith (1989, 1990)

$$d_{50} = \left\{(9\mu Q)/\left(\pi\rho_p z_c V_{tmax}^2\right)\right\}^{1/2}, \qquad (A1)$$

where μ is viscosity of gas, Q is flow through cyclone, and ρ_p is density of particle.

$$V_{tmax} = 6.1 V_i\left(ab/D^2\right)^{0.61} \times (D_e/D)^{-0.74}(H/D)^{-0.33} \qquad (A2)$$

$$z_c = (H-S) \text{ for } d_c < B$$
$$= (H-S) - \left[(H-S)/(D/B-1)\right] \times (d_c/B - 1) \text{ for } d_c > B \qquad (A3)$$

$$d_c = 0.47D\left(ab/D^2\right)^{-0.25}(D_e/D)^{1.4} \qquad (A4)$$

$$V_i = Q/(ab) \qquad (A5)$$

$$\eta = 1/\left(1 + [d_{50}/d]^{\beta}\right) \qquad (A6)$$

$$\ln(\beta) = 0.62 - 0.87\ln[d_{50}(\text{cm})] + 5.21\ln\left(ab/D^2\right) + 1.05\left[\ln\left(ab/D^2\right)\right]^2 \qquad (A7)$$

This material was prepared with the support of the U.S. Department of Energy (DOE), grant DE-FG22-87PC9922. Any opinions, findings, conclusions, or recommendations expressed herein are those of the authors and do not necessarily reflect the views of DOE.

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Received December 13, 1990; accepted March 11, 1991.