

The UNIVERSITY of NORTH CAROLINA at CHAPEL HILL

COMP 665 Imaging, Graphics, & Vision

Fall 2008

Problem Set #4 Solutions

December 3, 2008

1. “*Tensor Decomposition*” (1.0 points for each)

- A) 3 row vectors are necessary to represent 50% of the variance. In fact, they cover 51.641722% of the variance.
3 column vectors are necessary to represent 50% of the variance. In fact, they cover 51.359697% of the variance.
1 tube vector is necessary to represent 50% of the variance. In fact, it covers 54.275318% of the variance.
- B) 45 row vectors are necessary to represent 98% of the variance. In fact, they cover 98.397088% of the variance.
45 column vectors are necessary to represent 98% of the variance. In fact, they cover 98.437325% of the variance.
20 tube vectors are necessary to represent 98% of the variance. In fact, they cover 98.028093% of the variance.
- C) There are 9 non-zero entries in the core matrix: -32068.7381923, -313.632184551, -219.23979572, -65.7332539775, -2265.49230094, 572.048078197, -161.669300867, -3612.25399147, -1745.50872094.
- D) Mean Absolute Error of the truncated core matrix: 13.748760 (Note: This problem asks for the MAE between the original core tensor and the truncated one. If you compute the MAE of the reconstructed image, it should be 63.04. I accept this answer iff you clearly describe which MAE you’re calculating.)
- E) 1 principle component is necessary to represent 50% of the variance. In fact, it covers 68.684460% of the variance. For the PCA scheme, you need 48x48x1 numbers to reach 50% of the variance. By using the tensor decomposition, on the other hand, only 48x3+48x3+27x1 numbers are required to achieve similar quality. Obviously it’s a significant amount of reduction (87%).

2. “*Which Wavelet*” (1.0 points for each)

- A) Mean: 129.00170898
10 largest coefficients: [50.55078125 37.25 36.25 35.5 33.5 32.5 -31.875 -31.5 31.5 -31.375]

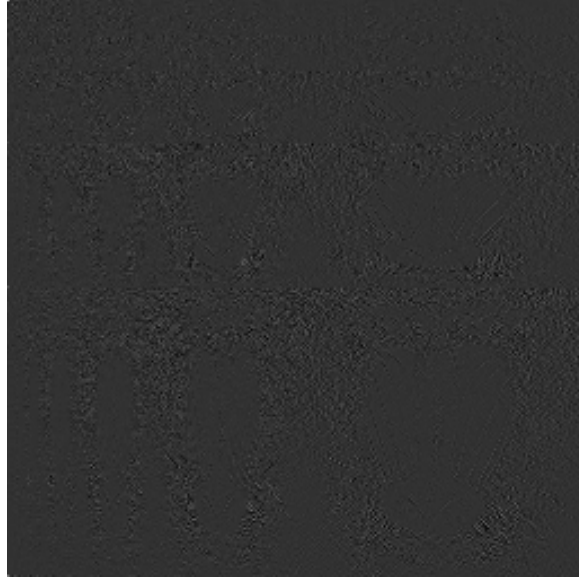


Figure 1: The wavelet-coefficient matrix generated using a standard 2D wavelet transform

B) Mean Absolute Error for the remapped reconstruction: 7.555893

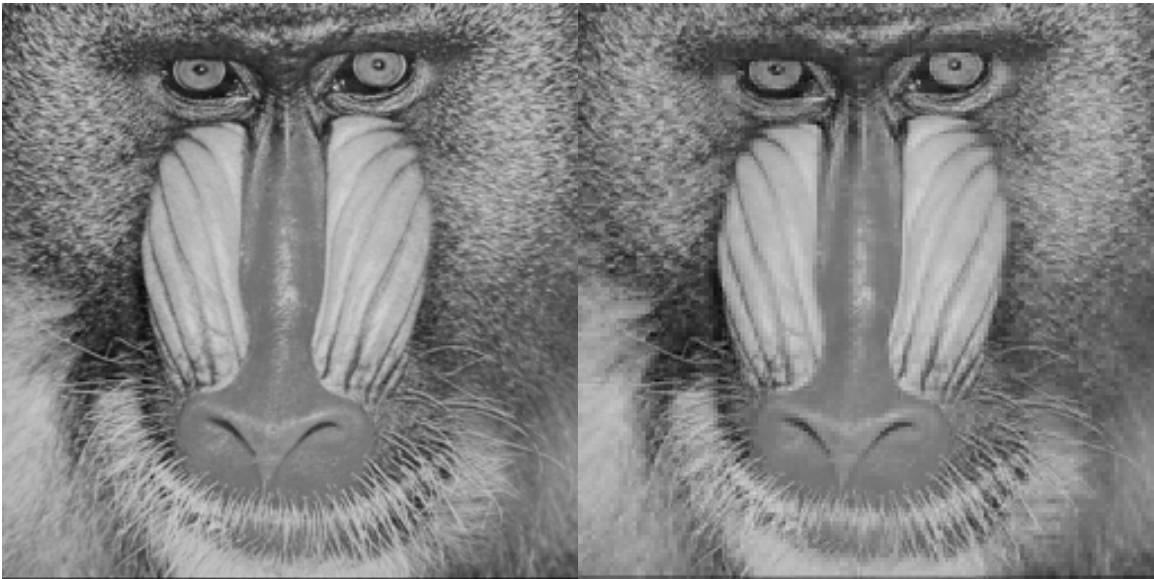


Figure 2: The original (left) and its modified reconstruction (right)

C) Mean: 129.00170898

10 largest coefficients: [73.5 73. 72.75 72.75 72.5 72.25 72. 71.75 71.25 71.25]

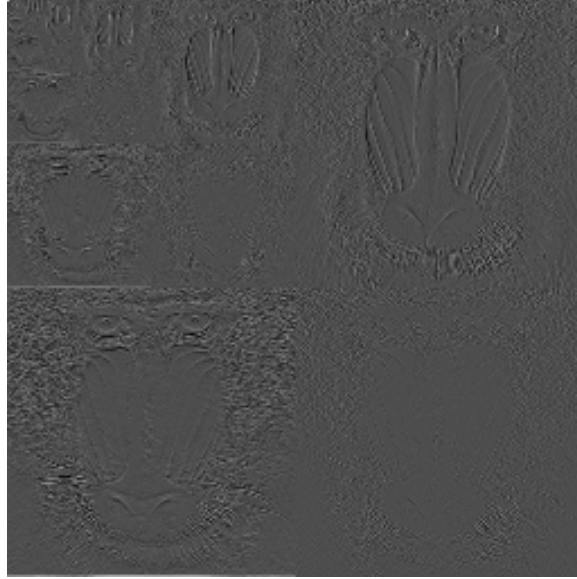


Figure 3: The wavelet-coefficient matrix generated using a non-standard 2D wavelet transform

D) Mean Absolute Error for the remapped reconstruction: 6.150978

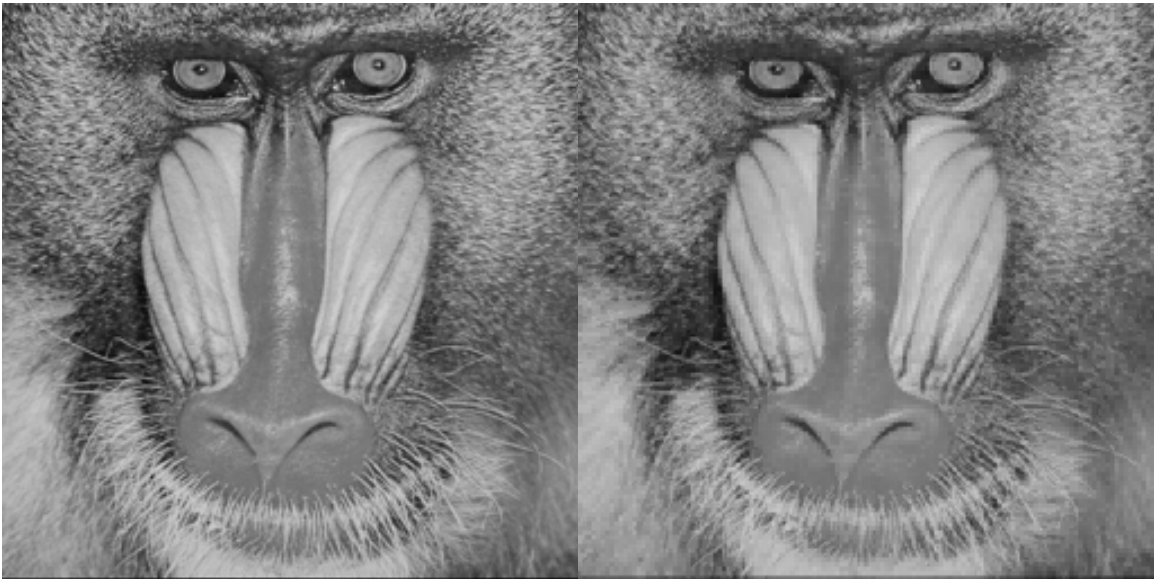


Figure 4: The original (left) and its modified reconstruction (right)

E) From the coefficient matrix, two different approaches gives distinct set of coefficients. For the standard wavelet transform, the LH and HL quadrants are also modified throughout the iterations (You can see stretched mandrills in different directions). For the non-standard one, only the HH quadrant is changed at each level. In terms of computational complexity, the two approaches have the same order $O(N^2)$ (N is the number of pixels in a row/column) but with different constants. Non-standard approach is slightly faster than the standard one.