Homework 1 Solution

1. a) 50 states → 4 possible states starting with an A
   \[ \log(50/4) = 2.64 \text{ bits} \]

   50 states → 1 state
   \[ \log(50/1) = 5.64 \text{ bits} \]

   b) \( 10^{20} \) atoms \( \rightarrow 10^{20} \) binary numbers
   \[ \log_2(10^{20}) = 265.75 \text{ bits} \]

   \[ 1 \text{ GB} = (2^{30}) \text{ bits} \]
   \[ \text{answer: } 2^{30} \times 8 \text{ bits } \approx 3.2 \times 10^7 \text{ numbers} \]

   c) suit : \[ \log_2(\frac{52}{13}) = 2 \text{ bits} \] (13 = 52 total cards / 4 suits)

   face : \[ \log_2(\frac{52}{12}) = 2.12 \text{ bits} \] (12 = 3 face cards × 4 suits)

   knowing face card is more info because 2.12 > 2

   both : you're down to only 3 choices now – J/Q/K of the given suit
   \[ \log_2(\frac{52}{3}) = 4.12 \text{ bits} \]

   d) given binary number with 72 bits, largest number that can be represented
   is \( 2^{72} - 1 \)

   how many decimal digits would this number have?
   \[ \log_{10}(2^{72} - 1) = 21.72 \approx 22 \text{ digits} \]

   hexadecimal digits?
   \[ \log_{16}(2^{72} - 1) = 18 \text{ digits} \]

2. a) 1024 MB, 93 GB, 2 GHz, 1440 x 900 pixels

   \[ \frac{\text{new number}}{\text{old number}} = \text{ratio} \]
   * don't forget to set your units to the
   \[ \frac{1 \text{ GB} = 1024 \text{ MB} = 1024 \times 1024 \text{ KB}}{\text{some quantity!}} \]

   \[ 1 \text{ GHz} = 1000 \text{ MHz} \]
d) should have looked something like this:

**Example w/ CPU speed:**

<table>
<thead>
<tr>
<th>Year</th>
<th>CPU Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>0.5 GHz</td>
</tr>
<tr>
<td>1996</td>
<td>2 GHz</td>
</tr>
<tr>
<td>2008</td>
<td>4 GHz</td>
</tr>
<tr>
<td>2018</td>
<td>8 GHz</td>
</tr>
</tbody>
</table>

- If trend continues, put a 1;
- Otherwise, put a 0.

- 15 GHz realistic in 10 years? Perhaps, although the current trend is toward multicore architectures.
- See a discussion on Moore's Law for interesting observations (which states that most computer specs double every 12-18 months).

### (3) a)

<table>
<thead>
<tr>
<th>Binary (16-bit)</th>
<th>Hexadecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0101</td>
<td>0x 0065</td>
</tr>
<tr>
<td>0000 0111 1101</td>
<td>0x 011B</td>
</tr>
<tr>
<td>1111 1111 1111</td>
<td>0x FFFF</td>
</tr>
<tr>
<td>-1</td>
<td>0x FFFF</td>
</tr>
</tbody>
</table>

\[
101 = 2^3 + 2^2 + 2 = 1 + 4 + 32 + 64 \quad \text{(binary)}
\]

\[
0x0065 = 5 \cdot 16 + 6 \cdot 1 = 5 + 64 \quad \text{(hexadecimal)}
\]

### b)

- Flip: 0100 0101 0010 1110
- Add 1: 0100 0101 0010 1111 = 2's complement

\[
2^4 + 2^3 + 2^2 + 2 = \text{decimal} = 16 + 8 + 4 + 2 = 28 \quad \text{(binary)}
\]

\[
0x 17711 = \text{hexadecimal}
\]

### c)

- \(0x \text{0000 0006} = 8 \cdot 16^0 + 2 \cdot 16^1 = 10 \)
- \(0x \text{0000 0101} = 1 \cdot 16^1 + 1 \cdot 16^2 = 257 \)
- \(0x \text{FFFF FFFF} = 16 \times 16^7 + 15 \times 16^6 + 15 \times 16^5 + \ldots + 15 \times 16^0 = -16 \)
- \(0x \text{FACE BABB = } -2469 \times 16^7 \) (Remember, the first A here is -10 \times 16^7.)