Supplemental Homework Set on Curvilinear Motion

1. Consider a particle 10 μm in aerodynamic diameter that is thrown upwards and to the right at a 45 degree angle with an initial velocity of 5 cm/s.

   a) Show that the distance the particle travels in the horizontal direction, x, is given by

   $$x = v_{i,x}\, \tau\, (1 - e^{-t/\tau}),$$

   and the corresponding distance, y, it travels in the vertical direction is given by:

   $$y = \left(v_{i,y}\, \tau + \tau^2 g\right)\left(1 - e^{-t/\tau}\right) - \tau g t \ .$$

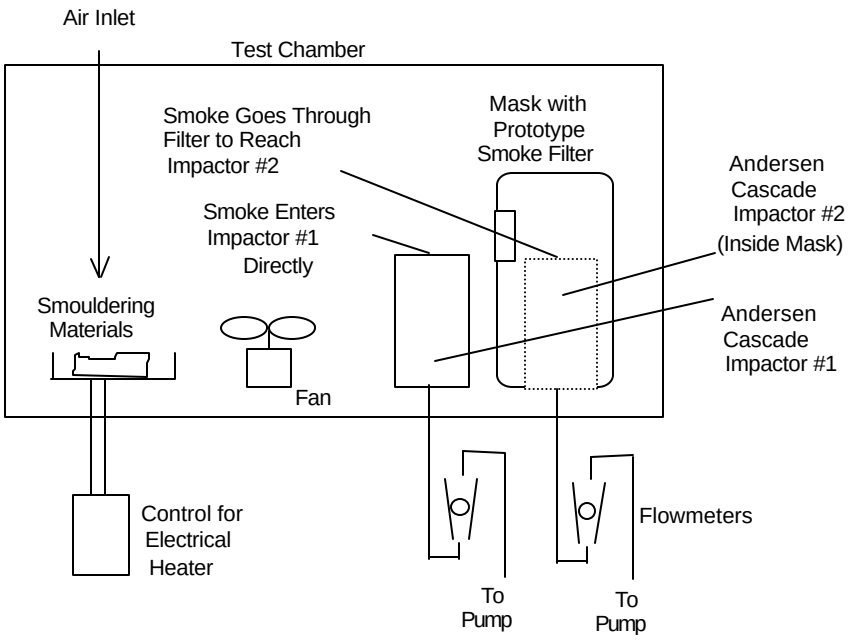
   b) Use these equations to plot the coordinates of the particle over a time of 0 to 4 milliseconds.

2. An entrepreneur is developing a mask that can be worn in a fire to protect the wearer against inhalation of smoke. The mask is comprised of a bed of fibers that are 10 μm in diameter, that has a solidity of 0.01, that has a cross-sectional area of 10 cm^2 and that is 0.5 cm thick. Assume an inspiratory flow of 60 L/min through the mask. Further assume that smoke particles are 1 μm in aerodynamic diameter.

   Some potential investors have asked you to evaluate how well this mask will work. Calculate how efficient the mask should be on the smoke particles. Would you advise investment in this mask? Why or why not?

3. To validate your calculations, you decide to test the mask using a smoke aerosol that you generate in a chamber shown in the diagram below. In the chamber you place an Andersen cascade impactor to measure the concentration of particles in each size range. You also place a mask in the chamber through which you draw smoke aerosol. Immediately downstream of the mask you have a second Andersen impactor to measure the concentration of particles in each size range that gets through the mask. Comparison of the masses that collect on corresponding stages of the two impactors will give you the fraction of particles in each size range that penetrates through the mask.

   The Andersen impactor is calibrated by the manufacturer for flow of 28.3 L/min according to the table below. However, for these tests, you decide that an inspiratory flow of 60 Lpm is more representative of breathing during exertion, as when trying to escape from a burning building. What will be the cut points for each impactor stage if each Andersen impactor is operated at 60 L/min instead of 28.3 L/min in your tests? For these tests, only the pre-selector, stage 0, and the first five stages of the Andersen impactor were used. The impactor normally operates with two additional stages, stages 6 and 7.



Aerodynamic Particle Sizes for Andersen Cascade Impactor at Air Flow of 28.3 L/min

Impactor Stage	Pre-selector	Stage 0	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Final Filter
Aerodynamic Size Range	> 10 µm	10 - 9.0 µm	9.0 - 5.8 µm	5.8 - 4.7 µm	4.7 - 3.3 µm	3.3 - 2.1 µm	2.1 - 1.1 µm	< 1.1 µm

4. Results of your mask performance tests are given below. These results came from a six minute sample at a flow of 60 L/min.

		Chamber Air, Impactor 1			Air Passed Through Mask, Impactor 2		
Stage	Size, µm	Tare-g	Exp.-g	Net-mg	Tare-g	Exp.-g	Net-mg
Preselector		37.3087	37.3109	2.2	32.3856	32.3857	0.1
Plate 0		23.3911	23.3925	1.4	23.6454	23.6454	0
Plate 1		23.6086	23.6109	2.3	23.2673	23.2674	0.1
Plate 2		23.4477	23.4497	2.0	23.5853	23.5853	0
Plate 3		23.6841	23.7021	18.0	23.3987	23.4004	1.7
Plate 4		23.5022	23.5629	60.7	23.5622	23.5853	23.1
Plate 5		23.4307	23.6030	172.3	23.3944	23.4918	97.4
Filter		0.3834	0.4480	64.6	0.3823	0.4277	45.4
Total:				323.5			167.8

a) The maximum 8-hour time weighted average for nuisance respirable particulates allowed by law is 10 mg/m^3. How does this compare with the concentration in your test chamber? How does this compare with the concentration you measured downstream of the mask? Assume for the moment that all particles are respirable.

b) Plot the cumulative size distributions by mass for the smoke aerosol in the chamber, and for the smoke aerosol that passes through the mask. Use log probability paper. How would you describe these size distributions? A blank sheet of this paper is on my door - please copy and return it for your use.

c) Plot the measured collection efficiency of the mask on the vertical axis against aerodynamic particle diameter on the horizontal axis.

d) Plot the theoretical collection efficiency of the mask, given the mask specifications from Problem 1, against aerodynamic particle diameter on the same plot as part (c).

5. What fraction of the smoke particles you used in your tests are (a) inspirable, (b) thoracic, and (c) respirable, according to the current criteria of the ACGIH?

6. You wish to take a representative sample of aerosol at 6 L/min from a duct 8 inches in diameter carrying 1000 cfm of air. The sampling probe must be 90 cm long to fit into the elbow where the sample is to be taken; see the diagram below. What inside diameter should the sampling probe have?