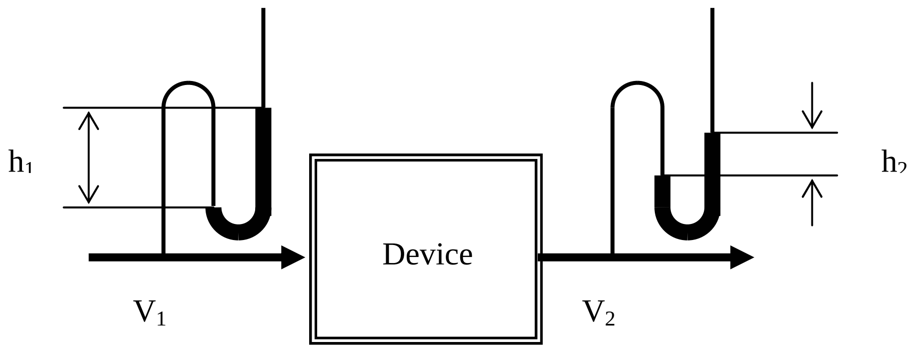
PRESSURE DROP, POWER, AND COST

Pressure Drop

Consider flow through a device as shown below. The pressure drop across the device, ΔP, is the difference in static pressures at the outlet and the inlet,

$\Delta P = h_1 - h_2$.



The total energy going into the device equals the total energy coming from the device, plus energy lost in the device.

Total Energy In = Total Energy Out + Lost Energy

We would like to develop an expression for ΔP that reflects what we know about energy in the system. Total energy is the sum of kinetic energy of the flowing fluid, KE, plus the potential energy, PE, due to the static pressure of the fluid. Energy is expressed per unit volume of fluid.

Total Energy = KE + PE .

Here,

$KE = \frac{1}{2}\rho_g V^2$ and

$PE = \rho_L g h$,

where

ρ_g = gas density,
V = gas velocity
ρ_L = density of fluid in manometer
g = acceleration of gravity
h = height of fluid in manometer

Now let the energy lost in the device be expressed as a constant, ΔH, times the kinetic energy of the incoming gas:

Energy lost in device = $\Delta H \ \frac{1}{2}\rho_g \ V_1{}^2$

Why should this expression for lost energy be reasonable? Pressure drop can be thought of as the force/area necessary to push fluid past the surfaces and obstacles in its way as it flows through the device. Applying Newton's Law for drag force in turbulent flow,

$$\Delta P = \frac{\text{Force}}{\text{Area}} = \frac{C_D \ \text{Area}\frac{1}{2}\rho_g \ V^2}{\text{Area}} = C_D \ \frac{1}{2}\rho_g \ V^2 \quad .$$

Thus, ΔH can be thought of as a sort of "drag coefficient" related to flow past or through the device. From this analysis, which applies for turbulent flow conditions, lost energy can be seen to be proportional to V^2. If flow is laminar, then lost energy would be proportional to V because for laminar conditions $C_D \propto 1/V$; however, in most cases that involve industrial flows, turbulent conditions exist.

After substitution, we find that

$$\frac{1}{2}\rho_g \ V_1{}^2 + \rho_L \ g \, h_1 = \frac{1}{2}\rho_g \ V_2{}^2 + \rho_L \ g \, h_2 + \Delta H \frac{1}{2}\rho_g \ V_1{}^2$$

If the velocity of the fluid flowing into and out of the device is the same, $V_1 = V_2$. Because pressure drop, ΔP is given by $h_2 - h_1$, substitution yields

$$\Delta P = \frac{V_1{}^2 \ \rho_g}{2 \, g \, \rho_L} \Delta H \ .$$

Values of ΔH can be determined experimentally by measuring ΔP with the other terms in the above equation known. These values have been tabulated for devices such as elbows, cyclones, and other objects that cause pressure drop in a stream of flowing fluid. Alternatively, for some shapes, ΔH can be calculated from theory.

Power

Recall that

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{\text{force} \times \text{distance}}{\text{time}}$$

Thus

$$\text{Power} = \frac{\text{force}}{\text{area}} \times \frac{\text{distance} \times \text{area}}{\text{time}} = \Delta P \;\, Q$$

where Q is volumetric flow. If we use mks units, then power is given in watts if pressure drop is in Pascals (N/m^2) and flow is in m^3/s. If we choose to use English units, so that pressure drop is expressed in inches of water and flow is expressed in cubic feet per minute (cfm), then

$$\text{Power}_{\text{watts}} = 0.118 \, \Delta P_{\text{inches of water}} \, Q_{\text{cfm}} \quad .$$

Cost

The power given in the equation above represents the power of the flowing gas stream. To convert this quantity to the electrical power necessary to run a fan that provides this power, we must divide by the mechanical efficiencies of the fan and the electrical motor, ME_{fan} and ME_{motor}. With this modification, and expressing power in kilowatts instead of watts, gives

$$\text{Power}_{\text{kw electrical}} = \frac{1.18 \times 10^{-4} \, \Delta P_{\text{inches of water}} \, Q_{\text{cfm}}}{ME_{\text{fan}} \, ME_{\text{motor}}}$$

From this equation, we can determine the cost of running the fan if we know the cost of electricity, typically about \$0.07/kwh, and the number of hours the fan operates. In addition, we can determine the horsepower for the electrical motor required, using the conversion that

$$\text{kw} \times 1.34 = \text{HP}.$$