SCRUBBERS FOR PARTICLE COLLECTION

Background

To understand how scrubbers work, we must first define some terms.

Single droplet efficiency, η_d, is similar to single fiber efficiency. It is the fraction of particles in the gas upstream of a spherical droplet that collect on that droplet as the droplet moves through the gas. Although the same mechanisms contribute to single droplet efficiency that contribute to single fiber efficiency, and include impaction, interception, and diffusion, impaction alone usually dominates. For particle collection due to impaction on a droplet, the following equation is often used:

$$\eta_d = \left(\frac{\text{Stk}}{\text{Stk} + 0.35} \right)^2 \tag{1}$$

where Stk is the Stokes number for the drop moving through gas that contains particles. The particles are assumed to be “embedded” in the gas; that is, they do not move relative to the gas except to cross streamlines by impaction. Inspection of Eq (1) shows that η_d is zero if Stk is zero, and approaches a value of unity as Stk becomes large.

$$\text{Stk} = \frac{d^2 \rho_p V_{d/g} C_C}{18 \mu d_d} \quad . \tag{2}$$

Here,

d	is particle diameter,
ρ_p	is particle density,
$V_{d/g}$	is the velocity of the droplet relative to the gas (see below)
C_C	is the Cunningham slip correction factor,
μ	is gas viscosity, and
d_d	is droplet diameter.

Because droplets are liquid, particle collection on liquid droplets will depend on some mechanisms that do not operate for collection on dry spheres or on dry fibers. For example, diffusiophoresis and Stefan flow can be important. If the droplets are water, Stefan flow will enhance particle collection under conditions when water vapor condenses on the droplets, but will reduce particle collection for evaporating droplets. Diffusiophoresis for condensing water vapor will tend to drive particles away from the droplet, and toward an evaporating droplet; however, this effect is usually less important than Stefan flow.

Liquid holdup, Hd, is the fraction of the scrubber volume that is filled with droplets. It is analogous to solidity for a filter. If, for example, all the droplets in a scrubber were made of ice, then the ice all melted, then Hd would be the volume of the water from the ice divided by the volume of the scrubber.

A difference between holdup and solidity is that holdup is a dynamic situation whereas solidity is a stationary situation. The value for scrubber holdup depends on the rate that liquid is fed into the scrubber and the velocity with which the droplets move through the scrubber.

$$Hd = \frac{\text{volume of droplets in scrubber}}{\text{volume of scrubber}} = \frac{Q_L \dfrac{Z}{V_{d/w}}}{A\ Z} = \frac{Q_L}{A\ V_{d/w}} \quad , \tag{3}$$

where

Hd is holdup, or volume fraction comprised of droplets,
Q_L is liquid volumetric flow into the scrubber,
$V_{d/w}$ is the velocity of the droplets relative to the scrubber wall (see below)
A is the cross sectional area of the scrubber, and
Z is scrubber length, measured perpendicular to A.

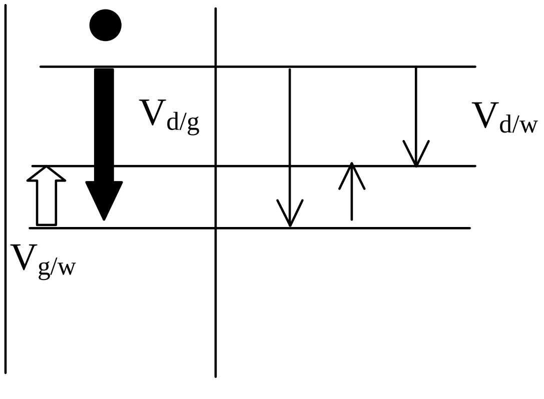
Droplet velocities within the scrubber can be expressed in several ways. Sometimes the important concept is the velocity of the droplets relative to the gas, $V_{d/g}$. This velocity would be important, for example, if we are interested in the impaction of particles that are in the gas on droplets that move through the gas.

At the same time that droplets move through the gas, the gas itself is moving through the scrubber with a certain velocity that should be taken relative to the scrubber wall, $V_{g/w}$. This velocity is important, for example, if we are interested in the residence time of the gas in the scrubber.

A final velocity of interest is the velocity of the droplets relative to the wall of the scrubber, $V_{d/w}$. This velocity is the vector sum of the velocity of the drops relative to the gas, and the velocity of the gas relative to the wall

$$\vec{V}_{d/w} = \vec{V}_{d/g} + \vec{V}_{g/w} \quad . \tag{4}$$

Consider a spray tower in which the gas enters at the bottom and leaves at the top. Water is sprayed downward into the top of the rising gas. If, for example, the droplets fall through the gas with a downward terminal settling velocity of 200 cm/s *relative to the gas*, and the gas itself flows upward through the scrubber with a velocity of 50 cm/s *relative to the scrubber wall*, then the downward velocity of the droplets as seen through a window in the side of the scrubber, or $V_{d/w}$, is - 200 + 50 = -150 cm/s or 150 cm/s in the downward direction. If the gas happened to flow upward at the exact same velocity that the droplets fell downward through the gas, then the droplets would appear stationary when seen through that window, or $V_{d/w}$ = 0. If the upward gas velocity had a higher numerical value than the downward droplet velocity,



then the drops would be blown out the top of the scrubber, even though they continue to settle downward through the rising gas. Relative velocity is important as we consider particle collection in scrubbers. Bear in mind that the important velocity for particle collection is the velocity of the droplets relative to the gas, or $V_{d/g}$ whereas the important velocity for droplet motion through the scrubber is the velocity of the droplets relative to the wall, or $V_{d/w}$.

Liquid Evaporation will occur if the gas stream is not saturated with vapor. Liquid that evaporates is not available to collect particles, so evaporation must be considered as we investigate scrubber performance. This study can be done using the psychrometric chart; see a link to such a chart on the course website. Similar charts are available in many books and at many websites.

Use of the psychrometric chart can be shown through an example.

> Consider a case where 100,000 cfm of air at 150 F, and relative humidity of 20%, is to be scrubbed. Assuming that the air becomes saturated, determine the exit gas temperature and the flow of water required to replace the water lost due to evaporation.

First, locate the point on the psychrometric chart that corresponds to 150 F, 20% RH. Read to the left to find absolute humidity of about 0.033 lb water/lb dry air. At this point (150 F, 20% humidity) read humid volume to be about 16.3 ft^3/lb dry air.

From this initial point, follow the adiabatic saturation line up and to the left to determine the dew point or wet bulb temperature, T_{wb}. For these conditions, T_{wb} = 103 F; 0.045 lb water/lb dry air; 15.2 ft^3/lb dry air. With these data we can solve the problem. The key is to express air conditions on a basis of pounds of dry air, as that quantity does not change as we add (or subtract) water vapor.

a. Determine pounds of dry air.

$$\frac{100{,}000\ \text{ft}^3}{\text{min}} \times \frac{\text{lb dry air}}{16.3\,\text{ft}^3} = 6134\,\frac{\text{lb dry air}}{\text{min}}$$

b. Determine flow after saturation:

$$6134\,\frac{\text{lb dry air}}{\text{min}} \times \frac{15.2\,\text{ft}^3}{\text{lb dry air}} = 93{,}300\,\frac{\text{ft}^3}{\text{min}}$$

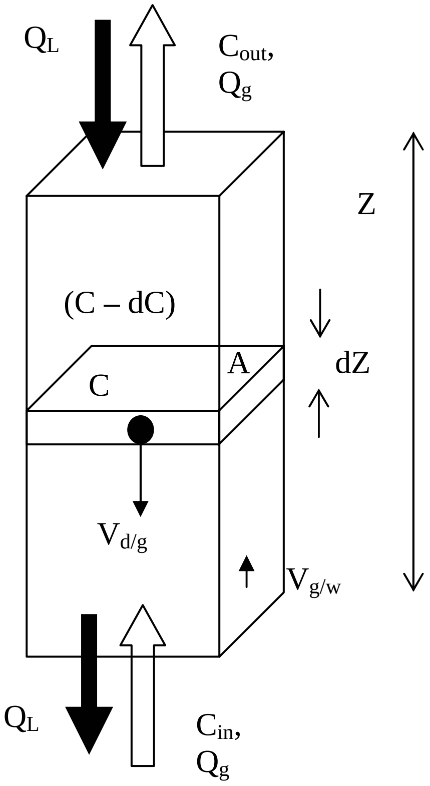
c. Determine water consumption:

$$6134\,\frac{\text{lb dry air}}{\text{min}} \times (0.045 - 0.033)\frac{\text{lb water}}{\text{lb dry air}} = 73.6\frac{\text{lb water}}{\text{min}}\ \text{or}\ 8.8\ \text{gallons / minute}$$

Thus, before we can have water droplets in the gas stream at all, we must supply enough water to saturate the gas. That process causes the gas stream to decrease in temperature from 150 F to 103 F, and to decrease in volume from 100,000 cfm to 93,300 cfm.

Spray Tower

The approach we will use to describe particle collection in a spray tower is similar to the one we used for fiber filters, and to what we used for gravitational settling chambers. The approach is based on the drawing shown below. Particle-laden gas enters at the bottom of the scrubber and moves vertically upwards. Liquid sprays into the top of the scrubber and falls downward as droplets, each of which falls at its terminal settling velocity.



As always, we need to make some assumptions:

1. Particle concentration, C, is uniform in any plane perpendicular to the direction of gas flow,

2. Gas velocity, V_g, is uniform throughout,

3. Droplets are spread evenly across the gas stream cross section, which has area “A”,

4. Droplets all have the same diameter, d_d,

5. No droplets collect on the walls,

6. No droplets agglomerate,

The last four assumptions represent “ideal” droplet behavior. If they are true (and they are usually not true) then each droplet acts in a “perfect” way for particle collection. Departure from ideal behavior will cause less particle collection than what we determine using an equation based on the ideal droplet assumptions.

A balance for the rate at which particles pass through the differential slice dZ yields the equation given on the next page. As for the equation we developed for particle collection in a filter, the term on the left side of the equation represents the rate that particles enter the slice from the air. The first term on the right side of the equation is for particles that leave in the air that flows from the back side of the slice. The second term on the right side is for particle accumulation within the slice, where accumulation is caused by particle collection on droplets. within the slice. The sign on the accumulation term is negative because particles are *removed* from the air in the slice; that is, they do not *accumulate* there. When thinking about the mass rate balance equation, one must be careful to recognize the role of the several relative velocities that are important in scrubbers.

$$\overbrace{Q_g\,C}^{\text{Rate dust flows into the slice}} = \overbrace{Q_g\,(C - dC)}^{\text{Rate dust flows out of the slice}} - \overbrace{\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{A\,dZ\;H_d}_{\text{droplet volume}}\;\underbrace{\left(\frac{\frac{\pi d_d^2}{4}}{\frac{\pi d_d^3}{6}}\right)}_{\text{collection area/volume}}}_{\text{collection area}}\;V_{d/g}}_{\text{volume of gas swept/time}}\;\eta_d}_{\text{volume of gas swept clean/time}}\;C}_{\text{mass of particles swept clean/time}}}^{\text{Rate dust accumulates in the slice}} \tag{5}$$

Here,

Q_g	is upward gas volumetric flow thorough scrubber,
C	is dust concentration in the slice
A	is scrubber cross-sectional area
Z	is scrubber height,
H_d	is holdup, the fraction of scrubber volume made up of droplets, see Eq (3),
d_d	is droplet diameter,
η_d	is single droplet efficiency, see Eq (1).

After substituting Eq (3) for holdup, Equation (5) becomes

$$\int_{C_{in}}^{C_{out}} \frac{dC}{C} = \int_0^Z \; - \frac{3}{2}\frac{Q_L}{Q_G}\frac{V_{d/g}}{V_{d/w}}\;\eta_d\;\frac{dZ}{d_d} \tag{6}$$

or

$$\mathrm{Pt} = \frac{C_{out}}{C_{in}} = \exp\left[-\frac{3}{2}\frac{Q_L}{Q_G}\frac{V_{d/g}}{V_{d/w}}\frac{Z}{d_d}\,\eta_d \right] = \exp\left[-\frac{3}{2}\frac{Q_L}{Q_G}\frac{V_{d/g}}{\left(V_{d/g} - V_{g/w}\right)}\frac{Z}{d_d}\,\eta_d \right] , \qquad (7)$$

or in terms of efficiency,

$$\eta = 1 - \exp\left[-\frac{3}{2}\frac{Q_L}{Q_G}\frac{V_{d/g}}{\left(V_{d/g} - V_{g/w}\right)}\frac{Z}{d_d}\,\eta_d \right] \qquad (8)$$

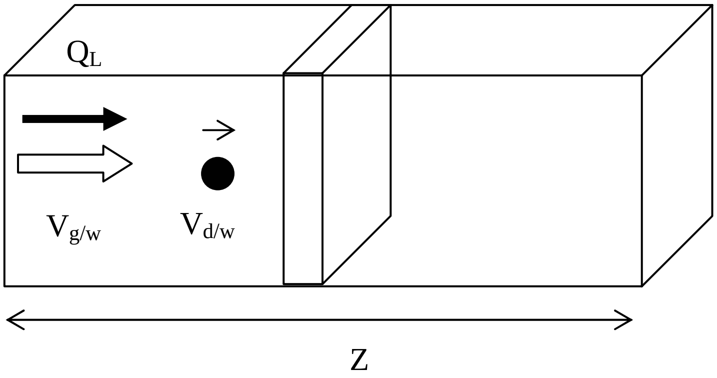
Note that efficiency is high when the ratio of water used to gas treated, Q_L/Q_G, becomes high. Efficiency also is high when the settling velocity of the droplets, $V_{d/g}$, approaches the upward velocity of the gas, $V_{g/w}$. In this case, holdup becomes high; that is, the tower contains more suspended drops. Efficiency is high if the tower is tall, and if single droplet efficiency is high.

The role of droplet diameter is complex. Eq (8) shows that efficiency increases as droplet diameter decreases, because for the same amount of liquid used smaller droplets present greater surface area for particle collection. Smaller droplets have lower settling velocity, $V_{d/g}$, which decreases the gas sweep rate and decreases efficiency, but increases holdup which increases efficiency. Smaller droplets have a lower Stokes number, which decreases single droplet efficiency, η_d. Analysis suggests that single droplet efficiency, η_d, is maximized when droplet diameter is about 1000 μm (1 mm) for particles of all sizes. Efficiency of the entire spray tower as given in Eq (8) is maximized when droplet diameter becomes so small that $V_{d/g}$, approaches the upward velocity of the gas, $V_{g/w}$. However, because droplets that are too small are likely to be blown out the top of the tower, and because spray nozzles produce droplets with a distribution of diameters, practical considerations suggest that droplets with median diameter of about 1000 μm may be most effective.

Eq (8) must be used with discretion. It was developed using assumptions that the droplets behave in an ideal way: that none evaporate, that all are distributed evenly over the scrubber cross section, that none collide with scrubber walls or with each other, etc. In an actual scrubber, none of these assumptions is likely to hold, with the result that scrubber efficiency is likely to be lower than that predicted. Predictions from Eq (8) should be regarded as the highest efficiencies reasonably possible.

Venturi Scrubber

Analysis of particle collection in a venturi scrubber is similar to that used to describe collection in a spray tower except that flow of gas and droplets is co-current rather than counter-current. Droplets enter the venturi at the throat, then accelerate in the gas stream due to drag force from the passing gas. Eventually, if the throat is long enough, the droplets will reach the full gas velocity.



Note:
"Positive" is toward the direction of gas flow.

As before, we can understand droplet motion using the concept of relative velocities. $V_{g/w}$ is the velocity of the gas relative to the venturi wall, and is taken to be constant through the venturi throat where the cross-sectional area is constant. $V_{d/w}$ is the velocity of the drops relative to the scrubber wall; that is, the apparent velocity of the drops as seen through a window in the side of the venturi. This velocity will change along the venturi length. At the point the drops are introduced through the wall of the venturi, $V_{d/w}$ will be zero. After the drops accelerate to the full gas velocity, $V_{d/w} = V_{g/w}$.

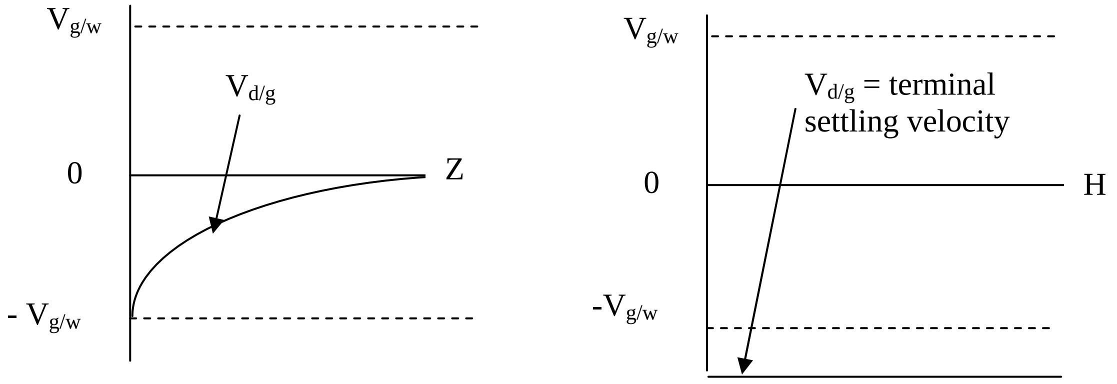
$V_{d/g}$ is the velocity of the accelerating droplets relative to the gas. At the point where the droplets are introduced, this relative velocity is highest. Once the droplets fully accelerate, they have no velocity relative to the gas and $V_{d/g}$ is zero.

These and other relationships are shown schematically in the plots below, which apply if a "positive" direction is taken as toward the right; that is, if gas flows from left to right. Each plot shows a parameter such as velocity on the vertical axis, and position along the scrubber on the horizontal axis. The left figure in each plot applies to conditions within a venturi scrubber, whereas the corresponding condition for a spray tower is on the right. Conditions in the spray tower are constant along the length of the tower, but velocities within the venturi change with length as the droplets accelerate.

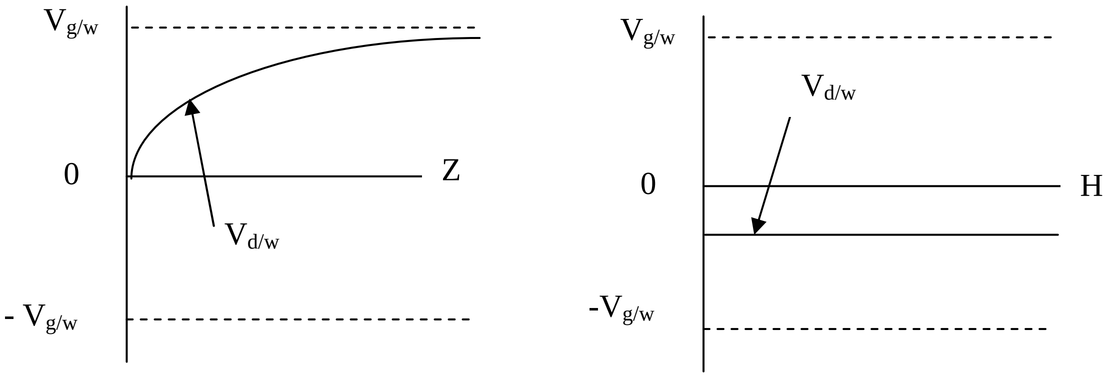
Recall that $V_{d/w} = V_{d/g} + V_{g/w}$ as given in Eq (4). In the venturi throat, $V_{g/w}$ is high and has a constant value. $V_{d/g}$ starts at $-V_{g/w}$, then increases to approach zero. Thus, $V_{d/w}$ must start at zero, then increase to reach $V_{g/w}$. From the standpoint of a particle "embedded" in the gas stream, droplets initially have a high, negative velocity; that is, they seem to the particle to move toward the left. After the droplets accelerate fully, the particles in the gas and the droplets have no relative velocity, as both move together with the gas.

Venturi **Spray Tower**

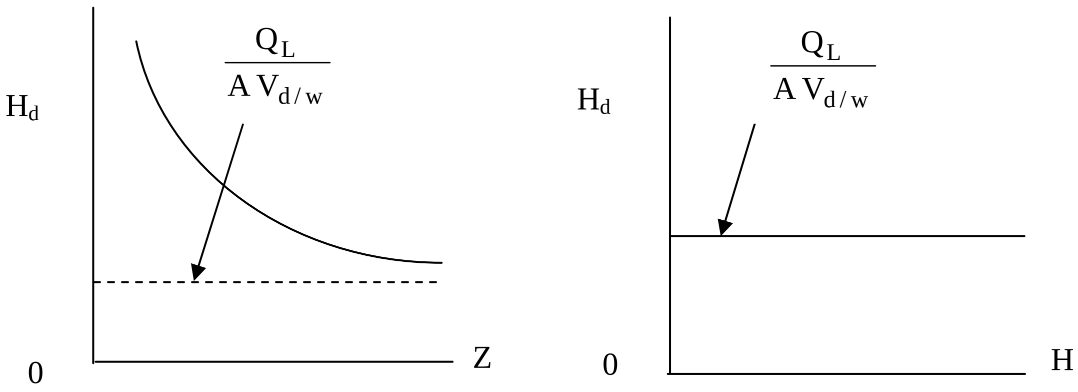
$V_{d/g}$: Velocity of the droplets relative to the gas.



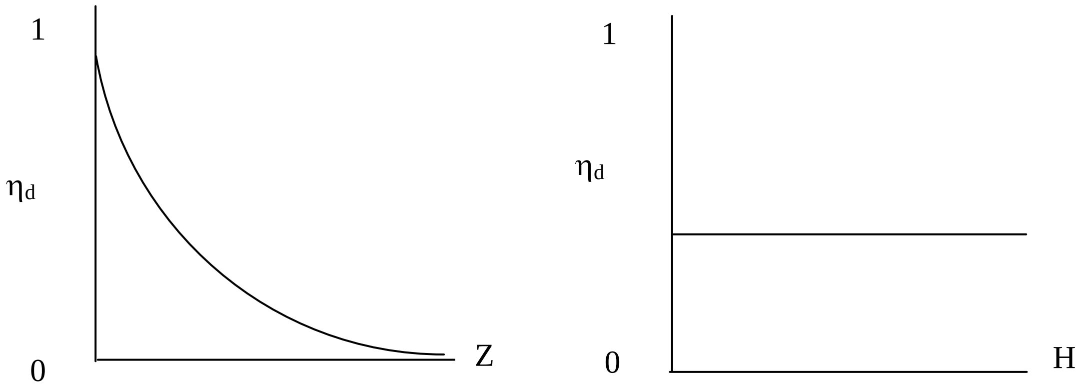
$V_{d/w}$: Velocity of the droplets relative to the scrubber wall.



H_d: Holdup of liquid in the scrubber (volume fraction comprised of droplets)



η_d: Single droplet efficiency



The equations that describe droplet motion down the venturi throat are complicated, because droplet motion is in the turbulent regime; that is, the Reynolds number for droplet motion through the gas in the venturi throat is initially much greater than unity. Several different authors have developed approaches to this problem, as described in the article by Rudnick *et al*. given on the link in our course website. Two approaches, that of Calvert, and that of Yung et al., are given in the course spreadsheet. The relevant equations for these approaches, taken from the Rudnick article, are given below.

Droplet Diameter – The diameter of droplets generated by pneumatic atomization has been studied for many years. Although several equations to predict mean droplet size are available, one by Nukiyama and Tanasawa has been widely used. This equation gives the Sauter diameter; that is, the surface-to-volume mean diameter.

$$d_d = \frac{0.0050}{\left| V_{g/w} - V_{di} \right|} + 0.918 \left(\frac{Q_L}{Q_G} \right)^{3/2} \tag{9}$$

where $V_{g/w}$ is given in meters per second, and V_{di} is the axial velocity of the droplets at the time of atomization. Droplet diameter, d_d, is given in meters in Eq (9).

Calvert Approach – Here, efficiency depends on parameters that govern droplet motion through the scrubber, plus a factor “f”, used to fit experimental data to the model. Controversy exists over the correct value of “f” to use, but in general a value of about 0.32 seems about right. The equation for penetration is given in Eq (10).

$$\mathrm{Pt} = \exp\left\{ -\frac{Q_L\,\rho_L\,d_d\,V_{g/w}}{55\,Q_g\,\mu\,K_i}\left[-2.8\ln\left(\frac{0.35+K_i\,f}{0.35}\right)-\left(\frac{0.49}{0.35+K_i\,f}\right)+1.4-4K_i\,f\right]\right\} \tag{10}$$

Here, K_i is the value of the Stokes number under conditions at the inlet to the venturi throat:

$$K_i = \frac{C_c\,\rho_p\,d^2\,V_{g/w}}{18\,\mu\,d_d} \tag{11}$$

Yung et al. Approach – The approach of Yung et al. is more complex. Here, the acceleration of droplets in the venturi throat is modeled using more realistic assumptions than used by Calvert. The equation for penetration, copied from the Rudnick et al. article, is:

$$\mathrm{Pt} = \exp\left\{ \frac{Q_L\,\rho_L}{Q_g\,\rho_g\,C_{Di}}\left(\frac{\left[\begin{array}{l} 4K(1-V^*_{de})^{1.5}+2.1(1-V^*_{de})^{0.5}- \\ \quad 3.55K^{0.5}(1-V^*_{de}+ \\ \quad 0.35/K)\tan^{-1}(K(1-V^*_{de})/0.35)^{0.5} \end{array}\right]}{0.35+K(1-V^*_{de})}\right) - \frac{\left(\begin{array}{l} 2.1+4K- \\ 3.55K^{0.5}(1+0.35/K)\tan^{-1}(K/0.35)^{0.5} \end{array}\right)}{0.35+K} \right\} \tag{12}$$

Note that the inverse tangents used in the above equations must be expressed in radians, not in degrees. The other terms in Eq (12) are:

$$V^*_{de} = 2\,(1-X^2+X\sqrt{X^2-1}) \tag{13}$$

$$X = 1+\frac{3L_t\,C_{Di}\,\rho_g}{16\,d_d\,\rho_L} \tag{14}$$

Here, L_t is the length of the throat and C_{Di} is the drag coefficient that applies for the droplets at the point of liquid injection, the beginning of the venturi throat, where $V_{d/g} = V_{g/w}$. The value of C_{Di} is determined from the "standard curve" of C_D versus Re for the droplets at the point where they are injected. If droplets are injected with a non-zero velocity, then the appropriate relative velocity between the gas and droplets at the point of injection must be used. Eqs (13) and (14) are also used to determine venturi pressure drop as described below.

The course spreadsheet includes equations for particle collection using both the Calvert approach and the approach of Yung et al. The article by Rudnick et al. compares the predictive power of the Calvert approach, the Yung et al. approach, and several other methods to predict venturi efficiency that are even more complex.

Pressure Drop – The pressure drop through a venturi scrubber is high by comparison to pressure drop across other particle collection devices. In a venturi, water drops are accelerated to the high velocity that exists in the venturi throat.

With the assumption that other sources of pressure loss are negligible compared to that required to accelerate the liquid droplets, the following equation can be developed. Consider the principle from physics that the impulse imparted to an object equals the momentum it attains. Impulse is force x time, whereas momentum is mass times velocity. Therefore,

$$F\,t = m\,V_{d/w} \tag{15}$$

If we divide both sides of Eq (15) by contact time, t, and by the cross-sectional area of the venturi throat, A, then we have an expression for pressure drop due to accelerating liquid drops.

$$\Delta P = \frac{F\,t}{A\,t} = \frac{m\,V_{d/w}}{A\ t}\,\frac{\rho_L}{\rho_L} = \left[\frac{m}{t\,\rho_L}\right]\left[\frac{V_{d/w}\,\rho_L}{A}\left(\frac{V_{g/w}}{V_{g/w}}\right)\right] \tag{16}$$

or

$$\Delta P = \left[Q_L\right]\frac{V_{d/w}\,\rho_L\,V_{g/w}}{Q_g} = \rho_L\,\frac{Q_L}{Q_g}V_{d/w}\,V_{g/w} \tag{17}$$

The highest velocity the droplets can attain relative to the wall is $V_{g/w}$ if the droplets fully accelerate to the gas velocity by the time they reach the end of the venturi throat. In practice, the droplets probably do not fully accelerate. If we define V^*_{de} such that

$$V^*_{de} = \frac{V_{d/w}}{V_{g/w}} \tag{18}$$

at the end of the venturi throat, where $V^*_{de} < 1$, then Eq (17) becomes

$$\Delta P = V^*_{de}\,\rho_L\,\frac{Q_L}{Q_g}V^2_{g/w}\ . \tag{19}$$

As a practical matter, a value of $V^*_{de} \cong 0.85$ can be used. Note that this assumption is that the drops attain 85% of the gas velocity by the time they reach the end of the venturi throat. Alternatively, V^*_{de} can be calculated from a force balance on the droplets during their acceleration with the result given in Eq (13) above.

A further refinement comes from recognizing that droplets accelerated in the throat at the cost of a certain pressure drop will return some of that pressure drop to the gas stream when the droplets decelerate in the diverging section of the venturi that follows the venturi throat. If the gas velocity in the throat is $V_{g/w}$ and the gas velocity at the end of the venturi diverging section is $V_{g/d}$ then a correction can be applied to Eq (19) for the regain in pressure drop due to droplet deceleration with the following, final result.

$$\Delta P \; = \; \rho_L \, \frac{Q_L}{Q_g} V_{g/w}^2 \; \left\{ V^*_{de} \left(1 - \frac{V_{g/d}}{V_{g/w}} \right) + \left(\frac{V_{g/d}}{V_{g/w}} \right)^2 \right\} \qquad (20)$$

The course spreadsheet includes pressure drops from Eqs (19) with $V^*_{de} = \beta$, which can be set by the user, with V^*_{de} from Eq (13), and from Eq (20) with V^*_{de} from Eq (13).

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