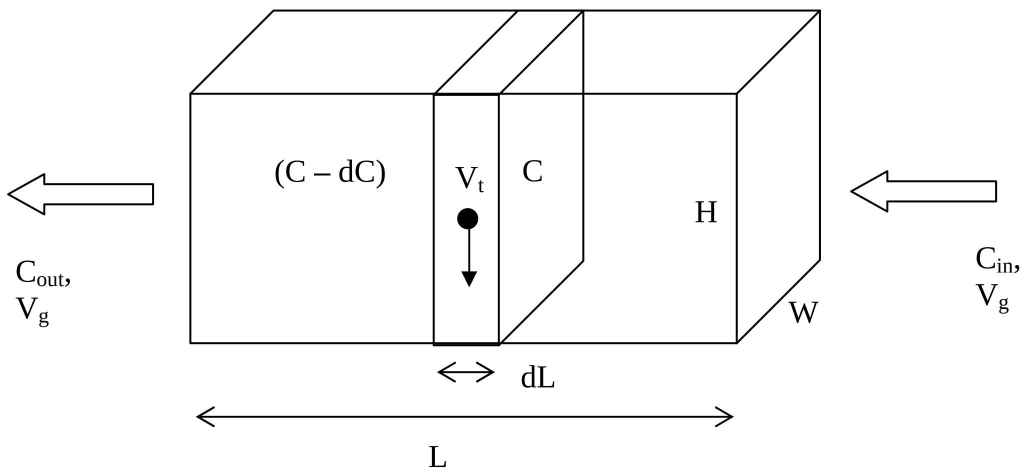
Gravitational Settling

Aerosol particles collect in a gravitational settling chamber as shown in the drawing below.



We can now develop an equation to predict the collection efficiency of the settling chamber with the help of the following assumptions:

1. Particle concentration, C, is uniform in any plane perpendicular to the direction of gas flow,

2. Gas velocity, V_g, is uniform throughout,

3. Particles are all at their terminal settling velocity, V_t,

4. No particles reentrain from the bottom of the chamber,

5. No particles interact with each other,

6. the “positive” direction is to the left and upwards.

A balance for the rate that particles flow through a differential volume with height H, width W, and length dL gives:

Rate In = Rate Out + Accumulation Rate

$$V_g\ H\ W\ C = V_g\ H\ W\ (C - dC) + (-V_t\ W\ dL\ C) \qquad (1)$$

Note that the “accumulation” rate is actually negative, as particles are actually depleted from the differential volume. Because the positive direction is defined as upwards, the terminal velocity carries a negative sign to show that particles move downward.

Integration yields

$$\int_{C_{in}}^{C_{out}} \frac{dC}{C} = \int_0^L -\frac{V_t\,dL}{V_g\,H}, \tag{2}$$

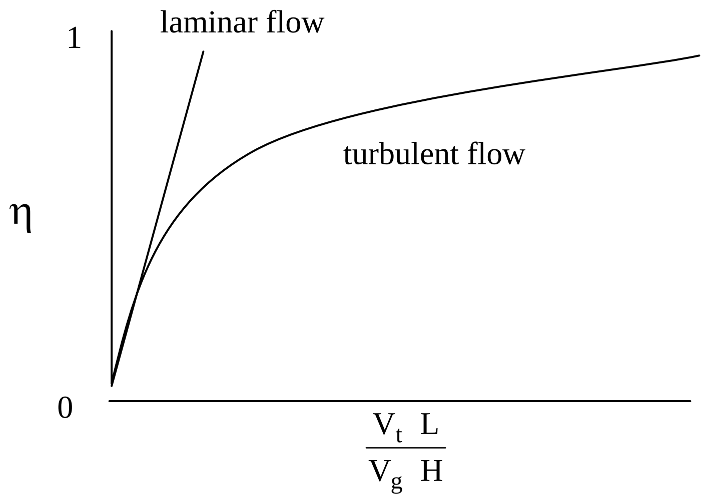
or

$$Pt = \frac{C_{out}}{C_{in}} = \exp\left[-\frac{V_t\,L}{V_g\,H}\right] \quad , \tag{3}$$

where Pt is fractional penetration. Because efficiency, η, equals 1 – Pt,

$$\eta = 1 - \exp\left[-\frac{V_t\,L}{V_g\,H}\right] \tag{4}$$

Efficiency, η, increases as the term in [ ] in Equation (4) increases; that is, as the chamber becomes longer and shorter, as particle terminal velocity increases and as gas velocity decreases.



How do you find values of particle terminal velocity? You can calculate these values remembering that Stokes's Law holds for particles up to about 70 μm in diameter, but for larger particles you must use the more complex approach that involves $CdRe^2$. You can also look up values in a table or from a plot, if one is available, as long as the conditions used to give the results in the plot are similar to your conditions.

The table below lists the time for particles of various sizes to settle a distance of 10 cm. From these data you can see that settling is very effective for larger particles, but is relatively unimportant for particles smaller than a few tens of micrometers in aerodynamic diameter.

Aerodynamic Diameter, μm	Settling Velocity, cm/s	Time to settle 10 cm
1	0.0030	55 minutes
5	0.076	2.2 minutes
10	0.30	33 seconds
30	2.73	3.7 seconds
50	7.57	1.32 seconds
70	14.8	0.68 seconds
100	30.3	0.33 seconds

Equation (4) above applies if the gas flow is turbulent. The first assumption in the list, that particle concentration is uniform in the direction perpendicular to gas flow, is valid if turbulence causes complete mixing of the uncollected aerosol. Whether the gas stream is, in fact, turbulent can be checked by calculating the Reynolds number for the gas flow. Note that this Reynolds number is different from the Reynolds number for particle settling through the gas. For the gas flow to be fully turbulent, the gas flow Reynolds number should be greater than about 10,000, as will ordinarily be the case for ducts the size used in industry.

In some cases, however, flow may be laminar. For example, gravitational settling will also occur in sampling tubes where velocities and tube sizes are relatively small, leading to small Reynolds numbers. In that case, the first assumption is no longer valid. Particles will "clear" from the air with a distinct boundary between the aerosol and the clean gas, as particles settle together toward the bottom of the duct. Analysis of the problem, and the relevant equations, are the same as we considered for turbulent flow, except that the concentration of particles in the air at the bottom of the duct, where they fall through the bottom of the differential volume, is always Cin, the concentration of the inlet aerosol. As a result,

$$\int_{C_{in}}^{C_{out}} dC = \int_0^L -\frac{V_t\, C_{in}\, dL}{V_g\, H} \qquad (5)$$

from which

$$Pt = \frac{C_{out}}{C_{in}} = 1 - \frac{V_t\, L}{V_g\, H} \quad \text{or} \quad \eta = \frac{V_t\, L}{V_g\, H} \qquad (6)$$

Note by comparing Equations (6) and (4) that under laminar conditions efficiency is calculated as the argument of the exponential term in the turbulent efficiency equation. Equation (6) suggests that under some conditions efficiency can be greater than unity, which is clearly impossible. The interpretation of Equation (6) should be that in this case, all particles of the given size deposit before they reach the end of the collector.

The figure above shows plot of efficiency against $\frac{V_t L}{V_g H}$ for both laminar and turbulent conditions. Note that collection efficiency under turbulent conditions is always less than it would be if the flow is laminar. This situation occurs because the particle concentration at the bottom of the differential volume is always C_{in} during laminar flow, and this concentration is always higher than the average concentration, C, within the collector itself. The effect of turbulence is to take some of the particles that are about to settle out, and mix them vertically back into the gas stream, with the result that efficiency under turbulent conditions is lower than it would be without this mixing.