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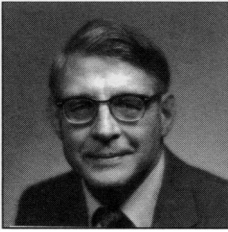
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## ***On Running in the Rain***

*Herb Bailey*



**Herb Bailey** ([herb.bailey@rose-hulman.edu](mailto:herb.bailey@rose-hulman.edu)) graduated from Rose Polytechnic Institute in 1945 and returned to teach mathematics at Rose-Hulman Institute of Technology in 1966. He retired in 1992 and has since been working on problems in mathematics and mechanics. For the past two years, he has been teaching to and learning from a dozen home schoolers ages 11 to 14. The teaching is a joy since there are no grades, tests, chairmen, deans, principals, or school boards involved.

### **Introduction**

Sally is heading home on foot without an umbrella and it is raining. Should she walk or run in order to stay as dry as possible? If she runs she will get less rain on her head and side during the trip but perhaps more on her front.

In the present note, we derive an expression for the total volume  $R$  of the rain absorbed in terms of the speed  $v_s$  of the traveler. We assume that  $v_s$  is a positive constant during the trip. It is shown that the minimum of  $R$  is either at an end point of the domain of  $v_s$  or at a point where  $R(v_s)$  does not have a derivative. This is a nice example for a calculus class to show that maximums and minimums are not always at the points where the slope of the function is zero.

This problem has doubtless been considered for many years. The first reference that we could find was posed as problem 2.1 in a 1975 book of physics problems entitled “The Flying Circus of Physics” [1]. It was later used as an example in a book on mathematical modeling [2]. Several articles in scientific journals, for example [3], [4], [5], [6], have also treated the problem. Peterson and Wallis [6] tested their calculations by traveling in a rainstorm with Wallis running and Peterson walking. The comparative weight increase of their sweat-clothes confirmed their calculations. Running in the rain has also generated a number of related websites, for example [7], [8], [9], [10], [11], [12].

All the references correctly conclude that in the case of vertical rain, or with a rain (wind) component into the face of the traveler, one should run at top speed to minimize wetness. The problem is not so simple with a component of wind at your back and is considered in [1], [2], [3], [4], [7], [8], [9], [12]. D. Craigen has an interesting site [7], [8], called DCPhysics that includes a JavaScript calculator to determine the amount of rain absorbed by a traveler. De Angelis [3] states that traveling at the same speed as that of the wind component on your back can be optimal. The other references all conclude that with wind at your back, you should always travel at a speed equal to this component of the wind speed. This advice is summarized in the following limerick [4] by D. Brown of York, England.

When caught in the rain without a mac,  
walk as fast as the wind at your back,  
but when the wind's in your face,  
the optimal pace  
is as fast as your legs can make track.

We show in fact that with wind at your back it is not always best to travel with this component of the wind speed. It is sometimes best in this case to run at top speed. This decision depends not only on the three components of rain velocity but also on the front, top and side areas of the traveler. We give examples involving two travelers starting for home together and show that the heavier one should run while the other should walk in order to stay as dry as possible.

### The equation

It is assumed that Sally travels in the  $(x, y)$ -plane along the positive  $x$ -axis. Let  $v_x$ ,  $v_y$ , and  $v_z$  be the  $x$ ,  $y$ , and  $z$  components of the rain velocity, where the  $z$ -axis is positive upward. Note that  $v_s$  is always positive,  $v_x$  and  $v_y$  may be positive or negative, and that  $v_z$  is always negative. Let  $A_x$ ,  $A_y$ , and  $A_z$  be Sally's front (or back), side (left or right), and top areas. These areas are shown in Figure 1 with Sally pictured as a rectangular prism. Let  $D$  be the distance to her home and  $\rho$  be the rain density (volume of rain per unit volume of air). Finally, let  $R_x$ ,  $R_y$ , and  $R_z$  be the rain volumes hitting areas  $A_x$ ,  $A_y$ , and  $A_z$  during her trip. Thus the total rain that she absorbs is given by  $R = R_x + R_y + R_z$ . Her goal is to minimize  $R(v_s)$ .

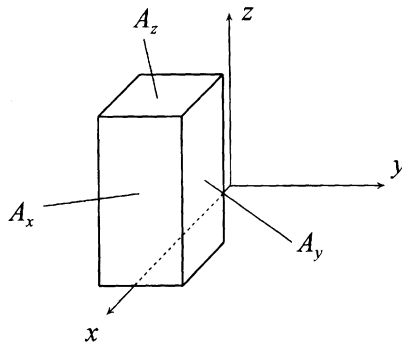


Figure 1.

The volume of rain hitting a given surface is the product of rain density, relative speed of the rain normal to the surface, area of the surface, and time. Since the trip time is  $D/v_s$ , the volumes of rain hitting her from the  $x$ ,  $y$ , and  $z$  directions are  $R_x = \rho|v_x - v_s|A_x \frac{D}{v_s}$ ,  $R_y = \rho|v_y|A_y \frac{D}{v_s}$ , and  $R_z = \rho|v_z|A_z \frac{D}{v_s}$ . Note that  $R$  is a continuous function of  $v_s$  and that  $R > 0$ . Splitting the domain of  $R$  into two parts gives

$$R(v_s) = \begin{cases} R_1(v_s) = \frac{D\rho}{v_s} [(v_x - v_s)A_x + |v_y|A_y + |v_z|A_z], & v_s \leq v_x \\ R_2(v_s) = \frac{D\rho}{v_s} [(v_s - v_x)A_x + |v_y|A_y + |v_z|A_z], & v_s > v_x. \end{cases}$$

Expanding these expressions gives

$$R_1 = D\rho \left[ -A_x + \frac{v_x A_x + |v_y|A_y + |v_z|A_z}{v_s} \right] \quad \text{and}$$

$$R_2 = D\rho \left[ A_x + \frac{-v_x A_x + |v_y|A_y + |v_z|A_z}{v_s} \right].$$

In order to find the minimum  $R$ , we consider two cases, Case I with  $v_x \leq 0$  and Case II with  $v_x > 0$ . Case II requires three sub-cases depending on  $Q$ , where  $Q = |v_y|A_y + |v_z|A_z - v_x A_x$ .

- Case I: Since  $v_x \leq 0$ ,  $R = R_2$  for all  $v_s$ . Moreover, since  $R_2$  is a decreasing function of  $v_s$  (with  $v_x \leq 0$ ) she should run at top speed.
- Case II: Since  $v_x > 0$ , we must determine the behavior of both  $R_1$  and  $R_2$ . First note that  $R_1$  is always a decreasing function of  $v_s$  and that the behavior of  $R_2$  depends on  $Q$ .
  - Case IIa:  $Q < 0$ . In this sub-case  $R_2$  is an increasing function of  $v_s$  and thus  $R$  is a minimum at  $v_s = v_x$ .
  - Case IIb:  $Q = 0$ . In this sub-case  $R_2$  remains constant for  $v_s \geq v_x$ . Thus  $R$  is minimized for any  $v_s$  such that  $v_s \geq v_x$ .
  - Case IIc:  $Q > 0$ . In this sub-case  $R_2$  is a decreasing function of  $v_s$ . Thus  $R$  is minimized when  $v_s$  is maximal.

Note for  $v_x > 0$  (wind at your back) that  $dR/dv_s$  does not exist at  $v_s = v_x$ , since the right- and left-hand derivatives are not equal. Thus if  $Q < 0$ , the minimum is at a point where the derivative does not exist. In addition, by using the expression for  $R$  with  $v_s > v_x$ , we see that the limiting value of  $R$  for large  $v_s$  is  $D\rho A_x$ . We assume that  $v_s$  is less than 30 ft/sec, since world class sprinters run at about 33 ft/sec. We also assume that  $v_x$  is less than the top speed of the traveler, that is, there is not a gale blowing at her back.

## Examples

We give four examples with reasonable values for the parameters to illustrate these results. The length units are feet and the time units are seconds. For all of the examples, let  $D = 1500$  and  $\rho = 6.42 \times 10^{-7}$  (this  $\rho$  corresponds to 0.25 inches per hour of rainfall when  $v_z = -9$  ft/sec.). The remaining values for the parameters for the four examples are:

Ex. 1:  $v_x = 1, v_y = 5, v_z = -9, A_x = 6, A_y = 4, A_z = 1$

Ex. 2:  $v_x = 5, v_y = 1, v_z = -9, A_x = 6, A_y = 4, A_z = 1$

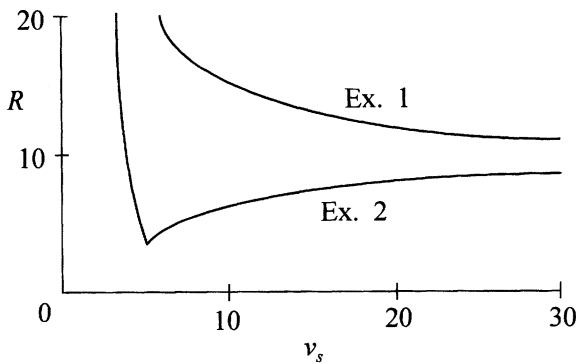


Figure 2.

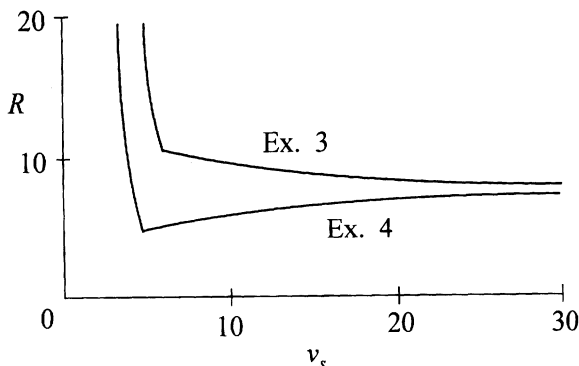


Figure 3.

Ex. 3:  $v_x = 5, v_y = 6, v_z = -9, A_x = 5, A_y = 4, A_z = 1$

Ex. 4:  $v_x = 5, v_y = 6, v_z = -9, A_x = 5, A_y = 2, A_z = 0.5$

The traveler is the same in Examples 1 and 2, the only difference being that the  $x$  and  $y$  components of rain velocity are interchanged. Figure 2 shows a plot of absorbed rain  $R$  (in cubic inches) as a function of her speed for these two examples. In Example 1, we have  $Q = |v_y|A_y + |v_z|A_z - v_x A_x > 0$  and she should head for home at top speed. In Example 2, we have  $Q < 0$  and she should travel at  $v_s = v_x = 5$ . For this example, the amount of rain absorbed is increased by more than 50% if she either halves or doubles the optimal pace.

Examples 3 and 4 (Figure 3) involve different travelers, but with the same components of rain velocity. For Example 3, we have  $Q > 0$  and the traveler should run at top speed. For the traveler in Example 4, we have  $Q < 0$  and the traveler should walk with  $v_s = v_x = 5$ . Thus with two travelers having different builds, starting at the same time, and heading to the same destination, we find that the heavier set one should run and the other should walk.

## A practical algorithm

For the traveler without calculator, laptop, tape measure, and anemometer, the following simple test will tell you the best speed. Just stand still ( $v_s = 0$ ) in the rain for a few seconds facing homeward and estimate the rain volumes  $R_x, R_y,$  and  $R_z$  on your front (or back), side, and top. If the rain hits your front or falls vertically, then  $v_x \leq 0$ , and you should run home at top speed. If  $v_x > 0$  and with  $v_s = 0$  then  $R_x = \rho v_x A_x T$ ,  $R_y = \rho |v_y| A_y T$ , and  $R_z = \rho |v_z| A_z T$ , where  $T$  is the duration of the test. In this case  $Q = (R_y + R_z - R_x)/\rho T$  and you should run at top speed if  $R_y + R_z - R_x > 0$  and walk with  $v_s = v_x$  if  $R_y + R_z - R_x < 0$ .

## Concluding remarks

We have assumed that Sally travels along the positive  $x$ -axis at constant speed and that the rain has constant velocity components  $v_x, v_y,$  and  $v_z$ . If the  $x$ -component of rain velocity is toward her face or if the rain falls vertically, then she should run at top speed to stay as dry as possible.

On the other hand, if the  $x$ -component is at her back, then her optimal pace depends on the parameter  $Q = |v_y|A_y + |v_z|A_z - v_x A_x$ , where  $A_x, A_y,$  and  $A_z$  are Sally's front

(or back), side (left or right), and top areas. If  $Q > 0$ , then her best choice is to again run at top speed. If  $Q = 0$  then her optimal pace is any speed greater than  $v_x$ . Finally, if  $Q < 0$  then she should travel at the same speed as the wind component at her back. The advice given in Brown's limerick is then improved with the following verse.

When the wind's in your face,  
Brown is right to make haste,  
but with wind at your rear,  
the best choice is less clear.  
You must measure your waist  
before setting the pace.

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