The Legacy of Augustin Cournot

Department of Economics Working Paper 99-05

James W. Friedman

Department of Economics, CB #3305
University of North Carolina
Chapel Hill, NC 27599-3305
e-mail: Jim_Friedman@unc.edu

November 16, 1999

Abstract

Broadly speaking, Cournot's legacy to economics consists of (i) showing economists how to use mathematics in a general way to develop economic theory, as opposed to using examples based on speci…c functional forms, (ii) his clear and sophisticated treatment of market demand, monopoly, competitive markets, and above all, oligopoly.

Among contemporary economists Cournot is mainly remembered only for his oligopoly theory; however, he appears to have had a formative in‡uence on Marshall and Walras as well as a lesser in‡uence on later mathematically inclined economists.

Antoine Augustin Cournot is most remembered for duopoly theory; in particular for homogeneous goods duopoly with output as the choice variable of each …rm in a model with a linear demand function and zero costs. Some economists are aware that Cournot actually analyzed n-…rm oligopoly, for arbitrary n, using both a fairly general demand function and general cost functions that di¤er from one …rm to another. His oligopoly solution concept has been seen in recent years as an early manifestation of the non-cooperative equilibrium concept of game theory due to Nash [47].

The oligopoly analysis is the tip of the iceberg of Cournot's contribution in Recherches sur les Principes Mathématiques de la Théorie des Richesses. Put brie‡y, Cournot [14] taught us how to use mathematics in economics and gave the …rst clear-cut, accurate analyses of the basic partial equilibrium markets, along with a well executed comparative statics analysis of the taxation of a monopoly. For monopoly and perfect competition Cournot's ideas and presentation re…ned the writings of his predecessors by their clarity and precision, but his treatment of oligopoly has, as far as I am aware, no predecessors. My purpose in the present essay is to look more closely at Cournot's e¤ect on and contributions to economics from a historical perspective.

For many years Cournot's work was ignored and, by the time of his death in 1877, he was apparently unaware of any in‡uence on economists. However, Cournot [14] started in‡uencing some leading economists around the 1860's. As mathematics was not generally integrated into the mainstream language and analysis of economics until the second half of the twentieth century, he was not widely known among economists, except for the prominence of his oligopoly theory in the 1920's and 1930's when much attention was being given to imperfectly competitive markets.

Some say that history should be rewritten from generation to generation, not because the facts change, but because new generations will …nd a history written from their own vantage point to be more illuminating and interesting in the light of their particular interests and background. It is in such a spirit that I write this essay. An evaluation of Cournot at the present time takes a much fuller cognizance of his

work as a precursor to non-cooperative game theory than would an evaluation even a quarter century ago. And an evaluation by someone whose professional career has been in game theory and its applications to economics, particularly its applications to oligopoly theory, may have a perspective that is di¤erent from that of an historian of economic thought or even of another theorist who does not use game theory extensively.[1] Thus this essay is a very personal appreciation of Cournot that contains no new facts. It would be disingenuous of me not to say that Cournot has been one of my greatest intellectual heroes in economics since …rst making his acquaintance forty years ago as a student of William Fellner whose book on oligopoly [25] was a great in‡uence on me. Furthermore, my view has been colored by historians of economic thought such as Schumpeter [56], Blaug [6], and Leonard [39].

Section 1 sketches the background into which Cournot's book …ts by brie‡y reviewing the use of mathematics in economics from the eighteenth through the mid-twentieth centuries. After placing his contribution in this context, Section 2 looks at Cournot's in‡uence on the early pioneers of mathematical economics among mainstream economists; people such as Marshall, Jevons, Walras, and Edgeworth. Then Section 3 examines the reception and fate of Cournot's oligopoly theory at the hands of Bertrand, Edgeworth, Fisher, and Wicksell, and its connection to game theory as seen through contemporary eyes. Conclusions appear in Section 4.

1 Economic Theory and Mathematical Economics

Over the past several decades the term mathematical economics has been falling into disuse. In the late 1950's most economic theorists did not use mathematics in their writings; a few mathematical economists did. Mathematics was neither required nor

[1] In a …ne, recent article Roger Myerson [46] places Cournot quite accurately among precursors of game theory; however, his focus is not on Cournot, but on Nash and the non-cooperative (Nash) equilibrium. Consequently the treatment of Cournot is brief and focused on the oligopoly equilibrium. Martin Shubik [58] is a brief biographical sketch which makes clear Cournot's role as the father of mathematical economics.

used in the core courses of many, probably most, of the best doctoral programs of that time.[2] In a 1947 review of a book aimed at teaching basic mathematics to economists Jacob Marschak [41] uses much of his space to argue the virtues of rudimentary mathematical education for economists. The tone of the article makes abundantly clear that his is a minority position. Indeed part of his review was prompted by a contemporary article of J. M. Clark [9] asserting that the "mathematical economists" owed it to the rest of the discipline to express their results in non-technical language. There was no hint that perhaps the rest of the discipline might consider learning enough mathematics to understand the mathematical economists. Since then it has become almost inconceivable to do research in economic theory without using mathematics and basic required courses for doctoral students now use mathematics routinely that would have been considered advanced …fty years ago; consequently, the distinction between the theorist and the mathematical economist no longer exists. As a conscious distinction it probably goes back to roughly 1870. As is well-known, Léon Walras [64] and Stanley Jevons [37] thought of themselves as mathematical economists as did Irving Fisher [28], who induced his wife's brother in law to translate Cournot [14] as Cournot [13] in 1897 and included with it a bibliography of mathematical economics which updated the one compiled by Jevons [37].

1.1 Mainstream Economics 1776 to 1848

In the early nineteenth century the great lights of economics included Adam Smith [59], Jean-Baptiste Say [55], David Ricardo [50], François Quesnay [49], and John Stuart Mill [44]. These were non-mathematical writers who did not attempt to con…ne

[2] In my own doctoral education at Yale, 1959 to 1962, the main sources in microeconomics were Hicks [34] (excluding mathematical appendices), Marshall [42] (excluding footnotes and appendices), along with the dauntingly di¢cult and modern Samuelson [54]. At about the same time at the University of Michigan the recently published book of Henderson and Quandt [33] was used as a textbook for an elective, advanced doctoral course in mathematical economics. I recall in the early 1960's hearing Bob Solow remark that the MIT undergraduates had better preparation in mathematics than the graduate students. The undergraduates in those days would all have had three semesters of calculus followed by one semester of di¤erential equations.

economics within the precision of mathematics. Ricardo made some use of numerical examples which smacked of mathematical theory using speci…c functional forms, but he did not appear to be building a consciously mathematical apparatus. Of course, there were a few mathematical writers as one can easily see by reading Fisher's bibliography or examining books such as Baumol and Goldfeld [3] or Theocharis [61]; however, Cournot towers above his mathematically inclined predecessors and contemporaries.[3]

In continental Europe during this time economics was often combined into faculties that included law. The natural mode of analysis and thought in law would work against the mathematization of economics. In England economics was a separate discipline from law and other topics. The Smith, Ricardo, Mill, Marshall traditions held sway and, although Marshall was a mathematics student who formulated his economics using mathematics, he relegated it to footnotes and appendices. In general, students and faculty in economics did not think they needed mathematics and they did not use it. Through the …rst half of the twentieth century the situation remained much the same. John Hicks could be added to the tradition; like Marshall, he wrote an in‡uential book [34] using a literary style in the text and supplementing this with a mathematical appendix.

1.2 19th Century Mathematical Economics

Cournot was born in 1801, trained at the Ecole Normale Supérieure, and earned his doctorate in 1829 with a dissertation in physics.[4] Here, then, we have a man trained in mathematics and physics whose primary knowledge of the application of mathematics to another discipline is the application to physics where theory is largely based on speci…c functional forms. Speci…c functional forms for basic economic relations are

[3] Among contemporary writers on mathematical economics, Dupuit should be singled out for [20] and other writings. He was of the …rst rank, but, in my view, a somewhat lesser …gure than Cournot.

[4] Edgeworth [23], Guitton [32], Nichol [48], and Shubik [58] are brief biographies. Moore [45] provides an extended picture based, in great measure, on Cournot [12].

not the norm; we do not know enough, for example, to give a speci…c form to a demand function, although we have evidence to suggest they are usually downward sloping. It was very imaginative of Cournot to bring the general mathematical approach, not used in physics, to the literary discipline of economics.

He was not a mainstream economist; his career was primarily outside of economics and the economists of his time paid little attention to him.[5] The …rst important economists to notice Cournot favorably and to publish works of theory using mathematics in an integral way were Jevons [37] and Walras [64] in the early 1870's. Mathematical writing remained in the minority for at least seventy …ve more years, although it steadily gained adherents; it became the mainstream mode in the 1960's and 1970's.

There had been some prior uses of mathematics in economics; however, the best of it relied on speci…c functional forms and, as a result, had no generality.[6] Perhaps the …nest work in mathematical economics prior to Cournot's birth was Bernoulli [4], published 1738, exactly a century before to Cournot [14], who presents and solves the St. Petersburg Paradox by postulating a logarithmic utility function. It is an insightful and brilliant paper and a stunning precursor to von Neumann-Morgenstern utility. Compared with Cournot [14], the scope of the paper is far less and the level of generality of the treatment of nearly any topic in Cournot is far greater. As we know now, all Bernoulli needed was an increasing, concave utility function. For the most part, other early mathematical theory does not come close to the Bernoulli standard.

Nearer to Cournot's time, von Thünen stands out as an important thinker; von Thünen [63] is a landmark in the development of mathematical economics. Schumpeter [56, page 466] credits von Thünen [63] as the …rst to use calculus in economic

[5] Apparently Cherriman [8] is the …rst review of Cournot [10]. See Dimand [18] and [19] for an appraisal of Cherriman and a reprint of his review. Theocharis [62] cites several notices and reviews of Cournot [10] prior to 1870.

[6] See Baumol and Goldfeld [3] for examples. Schumpeter [56, especially pages 954-963] surveys this terrain.

reasoning.[7]

As of 1870 there was probably no book in existence that presented economic theory using mathematics and that was written by someone recognized as a mainstream economist in his time. The mathematical economists of the time were such as the mathematician Cournot, the engineer Dupuit [20], the cleric Whewell [65], and the engineer Fleeming Jenkin [36]. Dupuit stands out to me as the greatest mathematical economist after Cournot and, with Cournot, the only one of …rst rank prior to Jevons, Walras, and the others great economists of the generation after Cournot.

Cournot had the great insight to see that economic relationships can be treated mathematically with functions that are relatively general and subject to mostly qualitative restrictions. He broke away from the speci…c functional form tradition of physics. At the same time, this mathematician with a physics doctorate knew and understood economics well enough to get his economics right. Consequently, what he wrote is a beacon to those with the background to understand. They can read him on demand, monopoly, or competitive markets and see that the mathematics is correctly, clearly, and succinctly expressing economics they recognize as substantively correct. In comparison with his contemporaries and his predecessors, for the most part either they used speci…c functional forms (e.g., Bernoulli [4]) or they translated literary text into mathematical language with little or no subsequent analysis (e.g., Whewell [65] and Jenkin [36]). Dupuit was an exception; he, like Cournot, engaged in economic analysis using mathematical structures. In Dupuit [20] the analysis is graphical; however, it is clear and exceedingly original. Cournot seems to me the greater of the two because he used calculus, derived new results with it, and the scope and originality of his achievements are distinctly larger.

[7] An English translation of [63] can be found in Dempsey [17].

1.3 Cournot's Achievement

The great general treatise on economics at the turn of the nineteenth century is Smith's Wealth of Nations [59]. It is instructive to compare Cournot's treatment of the concept of demand with that of Smith. Smith [59, chapter VII] has a clear concept of long run equilibrium price, which he calls natural price, and of demand at the long run equilibrium, which he calls e¤ectual demand; however, the concept of the demand function is not clearly formulated. That he has some sense of it is clear from his intuitive extended discussion of the dynamics of the market when out of equilibrium. At the same time, he is capable of writing in the demand chapter (page 78) "The price of monopoly is upon every occasion the highest which can be got." At best this is sloppy; at worst it shows the lack of a clear sense of the (modern) meaning of demand function. Cournot surely read Smith and probably others and could see a rigorous, clear, modern concept of the demand function underneath their murky prose. He then provides a proper, clear de…nition of the demand function. Cournot writes at the start of Chapter IV, Of the Law of Demand,

> Pour asseoir les fondements de la théorie des valeurs échangeables, nous ne remonterons pas avec la plupart des écrivans spéculatifs jusqu'au berceau de l'espèce humaine; nous n'entreprendons d'expliquer ni l'origine de la propriété, ni celle de l'échange ou de la division du travail. Tout cela appartient sans doute à l'histoire de l'homme, mais n'est d'aucune in‡uence sur une théorie qui ne peut devenir applicable qu'à une époque de civilisation très-avancée, à une époque où (pour parler le langage des géomètres) la part d'action des circonstances initiales est entièrement éteinte.

From here the concept of demand is discussed with crystalline clarity and then de…ned:

> Admettons donc que le débit ou la demande annuelle D est, pour chaque denrée, une fonction particulière F(p) du prix p de cette denrée.

> Connaître la forme de cette fonction, ce serait connaître ce que nous appelons la loi de la demande our du débit.

Cournot is building on the past and giving great clarity and precision to something that earlier writers understood in only a rather rough and intuitive way. Next is a cogent discussion of the reasons for avoiding speci…c functional forms after which the demand function, F, is de…ned to be continuous, monotone decreasing, bounded, and $pF(p)$ is speci…cally assumed to go to zero as $p \to 1$. As Schumpeter [56, page 960] eloquently says

> In order to rate at its full value Cournot's performance it is necessary to remind ourselves that at the time 'literary economists' experienced the greatest di¢culty in formulating the simple relation that became so familiar as 'Marshall's demand curve'; that, if we neglect Verri's forgotten contribution, Cournot created the theory of it; and that his treatment of monopoly was an even more striking feat of the same type, for nobody had anything useful to say on monopolistic pricing until Marshall published his masterly version of Cournot's theory.

One of the strongest overall impressions in reading Cournot [14] is the modernity of the book. He writes in a straightforward, clear style. Concepts and notation are properly de…ned; the underlying economics is completely sound. The heart of the book consists of Chapters IV through VIII, containing the explication of demand theory (Chapter IV), the theory of monopoly (Chapter V), taxation of a monopolist and the related comparative statics e¤ect of a tax on the equilibrium price (Chapter VI), oligopoly (Chapter VII), and perfect competition (Chapter VIII). The level, quality, and nature of his exposition is not below that used with undergraduates in American universities at present.

Cournot's analysis was partial equilibrium. He recognized this and, in Chapter XI, turns to the consideration of national income (revenue social), where he opens the

discussion by saying he has dealt in partial equilibrium "mais, en réalité, le système économique est un ensemble dont toutes les parties se tiennent et réagissent les unes sur les autres." He concludes that general equilibrium analysis is called for, but "ceci surpasserait les forces de l'analyse mathématique et de nos méthodes pratiques de calcul, quand même toutes les valeurs des constantes pourraient être numériquement assignées."

2 The First Wave of In‡uence

Although Cournot had a generally successful career, his economic writings brought him virtually no recognition in his lifetime. Following Bertrand's [5] famous book review where Cournot is reviled, the oligopoly chapter is held up to particular ridicule, and Bertrand sketches his famous price choice example, Cournot began to get attention for the oligopoly theory and he has been mainly famous for that ever since.[8] However, there was an earlier wave of appreciation and in‡uence on Léon Walras, Stanley Jevons, Francis Edgeworth, and Alfred Marshall. Thus Cournot was not unknown among some of the greatest economists of the second half of the nineteenth century. His in‡uence on them appears to have been broad and basic, because he showed them that mathematics was the natural and most e¤ective vehicle for the development and expression of economic theory. Irving Fisher [26] and [27] provides a review and evaluation of Cournot's work and in‡uence along with a picture of the degree of acceptance and activity in mathematical economics in 1898 and 1938, respectively. René Roy [51] reviews Cournot's place in mathematical economics and Roy [52] deals more broadly with his intellectual life and writing.

Cournot's method for demonstrating the potential of mathematics for economics was not to preach, but to lead by example. His contribution to demand was sketched above. His treatment of standard monopoly has not been improved upon in the

[8] For over a century there was no English translation of Bertrand [5]. There is now one by me in [16] and one by Jean Magnan de Bornier in [40].

more than 160 years since he wrote it. It forms the basis of the standard textbook exposition of our day. He shows us how to do comparative statics analysis by deriving the e¤ect on monopolistic equilibrium of the imposition of a tax. He provides the …rst analysis of oligopoly, showing that it di¤ers materially from both monopoly and competition. This treatment, like that of monopoly, remains a clear and basic treatment that is endlessly repeated in current textbooks. Indeed, I am sure many examples can be found where current textbooks provide an exposition much inferior to Cournot's. And he gave us an analysis of competitive markets of a quality with the other topics mentioned above. The …rst mainstream "mathematical economists," Marshall, Jevons, Walras, and Edgeworth, read Cournot and surely recognized that he was expressing economic theory mostly correctly and with a crystalline clarity that had not previously been seen. Only the oligopoly theory, which will be separately discussed in Section 3, was controversial. The rest dealt with mainstream economics and presented no surprises in terms of content. The form was novel.

On claims that Marshall made concerning what he knew on his own prior to the publication of Jevons [37], Walras [64], and Fleeming Jenkin [36], Whitaker, the chronicler of the early, unpublished writings of Marshall [43, pages 38-39], quotes from a letter written by Marshall to Walras in 1883,

> I cannot be said to have accepted Mr Jevons doctrine of '…nal utility'. For I had taught it publicly in lectures at Cambridge before his book appeared. I had indeed used another name viz: 'terminal value-in-use.' But following the lead of Cournot I had anticipated all the central points of Jevons book and had in many respects gone beyond him.

A little further on Whitaker [43, pages 44-45] says "There seems little reason to doubt Marshall's frequent claims that, to some extent under Cournot's in‡uence, he had developed the essential notions of demand-supply analysis and consumer and producer surplus before seeing Jenkin's 1870 paper in Recess Studies." [Jenkin [36].]

Walras [64, page 37] wrote "I am indebted to my father, Auguste Walras, for the fundamental principles of my economic doctrine; and to Augustin Cournot for the idea of using the calculus of functions in the elaboration of this doctrine." Thus both Marshall and Walras would have come upon Cournot at a formative time in their development and must have had the direction of their thought in‡uenced by him. Others, such as Jevons, Edgeworth, and Irving Fisher apparently found Cournot after the main lines of their education and professional development were set and had encouragement from his work.

Jevons [37] writes in the preface to the second edition (1879) of Cournot [14] "Cournot [took] the palpable facts known concerning the relations of price, production and consumption of commodities, he investigated these relations analytically and diagraphically with a power and felicity which leaves little to be desired. This work must occupy a remarkable position in the history of the subject. It is strange that it should have remained for me among Englishmen to discover its value."[9] As to Jevons' view of the proper role of mathematics in economics, in the same preface (page 52) he writes "I contend that all economic writers must be mathematical so far as they are scienti…c at all, because they treat of economic quantities, and the relations of such quantities, and all quantities and relations of quantities come within the scope of mathematics." Further insight into the intellectual culture of the economics profession in the 1890's comes from the 25 year old Irving Fisher [28, page 109] writing in 1892. "Has the mathematical method attained a …rm footing? Before Jevons all the many attempts at mathematical treatment fell ‡at. Every writer su¤ered complete oblivion until Jevons unearthed their volumes in his bibliography. Cournot certainly, Gossen possibly, now exert considerable in‡uence on economic thought." Fisher is not precisely correct; he was obviously unaware of Cournot's in‡uence on Marshall and Walras and of Dupuit's in‡uence on Marshall; however, their is much truth in

[9] Of course Jevons was not aware that Marshall had come upon Cournot in the 1860's. In the same passage Jevons makes clear that his discovery of Cournot is after the 1871 publication of his book.

his statement.

In Mathematical Psychics Edgeworth [21, page 83] begins the appendices to the book with a section called 'On Unnumerical Mathematics' by which he means mathematics using general functions. In defense of developing economic theory in this way he approvingly quotes Cournot's arguments and refers to him as 'the father of Mathematical Economics.' And in the Palgrave biography [23] he wrote "His Recherches is still the best statement in mathematical form of some of the highest generalizations in economic science."

3 The Second Wave of In‡uence

Cournot's oligopoly theory has excited very wide attention for over a century. He has had his champions including Amoroso [22], Edgeworth [21], and Wicksell [66] and his attackers including Bertrand [5], Edgeworth [24], and Chamberlin [7]. Fisher [28], [26, page 126], and [27] is a great fan of Cournot's, as evidenced by his arranging for the translation, Cournot [13]; however, his evaluation of the oligopoly theory in Fisher [26, page 126] is "Cournot's treatment of this problem is brilliant and suggestive, but not free from serious objections."

The controversy over Cournot's oligopoly theory stems from two things: the confusion of static and dynamic analysis and a lack of understanding of basic game theory. In current language and interpretation, Cournot postulated a particular game to represent an oligopolistic market in which n …rms were players, each …rm i had a strategy set $[0; 1)$ that was a set of possible output levels and a payo¤ function $¼_i(D) = D_iF(D) ¡ Á_i(D_i)$ where $D = (D_1; : : : ; D_n)$, F is the inverse demand function in the market, $Á_i$ is the total cost function of …rm i, and $D_i \in [0; 1)$ is the output of …rm i. The Cournot equilibrium coincides with the Nash [47] equilibrium of this game. Cournot's discussion suggests that the strategy sets of the players may not be $[0; 1)$. He discusses the …rms' behavior in a quasi-intertemporal way (If …rm 1

chooses D_1 then …rm 2 would choose D_2. However, if …rm 2 chooses D_2 then …rm 1 would choose D_1^0, and so forth.) Consequently, it is not clear what Cournot had in mind as the strategy sets. It is likely that he did not think with great precision about this issue.

3.1 Cournot as Nash's Precursor

Just as Adam Smith vaguely perceived the demand function but could not actually see it, Cournot vaguely perceived the Nash equilibrium, but did not quite see it. As Roger Myerson [46, page 1070] nicely puts it "to give him [Cournot] credit for the fundamental solution concept of noncooperative game theory would be to confuse one application of a methodology with its general formulation." Even more, the notion of strategic independence, that is, that each player can legitimately be regarded as choosing a strategy independent of the strategy choices of the other players, because a strategy speci…es what to do in every situation the player will encounter, is not recognized by Cournot. If Cournot had recognized this he surely would have articulated it and would have treated the reaction function discussion as a poetic …ction. Instead Cournot treats seriously his illegitimate pseudo-dynamic analysis as did a host of subsequent writers. Robert Leonard [39] eloquently makes the point that Cournot did not see the Cournot equilibrium the way we see it now; that he did not think in terms of a single-shot market situation in which his equilibrium is the obvious non-cooperative outcome. Instead it took the crisp clarity of game theory along with retrospective reinterpretation of Cournot to see precisely what Cournot ought to have meant and what we want for him to mean.

3.2 Early E¤ect of the Bertrand Critique

As we have seen, Edgeworth appreciated and respected Cournot even after deciding Cournot's oligopoly theory was ‡awed. Prior to the Bertrand review, Edgeworth [21, page 47] wrote in 1881

> Starting with complete monopoly, we shall …nd the price continually diminish as the number of monopolists increases, until the point of complete ‡uidity is reached. This gradual 'extinction' of the in‡uence of monopoly is well traced by Cournot in a discussion masterly, but limited by a particular condition, which may be called uniformity of price, not (it is submitted) abstractedly necessary in cases of imperfect competition.

After the Bertrand review Edgeworth changed his position.

The famous review by Bertrand [5] was written positioned on a saddle point that was a maximum of arrogance and a minimum of knowledge. Though an accomplished mathematician, Bertrand appears to have known nothing of economics. In a scathing review that reviled Cournot [14], he singled out the oligopoly chapter, the most original gem of a strikingly original and deep monograph, for special contempt. He says oligopolists will collude and, supposing they do not collude, then Cournot got the equilibrium wrong due to having erred in switching from price to quantity as the choice variable. Next he proceeds to an analysis of the linear demand, zero cost duopoly example using prices. Cournot clearly makes a conscious choice in Chapter VII to switch to outputs. He writes (page 59) "Au lieu de poser, comme précédemment, $D = F(p)$, il nous sera commode d'employer ici la notation inverse $p = f(D)$" which sounds very purposeful and which follows an earlier statement that there can be only one price in the market. This switch of variables is justi…ed by Cournot only for convenience and, later on he speaks of changing output (i.e., sales) by changing price; a contradiction of the notion of output as the choice variable. He does this in the context of an illegitimate pseudo-dynamic argument.

In his Lectures on Political Economy, published …rst in 1898 and subsequently revised twice, Wicksell [66, page 97, footnote] makes a point to disagree with Edgeworth's [24] dismissal of Cournot's oligopoly theory. He is not willing to totally endorse it, but concludes that the price model of Edgeworth and Bertrand is more problematic than that of Cournot. Speci…cally, he says the Bertrand-Edgeworth ob-

jection that outputs of rivals should not be taken as constant by a …rm in calculating its best output is valid, but much less objectionable than the parallel assumption about prices. Wicksell is repeating the failure to see the Cournot model as a single-shot game with one-time simultaneous output choice. To have one …rm choose its output reactively to choices of others requires either a sequential decision structure as in Stackelberg [60] or an intertemporal game in which players choose output levels in a sequence of time periods as in a repeated game (see, for example Abreu [1], Friedman [29], or Fudenberg and Maskin [31]).

3.3 Evaluation of the Bertrand Critique

Returning to Bertrand's criticisms, many economists, notable among them Chamberlin [7], thought that oligopolists would necessarily collude. From our present perspective it is easy to dismiss this criticism using a conventional game theoretic analysis, but to do this is an injustice to Chamberlin and misses something fundamental in the thinking of economists over most of the past. Intertemporal analysis is virtually absent from formal economic theory prior to about …fty years ago. Exceptions are capital theory which is impossible to treat other than intertemporally and a few marginal excursions into two period analysis such as Fisher's time preference. At the same time, informal intertemporal arguments abound. Often their aim is to justify an essentially static, timeless equilibrium. Competitive equilibrium is often discussed in these terms; if there is excess demand, then price is argued to rise, amount demanded to fall, and amount supplied to rise until equilibrium is reached. Such an informal argument reinforces the equilibrium nature of the equilibrating price and helps to convince the reader that equilibrium prices are to be expected.

Cournot, similarly, used an ad hoc dynamic argument to the same end in his reaction function diagrams, …gures 2 and 3. These arguments tempt the reader to think of them as literal intertemporal actions taking place in real time. In the competitive market case the view that all participants take prices as given and do not think that

future prices may di¤er has been widely accepted. However, when a market is driven by the actions of two …rms, it becomes implausible that when one …rm changes the quantity it o¤ers to the market that the rival …rm will not want to make some adjustment. And it is reasonable to believe that each …rm will try to guess what to expect from the other …rm. Indeed when such a path of inquiry is followed, one can arrive at the folk theorem results that have had prominence in recent decades. The instincts of Chamberlin and others who believed that oligopolists would necessarily collude are instincts that, when formalized, would lead in this direction.

I raise this not to say that Chamberlin was right and Cournot wrong, but to say that, at bottom, the Chamberlinian criticism of Cournot has its roots in something important. The point is that Chamberlin has implicitly in mind an interesting game that is di¤erent from the interesting game that Cournot analyzed. Insofar as we stay in a single-shot game context, Cournot is largely correct. He can be faulted where he talks about adjusting a price and for indulging in the reaction function/stability discussion as if there is a real time process going on, but the basic analysis is correct.

We now realize that understanding collusion in the absence of legally binding agreements requires working with the non-cooperative equilibrium, the precursor of which is Cournot's equilibrium; without an institution to enforce binding agreements, the players must choose a Nash equilibrium that supports payo¤s that do not coincide with single-shot equilibrium payo¤s. Typically, we have in mind payo¤s on the payo¤ possibility frontier when thinking about collusive outcomes supported by (subgame perfect) non-cooperative equilibria. This line of development is due mainly to Friedman [29], Selten [57], Rubinstein [53], and Fudenberg and Maskin [31]; it relies on Selten's concept of perfect equilibrium which is a much more subtle concept than Nash equilibrium which it re…nes and it explicitly incorporates time and intertemporal payo¤ maximization. All of this is much beyond the scope of Cournot's oligopoly theory and this work was done over a century later utilizing many important developments in economic theory that had come in the interim.

On Bertrand's contention that price is the appropriate decision variable, for many years that was the general viewpoint among economists. The defenders of Cournot would argue that the essence of oligopolistic interaction is elegantly captured by the Cournot model despite use of the wrong decision variable. After the di¤erentiated products literature got going with Hotelling [35] and Chamberlin [7] this point had more force. Di¤erentiated products seemed to wipe out the implausible discontinuities that drive the Bertrand example. While the discontinuities are theoretically clear and present, real markets do not appear to exhibit them; thus di¤erentiated products provides a …ne theoretical basis for their absence and it permits one to go back to Cournot to …nd the most simple model that shows a "reasonable" looking oligopolistic interaction.

However, even the notion that price is the correct decision variable does not stand up to careful scrutiny. If you think about what "correct decision variable" should mean I think you must start by acknowledging that any …rm must, of necessity, choose both the amount it will produce and the price at which it o¤ers output to the market. It must also be recognized that a …rm's sales and its output are not the same, even if they turn out to be the same in equilibrium. That is, the …rm produces, o¤ers its produce at some price, and sells whatever the market will take at that price, up to a maximum of the amount produced. A complete model would have …rms choosing output and price and then would postulate how sales and pro…ts are related to the output and price vectors. If price is the correct decision variable, then such a complete model would, at equilibrium, have output equal to sales for all …rms and would have the equilibrium prices equal to what the equilibrium prices would be in a conventional model where prices were the only decision variables.

Such an analysis was done and it turns out that either one of the two variables can be the "correct" decision variable. Part of this was …rst seen by Kreps and Scheinkman [38] in a linear-quadratic homogeneous goods duopoly where output is chosen and made public. After that prices are selected. The Cournot quantity equi-

librium prevails. Benassy [2] and Friedman [30] carry out an analysis that di¤ers from Kreps and Scheinkman by using a di¤erentiated products model, an arbitrary number of …rms, much more general functional forms, and much more general treatment of what happens to consumers who want to buy from one …rm, but …nd that …rm sold out (i.e., of contingent demand). They …nd that if the two variables are chosen sequentially with the choices of the …rst variable revealed before the second is selected, then the one chosen …rst totally drives the model strategically. Thus, if output is chosen …rst, made common knowledge, and then prices are selected, the equilibrium prices and outputs are the same as in an output-only model. These results can be interpreted as saying that the variable chosen …rst is less ‡exible and, as a consequence, its choice drives the equilibrium in the strategic sense.

In 1897 Edgeworth [24] gave a much fuller and more complete analysis of the price choice, homogeneous goods duopoly model initiated by Bertrand and, when collecting and reprinting much of his output in 1925 wrote in an added introductory comment (page 111) "Still in 1897 much of Cournot's construction remained standing; Now the demolition of Cournot's theory is generally accepted." This judgment of Edgeworth does not stand the test of time. Even the discontinuity that the Bertrand example requires at the equilibrium price pro…le appears to be an artifact of the linear demand, zero marginal cost assumptions (see Dastidar [15]). In my view the judgment of Edgeworth is the opposite of correct. The Cournot oligopoly model is, for all practical purposes, totally rehabilitated and the Cournot equilibrium anticipates the Nash equilibrium.

4 Conclusions

At a personal level Cournot experienced disappointment with the 1838 book. It attracted no visible attention and was followed in 1863 by [10] and in 1877 by [11]. Both works were non-mathematical; Cournot concluded that presenting economics

mathematically would not succeed. He was unaware that the young Alfred Marshall was learning from him, perhaps unaware that he had made a deep impression on the young Walras who would have great professional in‡uence, and that in the 1860's he was on the verge of having a great deal of direct in‡uence and of being an inspiration to a rising generation of mathematical economic theorists. He made the case for using mathematics in economic theory by the most potent method; he did it in a …rst rate way.

Among the …ne contributions in Cournot [14] the oligopoly analysis stands out for its complete and striking originality and because it has assumed huge importance with the ‡owering of game theory. Robert Leonard [39, page 509] summarizes the Cournot-Nash relationship aptly: "Seeing Cournot as a precursor, and Nash as a successor, required that both be interpreted in a manner that rendered them consonant, so that, even though Nash had never read Cournot, we could still thereafter speak of the Cournot-Nash equilibrium."

The oligopoly analysis alone would give Cournot immortality; however, my own judgment is that the use of general mathematical techniques, without resort to spe-ci…c functional forms, to prove general propositions about economic relations is the greatest contribution he made. It is crucial that the economics he developed was, on the whole, sound economics and that some of it expressed results that were both central and were along lines that had been previously regarded as true.

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