Incentive Auctions and Information Revelation[1]

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Abstract

We study an incentive auction where multiple principals bid for the exclusive services, or effort, of a single agent. Each principal has private information about her valuation for these services and the agent has private information about his disutility of providing them. We characterize the equilibrium of this auction and examine the agent's incentives to reveal information about his type. We show that the effort level taken by the agent is smaller than in the standard auction for a known agent type and greater than in the single-principal, single-agent model.

1 Introduction

We examine a model where several principals bid for the exclusive services, or effort, of a single agent. Each principal has private information about her valuation for these services and the agent has private information about his disutility, or cost, of providing them. To compete for the agent, the principals offer sealed bids consisting of menus of incentive contracts. The agent then chooses the best contract offered by the principals. We characterize the equilibrium of this auction and examine the agent's incentives to reveal information about his type. Applications that fit this model include: Jurisdictions bidding to attract or keep a specific large firm, sports franchises competing for star athletes and coaches, companies using performance bonuses to attract or retain top executives, and publishers offering royalty based book contracts.

Jurisdictions bidding for firms have private information about their valuation for increased employment, reduced pollution, expansion of the tax base, plant size and the associated agglomeration effects, while the firm has private information about its profitability. Typically jurisdictions offer incentive packages that include income and sale tax reductions, employment based tax credits, land preparation and workers' training subsidies, and relaxation of existing environmental and zoning laws. For example, in 1993 Alabama won a bidding war among 34 states for a Mercedes Benz automobile factory by offering incentives worth an estimated 253 million dollars. Mercedes received county and city sales tax abatements for the cost of machinery and construction materials. If the number of employees exceeds the original target of 1500 it can also receive up to a 5% rebate on its state corporate income taxes. Furthermore, Alabama has committed to pay up to 60 million dollars in present and future worker training costs; if employment is sufficiently high this amount could be increased.[2]

Sports franchises often compete for free agent athletes; the baseball player Mike Piazza, who was signed by the New York Mets in the Fall 1998, is a typical example.

Piazza is considered one of the best catchers in the game and several teams vied for his services. The contract with the Mets includes bonuses for his ranking in the National League Most Valuable Player balloting, going from $125,000 for winning and dropping by $25,000 for each lower place. The contract also includes $100,000 for winning the World Series MVP award, $50,000 for making the All Star team, $50,000 for winning either the Gold Glove or the Silver Bat award, etc. Another typical sport example is the contract of Philip Fulmer, the head football coach at the University of Tennessee. Fulmer receives a 24% bonus over his base salary for winning the collegial national championship, 16% for winning an Alliance Bowl game, 12% for the Southeast Conference Title, and 8% for a non-Alliance bowl.

More generally, firms compete to attract and retain key employees by offering performance related rewards. For example, in a study of medium size companies in the UK the consulting firm Coopers & Lybrand (1997) shows that important factors in retaining vital employees include performance related promotion and salary increases. The employees included in the study have marketable skills and are attractive catches for other medium and large firms. Companies in the study are "evenly divided between developing their own executives (45%) and recruiting from outside (47%)." The most cited reason for success in luring an employee away from his current job are better pay and conditions, better career and promotion opportunities, and the chance of acquiring a wider range of experiences. To recruit and retain top executives, it is also typical to offer contracts that include large stock options.

In the applications we want to model, it is natural to assume that neither side of the market has all the bargaining power. For example, even if several states are competing for a plant of a single car manufacturer, it seems unlikely that the firm would be able to pick its optimal auction. Furthermore, collusive agreements among states are uncommon and difficult to enforce, despite the calls by many policy makers to put an end to bidding wars. Modeling bargaining over the auction form is interesting, but involves delicate and unresolved theoretical issues about multiparty bargaining under incomplete information. We sidestep these issues and instead analyze an analog of a first price sealed bid auction in which bids are incentive schemes. This auction divides the surplus among the principals and the agent and seems a natural first step towards understanding competition with nonlinear incentive contracts when there is private information on both sides of the market. As in the standard single-principal, single-agent model, we can think that each principal offers the agent a menu of options. Since the agent will select his most preferred option, the agent's compensation may appear largely fixed in the final contract. There are a few papers where principals with no private information compete with nonlinear contracts to attract agents. Champsaur and Rochet (1989), Spulber (1989), Ivaldi and Martimort (1994), Stole (1995) Armstrong and Vickers (1998), and Rochet and Stole (1999) examine differentiated duopoly models, where firms compete by offering a nonlinear price schedule to consumers with unknown preferences. Stole (1991), Martimort (1992), and Mezzetti (1997) examine common agency models, where the agent has private information and can work for both principals simultaneously. Biglaiser and Mezzetti (1993) and Sobel (1996) analyze models where principals compete for the exclusive services of an agent whose ability is unknown.

In the standard auction model there is a single good, which is sold at a single price. Furthermore, once the good is sold the seller has no influence on the buyer's payoff (e.g., Myerson (1981), Riley and Samuelson (1981), and Milgrom and Weber (1982)). This model can be extended to the case in which the bidders are principals competing for the exclusive services of an agent who does not have private information. We show that in this extension all principals offer contracts requiring the agent to exert the first best level of effort. On the contrary, in the standard single-principal, single-agent model, only the lowest cost agent takes the first best level of effort while higher cost agents are induced to choose less than the first best (e.g., Baron and Myerson (1982), Maskin and Riley (1984), and Laffont and Tirole (1986)).[3]

The equilibrium effort levels in our incentive auction are different from both of these models. A class of principal types, including the one with the lowest valuation for the agent's effort, act as if there was a separate auction for each agent type; that is, all the incentive compatibility constraints are slack. All these principals induce the agent to take the first best effort level, and bid as in the standard auction. We give conditions under which this class includes all principal types. When these conditions fail, there exists another class of principal types that treat their bids as being connected; that is, some incentive compatibility constraints bind. These principals ask the low cost agent to take the first best level of effort and the high cost agent to take less than the first best level. This is analogous to what happens in the single-principal, single-agent model, but the efforts of all agent types are at least as high in the incentive auction; competition among principals reduces the effort distortions associated with contracting under private information. In the limit, as the number of principals goes to infinity, the inefficiency due to the agent's private information disappears and the agent receives all the surplus.

Milgrom and Weber (1982) showed that the auctioneer always wants to reveal his private information in a first price sealed bid auction. In our auction a high cost agent receives no information rents, thus he prefers that his type be made public, because effort would increase to the first best level and result in higher bidding for his services. On the other hand, a low cost agent prefers that no information be revealed, since his effort would remain at the first best level, but he would lose information rents.

Our model has features in common with standard procurement, but in procurement there is a single principal (e.g., the department of defense) and multiple agents (e.g., defense contractors). Procurement auctions were studied by Laffont and Tirole (1987), McAfee and McMillan (1987), and Riordan and Sappington (1987). Under the assumption that the principal is able to implement an auction form that maximizes her expected payoff, they showed that if the agent's types are identically and independently distributed then the “separation property” holds: competition only affects the winning agent's compensation. In the case of a project of fixed size this means that the winning agent's cost is the same as if he had no competitors; if the project's size is variable, then competition does not affect the equilibrium quantity. In these models, the principal has no private information and takes no action. In our model there are multiple principals bidding for a single agent, and both the principals and the agent have private information. One of our main results is that the "separation property" does not hold; besides affecting the agent's compensation, competition among principals has a "real," efficiency enhancing effect on the agent's effort.

In the next section we introduce the incentive auction, and in Section 3 we present benchmark solutions for the standard auction for a known agent type and for the single-principal, single-agent model. In Section 4, we derive the equilibrium of the incentive auction and in Section 5 we study the issue of information revelation. Section 6 contains a discussion of possible extensions and offers some concluding remarks.

2 The Incentive Auction Model

There are N principals competing for the exclusive services of a single agent. The agent undertakes an action, x, that affects the winning principal's payoff; we refer to this action as the agent's effort level. The agent is one of two types $\mu_j;\ j = L; H$, with $\mu_L < \mu_H$. The agent's type is private information. The prior probability of type μ_L is $p > 0$; and is common knowledge. Agent μ_j's disutility of effort function $C(x; \mu_j)$ is a twice differentiable, strictly increasing, strictly convex function of x, with $C(0; \mu_j) = 0$ for all μ_j. Moreover, denoting partial derivatives with subscripts, $C_x(x; \mu_L) < C_x(x; \mu_H)$ and $C_{xx}(x; \mu_L) \quad C_{xx}(x; \mu_H)$; that is, the marginal cost of effort is increasing in μ_j and so is its curvature.[4] Let the agent's transfer from the winning principal be w, and his utility if he does not participate be zero; μ_j's payoff function is

$$U_j = w - C(x; \mu_j) \tag{1}$$

Principal i's valuation for the agent's effort is $R(x; t_i)$; where t_i is a privately known random variable. $R(\cdot)$ is twice differentiable, strictly increasing and concave in x, differentiable and strictly increasing in t_i, and satisfies $R(0; t_i) = 0$ and $R_x(0; t_i) > C_x(0; \theta_j)$ for all t_i and θ_j. Furthermore, the cross partial is positive, $R_{xt}(\cdot) > 0$; that is, the marginal valuation is increasing in t_i. Principal i's payoff function is

$$\Pi_i = R(x; t_i) - w \tag{2}$$

Each principal's type is drawn independently from distribution $G(t)$ with support $[a; b]$. $G(t)$ has a density $g(t)$ that is strictly positive everywhere. Principals compete for the exclusive services of the agent in a sealed bid auction. Each principal's bid is a menu of schedules specifying the transfer as a function of the observable and verifiable effort level x.

In the application of our model to jurisdictions competing to attract a firm, we can interpret the effort level x as the amount by which the plant size or employment level chosen by the firm exceeds the profit maximizing level. In this case, $C(\cdot)$ is the reduction in profit and $R(\cdot)$ the increase in the jurisdiction's benefit (e.g., due to agglomeration effects) associated with an increase in plant size or employment above the profit maximizing levels. Usually the firm has private information about its cost, and the jurisdictions have private information about their valuations. Furthermore, a jurisdiction's valuation for plant size and employment level depends on local political conditions which are largely independent across jurisdictions. Thus, it is reasonable to assume that the jurisdictions' types are independently distributed. The assumption that jurisdictions compete in a sealed bid auction is more debatable, but not unrealistic; in practice, the details of a jurisdiction's bid are not observable to other jurisdictions. To assume that the firm does not care about the specific location of the competing jurisdictions is justifiable when all the jurisdictions are similar in terms of geographic features and labor market conditions. For example, automobile companies building new plants in the U.S. focus their search of sites for new factories on the middle states of the Southeast and the lower Midwest, where shipping costs are minimized and unionization rates are low.

We restrict attention to the case in which principals offer two incentive compatible transfer schedules, one for each type of agent. Because effort is verifiable, there is no loss of generality in considering only functions that make transfers conditional on effort targets. More precisely, each principal t offers a pair of contracts $\{[w_L(t), x_L(t)], [w_H(t), x_H(t)]\}.$ If the agent selects contract $[w_j(t), x_j(t)]$, then he receives $w_j(t)$ if and only if his effort is at least $x_j(t)$; if the agent's effort is less than $x_j(t)$, he receives a transfer of zero. After the principals offer their contracts to the agent, the agent chooses his most preferred contract and his effort. The agent is then compensated by the chosen principal according to the contract that he selected. We examine perfect Bayesian equilibria of this game. As is common in auction models, we restrict attention to symmetric equilibria.

Before proceeding with our analysis, we briefly discuss two important modelling issues. First, we know from Epstein and Peters (1999) that the right application of the revelation principle when there are competing principals must allow for each principal to try and elicit information from the agent about the other principals' offers. In other words, one should include the other principals' contract offers in the description of an agent's type. Moreover, a principal may want to include in her incentive scheme a counteroffer to the other principals' counteroffers to the principals' counteroffers, and so forth. Thus, the type space of the agent that must be used to properly apply the revelation principle is quite complex. At this point, it is not known how to use this general revelation principle in applications, and whether there are circumstances under which one can restrict attention to standard revelation mechanisms without loss of generality. Our view is that even if it was possible, it would not necessarily be reasonable to apply the general revelation principle in our setup.

Second, in a single-principal, single-agent model, Maskin and Tirole (1990) show that if preferences are quasi-linear, as in our model, then a principal will not lose from offering a mechanism which identifies her true type before the implementation stage.

This implies that in our incentive auction, for any given set of incentive schemes offered by her opponents, a principal will not lose from making an offer that fully reveals her type to the agent. Thus, fully revealing her type is a weakly dominant strategy for each principal.

3 The Benchmarks

We first derive the solution to the auction when the agent's type is known. Next, we present the solution when a single principal faces an agent with private information. These benchmarks build intuition for the solution to the general problem.

3.1 The Single Price Auction

In this subsection, we assume that principals know that the agent's true type is θ_j. In the auction for a known agent's type, principal t's maximization problem is

$$\max_{x_j;U_j} [R(x_j;t) - C(x_j;\theta_j) - U_j]G^{N-1}(t_j^{-1}(U_j))$$

where $t_j^{-1}(U_j)$ is the inverse of the bidding function for agent θ_j. Since our assumptions guarantee an interior solution, the first order condition with respect to x_j is

$$R_x(x;t) - C_x(x;\theta_j) = 0 \tag{3}$$

Let $x_j^F(t)$ be the solution to equation (3); $x_j^F(t)$ equals the first best effort level if the agent is type θ_j and the principal's type is t. Thus, each principal always requires the agent to exert the first best level of effort. Note that $x_j^F(t)$ is increasing in t, due to $R_{xt}(x;t) > 0$, and decreasing in θ_j, since the agent's marginal cost of effort is increasing in θ_j. The first order condition with respect to U_j is

$$\frac{dU_j}{dt} = (N-1)[R(x;t) - C(x;\theta_j) - U_j(t)]\frac{g(t)}{G(t)} \tag{4}$$

Multiplying both sides by $G^{N-1}(t)$, integrating and using equation (3) yields

$$U_j(t) = R(x_j^F(t); t) - C(x_j^F(t); \theta_j) - \frac{\int_a^t R_t(x_j^F(\tau); \tau) G^{N-1}(\tau) d\tau}{G^{N-1}(t)} \tag{5}$$

The equilibrium wage function then is

$$w_j(t) = R(x_j^F(t); t) - \frac{\int_a^t R_t(x_j^F(\tau); \tau) G^{N-1}(\tau) d\tau}{G^{N-1}(t)} \tag{6}$$

We can now state the following lemma:

Lemma 1 The symmetric equilibrium utility and wage functions satisfy equations (5) and (6).

Equations (5) and (6) have the usual interpretation: each principal bids less than her valuation of the agent. The last term in each expression corresponds to the amount by which principal t shades her bid below her valuation. In an auction for a single object, principal t's valuation is $v = t$, and thus the rate of change of the valuation with respect to t equals 1. In our model, agent θ_j's value added when working for principal t is $R(x_j^F(t); t) - C(x_j^F(t); \theta_j)$. Thus, the rate of change in the valuation with respect to t is $R_t(x_j^F(t); t)$.

3.2 The Single-Principal Solution

Now we examine the case when there is a single principal t who faces an agent whose type is unknown. The principal will offer the set of incentive compatible, individually rational contracts which maximize her expected payoff. The incentive compatibility constraints are

$$w_L(t) - C(x_L(t); \theta_L) \geq w_H(t) - C(x_H(t); \theta_L) \tag{IC_L}$$

$$w_H(t) - C(x_H(t); \theta_H) \geq w_L(t) - C(x_L(t); \theta_H) \tag{IC_H}$$

Since the agent's reservation utility is equal to zero, the individual rationality constraints are

$$w_j(t) - C(x_j(t); \theta_j) \geq 0 \tag{IR_j}$$

As is well known, if p is not too large, then in the solution of the single-principal problem (IC_L) and (IR_H) bind, while (IC_H) and (IR_L) are slack. The low cost agent, θ_L, exerts the first best level of effort and receives information rents. The high cost agent exerts less than the first best level of effort and receives no information rents.[5]

Define $x_j^S(t)$ as θ_j's effort level when he faces principal t, and $w_j^S(t)$ as the transfer. Let

$$p^*(t) \equiv \frac{R_x(0; t) - C_x(0; \theta_H)}{R_x(0; t) - C_x(0; \theta_L)}$$

If $p \leq p^*(t)$, then $x_H^S(t) \geq 0$ is the solution to the first order condition

$$R_x(x_H; t) - C_x(x_H; \theta_H) = \frac{p}{1 - p}[C_x(x_H; \theta_H) - C_x(x_H; \theta_L)] \tag{7}$$

Lemma 2 If $p \leq p^*(t)$, the pair of contracts offered by principal t satisfy the following conditions: (i) (IR_H) and (IC_L) bind; (IC_H) and (IR_L) are slack; (ii) $x_L^S(t) = x_L^F(t)$; (iii) $x_H^S(t) < x_H^F(t)$, and (iv)

$$w_L^S(t) = C(x_L^S(t); \theta_L) + [C(x_H^S(t); \theta_H) - C(x_H^S(t); \theta_L)] \tag{8}$$

The proof is standard and is omitted. Equation (7) shows that agent θ_H takes less than the first best level of effort.

4 The Analysis of the Incentive Auction

We first present some preliminary lemmas. Then we introduce sufficient conditions under which the incentive auction can be treated as two independent auctions for the agent types. Finally, we study the symmetric equilibrium in the general case.

Given a pair of incentive compatible contracts $[(w_L(t); x_L(t)); (w_H(t); x_H(t))]$ offered by a type t principal, let $U_j(t)$ be the utility that agent j receives from that principal. The following lemmas extend standard results in auction theory.

Lemma 3 In equilibrium, the agent's utility $U_j(t)$ is an increasing function of the principal's type t.

Proof. See the Appendix.

Lemma 4 Principal type a requires agent type μ_j to choose $x_j^F(a)$ and sets his transfer to $w_j(a) = R(x_j^F(a); a)$:

Lemma 3 shows that the agent's utility must be increasing in the principal's type. This has two implications. First, if a principal wins one agent type, then he also wins the other type. Second, the principal who values the agent most always gets him. The idea behind this result is simple. For the agent to choose to work for her, the contract offered by a principal must give the agent more utility than any other competing principal's contract. Let t_j and t_k be the types of principals j and k, with $t_j < t_k$. Suppose agent μ_j takes t_j's offer, while agent μ_k takes t_k's offer. Clearly, t_j must give μ_j at least as much utility as μ_j can get from t_k: Since the agent generates more revenue with t_k than with t_j at any given effort level, t_k could increase her payoff by mimicking the offer made by t_j to μ_j; she would attract both agents without violating the incentive compatibility constraints.

The logic behind Lemma 4 is similar to the one behind the result that under Bertrand competition price equals marginal cost. Loosely speaking, no interval of principal types above principal a will offer a transfer less than $R(x_j^F(a); a)$, because if they did an interval of lower types including a would outbid them. On the other hand, type a would not want to offer a transfer exceeding $R(x_j^F(a); a)$. Thus, the agent gets all the surplus available from the lowest type of principal. A corollary of Lemma 4 is that if all principals are of the same known type t; then, as in Bertrand competition, the agent extracts all the available surplus; that is, he exerts the first best effort level $x^F_j(t)$ and receives a transfer of $R(x^F_j(t);t)$.

In our model an equilibrium always exists, since the principals can contract over all arguments that enters their profit functions, namely x and w. In particular, if all principals are of the same known type, they offer a contract that gives all the surplus to each agent type. On the contrary, Rothschild and Stiglitz (1976) showed that an equilibrium may not exist in a competitive insurance market when an agent has private information about his probability of a loss. In their model, the agent's private information directly enters the firms' profit functions. This makes it impossible for the firms to make nonnegative expected profits while offering contracts that allow each agent type to extract all the surplus.

Lemmas 3 and 4 imply that all agent types exert positive effort. Note that this is different than in the single-principal model, where the high cost agent will exert zero effort if the probability of him occurring is sufficiently small. Lemma 4 also implies that both incentive compatibility constraints are slack for the equilibrium contracts offered by principal a.

We now introduce sufficient conditions under which the incentive auction can be treated as two independent auctions for the agent types. Let $\phi^F_{j;k}(t)$ be the difference in agent j's utility from principal t between telling the truth and claiming that his type is k, when the first best effort levels are required and compensation is as in the standard auction. From (IC_L), (IC_H) and (5), we obtain

$$\begin{aligned}\phi^F_{j;k}(t) = {} & R(x^F_j(t);t) - C(x^F_j(t);\theta_j) - R(x^F_k(t);t) + C(x^F_k(t);\theta_j) \\ & - \frac{\int_a^t R_t(x^F_j(\tau);\tau)G^{N-1}(\tau)d\tau}{G^{N-1}(t)} + \frac{\int_a^t R_t(x^F_k(\tau);\tau)G^{N-1}(\tau)d\tau}{G^{N-1}(t)} \qquad (9)\end{aligned}$$

for all j, k with $j \neq k$:

Lemma 5 For all $t \in [a;b]$, $\phi^F_{H;L}(t) > 0$. If $\phi^F_{L;H}(t) > 0$ for all $t \in [a;b]$, then for all j and all $t \in [a;b]$ (i) $x_j(t) = x^F_j(t)$, and (ii) $w_j(t)$ satisfy equation (6).

Proof. See the Appendix.

This lemma shows first that if both agent types take the first best effort and are compensated as in the standard auction then (IC_H) is always slack; that is, the high cost agent does not want to mimic the low cost agent, he strictly prefers contract $[w_H(t); x_H^F(t)]$ to contract $[w_L(t); x_L^F(t)]$. Second, it shows that if (IC_L) is also slack (i.e., if the low cost agent does not want to mimic the high cost agent either), then we can treat the bidding as two independent auctions, one for each agent type. In such a case the presence of θ_L does not influence the bidding for θ_H, and vice versa; the agent's private information does not affect the effort levels and transfers that the principals offer him. Thus, in this special case the "separation property" fails in a very strong sense. Contrary to the single-principal case, in the presence of competing principals all agent types undertake the first best effort level. If $\phi_{L;H}^F(t) < 0$ for some t, then the bidding cannot be treated as a standard auction; some principals need to account for the incentive compatibility constraints. As we will see in Corollary 1, even when incentive constraints bind the "separation property" does not hold; effort levels are not the same as in the single-principal model.

It is easy to find examples where $\phi_{L;H}^F(t)$ is always greater than zero, and where it is not. Let $R(x; t) = (A + \alpha t)x$, where $A > \theta_H$, $C(x; \theta_j) = \theta_j x + x^2/2$, $G(t)$ be uniformly distributed on $[0; b]$, and suppose that $N = 2$. For this example,

$$\phi_{L;H}^F(t) = \frac{(\theta_H - \theta_L)(\theta_H - \theta_L - \alpha t)}{2}$$

If $\alpha b < \theta_H - \theta_L$, then $\phi_{L;H}^F(t) > 0$ for all t and the principals bid as if there were two independent auctions. If $\alpha b > \theta_H - \theta_L$, then $\phi_{L;H}^F(t) > 0$ if and only if $t < (\theta_H - \theta_L)/\alpha$. Thus, in this case bidding for agents θ_L and θ_H is interconnected for an interval of principal types. In this example, if the cross partial $R_{tx} = \alpha$ is small, if $\theta_H - \theta_L$ is large, or if b is close to zero, then $\phi_{L;H}^F(t) > 0$,while the opposite is true if α is large, if $\theta_H - \theta_L$ is small, or if b is large. The following lemma shows that this is a general result.

Lemma 6 (i) For any given θ_H and θ_L, there exists ε such that if $R_{tx}(x;t) < \varepsilon$ for all x and t, then $\phi^F_{L;H}(t) > 0$ for all t. (ii) There exists $b^* > a$ such that if $b < b^*$, then $\phi^F_{L;H}(t) > 0$ for all $t \in [a;b]$ and as b converges to a the agent receives the first best surplus level. (iii) For all $t \in [a;b]$, $\lim_{N \to \infty} \phi^F_{L;H}(t) > 0$ and the agent's utility converges to the first best surplus. Finally, (iv) $\lim_{\theta_H \to \theta_L} \phi^F_{L;H}(a) = 0$ and for all $t > a$ $\lim_{\theta_H \to \theta_L} \phi^F_{L;H}(t) < 0$.

Proof. See the Appendix.

Parts (i)-(iii) of Lemma 6 provide sufficient conditions under which the bidding for the agent can be treated as two independent standard auctions for the agent types. These conditions are: (i) marginal revenue depends very little on the principal's type; (ii) principals are very similar; (iii) there is a large number of competing principals. Part (iv) of the lemma shows that if the agent's types are very close, then bidding for the two types is interconnected and cannot be treated as two independent auctions; that is, principals cannot ignore the incentive compatibility constraints.

Part (i) follows because the first best effort level of each agent type is essentially independent of the principal's type when $R_{tx}(x;t)$ is small. This implies that the sum of the last two terms of (9) is essentially zero for all t; that is, the amounts by which a principal shades her bid below total surplus in the standard auctions for a low and for a high type are essentially the same. Thus, the sign of (9) is positive, since it is the sign of the difference, when the agent is θ_L, between the first best total surplus and total surplus when effort is $x^F_H(t)$.

Part (ii) holds because if the principals' valuations become close to one another then, as in Bertrand competition, the agent extracts all the available surplus. Similarly, part (iii) follows because in the limit, as the number of competing principals increases, the agent extracts all the surplus.

The intuition for part (iv) of the lemma is that when principals offer the standard auction contracts as the difference between agent types becomes smaller, the gain to θ_L from mimicking θ_H increases, and hence it is more likely that incentive compatibility is violated.

We now examine the symmetric equilibrium of the incentive auction in the general case. By Lemmas 3 and 4 the individual rationality constraints are always slack. Thus, they can be ignored by the principals when deciding their optimal bids. Put differently, in order to ensure "participation" by the agent a principal must offer him more than what other principals are offering. Thus, the participation constraint or the outside reservation utility of the agent is endogenous in our model. As in standard auctions, an increase in the bid or utility offered to the agent reduces a principal's payoff when winning, but increases the probability of winning.[6]

Since $U_j(t)$ is increasing in t by Lemma 3, the probability that the principal wins the bidding for agent j is $G^{N-1}(t_j^{-1}(U_j))$, where $t_j^{-1}(U_j)$ is the inverse of the bidding function for agent j. We will show in Lemma 7 that (IC_H) is always slack. Hence, if we ignore the (IC_H) and the (IR_j) constraints and replace transfers by utility levels, principal t's maximization program is:

$$\begin{aligned}\max_{x_L, x_H, U_L, U_H} \quad & p[R(x_L, t) - C(x_L, \theta_L) - U_L]G^{N-1}(t_L^{-1}(U_L)) \\ & +(1-p)[R(x_H, t) - C(x_H, \theta_H) - U_H]G^{N-1}(t_H^{-1}(U_H)) \qquad (10)\end{aligned}$$

subject to

$$U_L - U_H + C(x_H, \theta_L) - C(x_H, \theta_H) \geq 0 \qquad (11)$$

Let $\lambda(t)G^{N-1}(t)$, with $\lambda(t) \geq 0$, be the Lagrangian multiplier associated with (11); we can interpret it as the shadow price of meeting the incentive compatibility constraint. Define the following functions on [a,b],

$$\varphi_L(s) = \int_a^s \frac{p(N-1)g(u)}{[p-\lambda(u)]G(u)}du \qquad (12)$$

$$\varphi_H(s) = \int_a^s \frac{(1-p)(N-1)g(u)}{[(1-p+\lambda(u)]G(u)}du \qquad (13)$$

Theorem 1 The equilibrium in the incentive auction is characterized by the following system of equations:

$$[U_L(t) - C(x_H(t);\theta_H) + C(x_H(t);\theta_L) - U_H(t)]\lambda(t) = 0 \tag{14}$$

$$R_x(x_L(t);t) - C_x(x_L(t);\theta_L) = 0 \tag{15}$$

$$(1-p)[R_x(x_H(t);t) - C_x(x_H(t);\theta_H)] + \lambda(t)[C_x(x_H(t);\theta_L) - C_x(x_H(t);\theta_H)] = 0 \tag{16}$$

$$U_j(t) = R(x_j(t);t) - C(x_j(t);\theta_j) - \int_a^t \{[R_x(x_j(s);s) - C_x(x_j(s);\theta_j)]\frac{dx_j}{ds}(s) + R_t(x_j(s);s)\} e^{\varphi_j(s)-\varphi_j(t)} ds \text{ for } j = L;H \tag{17}$$

Proof. We solve for equilibrium under the assumption that (IC_H) is slack; we then show that our solution satisfies this assumption. Equation (14) is the complementary slackness condition. Equations (15) and (16) are the first order conditions of the maximization program of principal t with respect to $x_L(t)$ and $x_H(t)$. The first order conditions with respect to $U_H(t)$ and $U_L(t)$ are

$$\frac{dU_H}{dt} = \frac{1-p}{1-p+\lambda(t)}(N-1)[R(x_H;t) - C(x_H;\theta_H) - U_H]\frac{g(t)}{G(t)} \tag{18}$$

$$\frac{dU_L}{dt_L} = \frac{p}{p-\lambda(t)}(N-1)[R(x_L;t) - C(x_L;\theta_L) - U_L]\frac{g(t)}{G(t)} \tag{19}$$

Using (12) and (13) as the integrating factors yields the differential equations

$$\frac{d}{dt}[U_H e^{\varphi_H(t)}] = \frac{1-p}{1-p+\lambda(t)}[R(x_H;t) - C(x_H;\theta_H)](N-1)\frac{g(t)}{G(t)}e^{\varphi_H(t)} \tag{20}$$

$$\frac{d}{dt}[U_L e^{\varphi_L(t)}] = \frac{p}{p-\lambda(t)}[R(x_L;t) - C(x_L;\theta_L)](N-1)\frac{g(t)}{G(t)}e^{\varphi_L(t)} \tag{21}$$

Since dU_j/dt is strictly increasing in t, $1 > p - \lambda(t) > 0$ for all t. Integrating (20) and (21), using the fact that the constant of integration $U_j(a)$ is equal to $R(x_j;a) - C(x_j;\theta_j)$ by Lemma 4, and then integrating the r.h.s. by parts, we obtain equations

(17). Lemma 7, which is proved in the Appendix, shows that (IC_H) is slack.[7]

Lemma 7 The incentive constraint (IC_H) is slack at the solution of (14)-(17).

This completes the proof. ■

The complementary slackness condition, equation (14), says that the incentive compatibility constraint (IC_L) for principal t is either slack and $\lambda(t) = 0$, or it binds and $\lambda(t) > 0$. Equation (15) shows that the low type agent always takes the first best effort level. Equation (16) shows that the high type agent also takes the first best effort if $\lambda(t) = 0$, while if $\lambda(t) > 0$ (i.e., if the incentive constraint (IC_L) binds) then principal t asks the high type agent to take less than the first best effort level. Note that $\lambda(t)$ is not necessarily monotone in t, and the set of principal types requiring type θ_H to take the first best effort level need not be a connected set. Equations (17) shows the equilibrium utility that each agent receives from principal type t. The agent receives the total surplus that he generates with the principal at the equilibrium effort level, $R(x_j(t);t) - C(x_j(t);\theta_j)$, less the amount by which the winning principal shades her bid below the total surplus.

To compare the incentive auction to the standard auction, consider first the case in which all principals below type t ask the agent to undertake the first best effort level, $\lambda(s) = 0$ for all $s < t$. Then marginal revenue, $R_x(x_j(s);s)$; equals marginal cost, $C_x(x_j(s);s)$; for all $s < t$, and by equation (16) equilibrium efforts equal the first best levels. Moreover, $e^{\varphi_j(s) - \varphi_j(t)} = G^{N-1}(s)/G^{N-1}(t)$, and by equations (17) principal t offers the same incentive scheme as in the standard auction. Note also that the first order conditions (18) and (19) reduce to the first order condition for the standard auction, equation (4). Thus, if $\lambda(t) = 0$ for all $t \in [a;b]$, then all the principals bid independently for the two types of agent as in the standard auction.

Now consider the case when $\lambda(s) > 0$ for some interval $[s';s'']$ of principals below type t. By equation (18) the rate of change in $U_H(s)$ is decreasing in $\lambda(s)$ and thus principal t offers the high cost agent less than in the standard auction. On the other hand, equation (19) implies that the rate of change in $U_L(s)$ increases with $\lambda(s)$ and that principal t offers the low cost agent more than in the standard auction (see Theorem 2 below). This shows that no principal $t \geq s''$ can treat the bidding for the two agents as two independent standard auctions.

To compare the equilibrium efforts in the incentive auction and in the single-principal, single-agent model use (16) to implicitly define x_H as a function of λ. Since θ_L's utility is increasing in t, $p > \lambda(t)$. Comparing (7) and (16) and using the concavity of $R(\cdot) - C(\cdot)$ in x, and the fact that marginal cost and its slope are increasing in θ, shows that the distortion in the high cost agent's effort is less than in the single-principal, single-agent model. Thus, we have the following corollary.

Corollary 1 The equilibrium effort levels in the incentive auction satisfy $x_H^F(t) \geq x_H(t) > x_H^S(t)$ and $x_L^F(t) = x_L(t) = x_L^S(t)$ for all t.

The presence of multiple principals not only affects the agent's compensation, it also reduces the distortion in the agent's effort level due to private information; the "separation property" does not hold. By Lemma 6, in the limit, as the number of principals approaches infinity, the agent takes the first best effort level and obtains all the surplus.

We now demonstrate that under the incentive auction: (i) the payoffs of both agent types are higher than under the single principal model; (ii) θ_L's payoff is higher while θ_H's payoff is lower than under two separate, independent auctions.

Let $U_j^S(t)$ be agent θ_j's utility from principal t in the single-principal model, and $U_j^A(t)$ be the agent's utility in a standard auction model when his type is known.

Theorem 2 The agent's utilities can be ranked as follows: (i) $U_j(t) > U_j^S(t)$ for all j; (ii) $U_L(t) \geq U_L^A(t)$; (iii) $U_H^A(t) \geq U_H(t)$.

Proof. Using Lemma 2, we can see that θ_H's payoff in the single principal model is equal to zero; μ_L's payoff is

$$C(x_H;\mu_H) \text{ ¡ } C(x_H;\mu_L) \tag{22}$$

Equation (22) represents μ_L's information rent; it is increasing in the effort taken by μ_H. By Corollary 1, the efforts are higher in the incentive auction than in the single-principal model for each principal t. Also, by Lemmas 3 and 4 μ_H receives a positive payoff in the incentive auction. Since μ_L receives at least the payoff of μ_H plus (22), μ_L's payoff is higher in the incentive auction than in the single-principal model. This proves part (i).

If the incentive constraint does not bind for principal types less than t; then the agents' payoffs from all principals less than t are the same as in the standard auction. Suppose that the incentive constraint does bind for some subinterval of principals in $[a; t]$. By equations (17) μ_L receives the payoff in the standard auction plus some information rent, while μ_H receives a lower payoff in the incentive than in the standard auction. To see this, simply note that the first order conditions (18) and (19) imply that $U_L(t)$ is increasing while $U_H(t)$ is decreasing in ¸. ■

5 Incentives for Information Disclosure

We now examine the agent's incentives to reveal information prior to the bidding. Suppose that a stage is added to the game before the bidding, where the agent can costlessly prove his true type to all the principals. We restrict attention to the only interesting case, when $Á^F_{L;H}(t) < 0$ for a set of principal types with positive measure, so that the agent's expected payoff in the incentive auction differs from his expected payoff in the standard auction. The next corollary follows from Theorem 2.

Corollary 2 The policy of disclosing his true type increases μ_H's expected utility, while it decreases μ_L's expected utility. Thus, if the agent can costlessly prove his true type, then in equilibrium his information will always be revealed.

Agents disagree on whether an information revelation stage should be added before the auction. By Theorem 2, a high cost agent would always reveal his type since his expected utility is higher in the standard auction than in the incentive auction. Thus, indirectly, the low cost agent type would also be revealed in equilibrium. This contrasts with the standard single-principal, single-agent model where the agent never gains from revealing his type, since the only utility that he receives is the information rent. On the other hand, Milgrom and Weber (1982) show that in a standard first-price, sealed-bid auction the policy of publicly revealing the seller's information raises the expected price when bidder's valuations are affiliated. In our model, the effort level that the agent takes depends on his type. Thus, although their types are independently distributed, the principals' revenues from the agent depend, through the effort levels, on the agent's type. In this sense, the agent's type is a common component of the principals' payoffs from the agent, which then are positively correlated (affiliated). Type θ_H prefers that information becomes public, so as to raise the bidding for his services, as in Milgrom and Weber; on the contrary, type θ_L obtains information rents and prefers that no information be transmitted to the principals.

Now, we examine the situation when the agent cannot prove his true type, but he can send a costless message. We focus our attention on the existence of pooling and separating equilibria.

Theorem 3 If the agent can send a costless message, then: (i) a pooling equilibrium always exists; (ii) a separating equilibrium exists if and only if

$$E[\phi^F_{L;H}] \equiv \int_a^b \phi^F_{L;H}(t)dG^N(t) \geq 0 \tag{23}$$

Proof. See the Appendix. ■

It is clear from the proof of Theorem 3 that on a pooling equilibrium path principals offer the same contracts as in the incentive auctions, while on a separating equilibrium path principals bid for each agent type as if there were two separate standard auctions. Condition (23) always holds if $\phi^F_{L,H}(t) \geq 0$ for all t; that is, it holds if the low cost agent never wants to mimic the high cost when each principal offers the same contracts as in two separate, standard auctions for each type. It also can hold when $\phi^F_{L,H}(t) < 0$ for some t, provided that “on average" the low cost agent does not want to mimic the high cost agent.

6 Conclusions

This paper is a contribution to the study of incentive bidding by multiple, privately informed principals for a single, privately informed agent. We view it as a first step towards a more comprehensive study of nonlinear pricing in markets with several privately informed parties on both sides.

We have shown that principals who have relatively low value for the agent's effort bid as if there were a separate auction for each agent type. The agent is induced to take the first best effort level with these principals. On the other hand, the bids for different agent types are often interconnected when principals have relatively high valuations for the agent's effort. In this case, incentive constraints bind and the high cost agent takes less than the first best level of effort; however, the agent takes more effort than if there were a single principal. Thus, competition reduces distortions due to private information.

In recent years, many commentators have argued that states bid too much to attract firms. Many have called for a moratorium in these bidding wars. We can think of such a moratorium as the collusive agreement that the principals would reach if they could coordinate their contract offers. Collusion is enhanced if transfers are possible, but even if transfers are allowed the principals' benefit in a collusive agreement cannot exceed the benefit obtained when the agent is assigned to the highest type principal and the menu of contracts offered coincides with what the highest type principal would offer in the standard single-principal, single-agent model.[8] Think of this as the optimal outcome for the principals. Whether it can be implemented by a Bayesian mechanisms depends on how one models the principals' outside option utility.

In a previous version of this paper, we showed that such a mechanism exists if the principals' reservation utility is zero. Assuming a zero reservation utility is tantamount to assuming that a principal that does not participate in the collusive agreement can be prevented from bidding for the agent. Clearly, this is only possible in special circumstances. If the deviating principal cannot be prevented from bidding, then we must specify what happens if a deviation occurs. There are two natural possibilities. First, the remaining principals could agree to bid jointly against the deviating principal. This is a model that is difficult to analyze because it involves an incentive auction between asymmetric bidders. Second, following a deviation all cooperation could break down and the principals would then participate in the incentive auction. In this case, the reservation utility of a principal is her expected payoff in the incentive auction. We can show that the optimal outcome for the principals need not be implementable. However, if the number of principals is sufficiently large, then it is implementable. This is because competition in the incentive auction increases with the number of principals, and this reduces the outside option of a principal and makes her less inclined to defect from a collusive agreement.

In any collusive agreement, effort is farther away from the efficient level than in the incentive auction; thus, a moratorium on bidding wars could reduce overall efficiency. Of course in a second best world reducing one distortion (allowing bidding competition) does not necessarily improve welfare. It is also true that political leaders do not necessarily maximize their jurisdiction's welfare when bidding for firms. For example, political leaders may have an incentive to overbid for a firm in order to gain reelection (see Biglaiser and Mezzetti (1997)).

In our incentive auction, neither side of the market has all the bargaining power; the auction is not optimal for either the principals or the agent. We have discussed the optimal auction for the principals. Which auction is optimal for the agent is less clear. If the agent's type is common knowledge, then it is well known that a first or second price auction with a properly selected reserve price is optimal. However, when his type is private information the low cost agent will not want to reveal it. The possibility that the agent could use his choice of an auction form to signal his type to the principals complicates the analysis.

In this paper, private information on the two sides of the market is modeled differently: there is a continuum of principal types, but only two types of agents. It would be interesting to extend our model to the case of a continuum of agent types. We have done some preliminary work in this direction and uncovered some interesting new features. First, with a continuum of agent types it is never the case that equilibrium bids are the same as in the standard auction; this is because these bids violate the incentive compatibility constraints. This result is consistent with Lemma 6, which shows that when bids are the standard auction bids the incentive constraint for the low agent type is not satisfied if the two types are sufficiently close. Second, if the effort function offered by a principal is strictly monotonic in the agent type (i.e., there is no bunching), then both the lowest and the highest type agents exert the efficient effort levels.[9] Finally, as in Rochet and Stole (1999), bunching of types in the effort function is possible. This creates some additional technical difficulties which we will pursue in future work.

Appendix

Proof of Lemma 3. We first show that the probability that a principal attracts an agent is the same for both types. Next, we demonstrate that $U_j(t)$ is increasing. The expected utility of principal i when her type is t is

$$\sum_{j=L,H} p_j[R(x_j;t) - C(x_j;\theta_j) - U_j]\Pr(U_j > \max_{k\neq i} U_j(t_k))$$

where $U_j(t_k)$ is the utility offered to θ_j by principal k of type t_k and $\Pr(U_j > \max_{k\neq i} U_j(t_k))$ is the probability that principal i wins agent θ_j:

We first show that for any two disjoint intervals T_h and T_k it cannot be the case that principals in T_h attract θ_h, while principals in T_k attract θ_k: Suppose the contrary occurs; let principal $t_h \in \arg\max_{\tau\in T_h} U_h(\tau)$ and $t_k \in \arg\max_{\tau\in T_k} U_k(\tau)$. If t_h deviated and offered the same incentive scheme to θ_k that is offered by t_k, then she would attract θ_k when faced with principals in T_h. Moreover, this deviation would not influence her payoff from θ_h or violate any incentive constraints. Since there is a positive probability that t_h faces a type in T_k, for this deviation not to be profitable it must be that in the supposed equilibrium

$$R(x_k(t_k);t_h) - C(x_k(t_k);\theta_k) - U_k(t_k) \quad 0 \tag{24}$$

Furthermore, since t_h attracts θ_h, it must be the case that

$$R(x_h(t_h);t_h) - C(x_h(t_h);\theta_h) - U_h(t_h) \geq 0 \tag{25}$$

We can write similar inequalities for principal t_k

$$R(x_h(t_h);t_k) - C(x_h(t_h);\theta_h) - U_h(t_h) \quad 0 \tag{26}$$

$$R(x_k(t_k);t_k) - C(x_k(t_k);\theta_k) - U_k(t_k) \geq 0: \tag{27}$$

By inequalities (24) and (27), $t_k \geq t_h$, and by inequalities (25) and (26), t_h t_k: This contradicts the assumption that T_k and T_h are disjoint intervals. This demonstrates that with probability one both agent types accept a contract from the same principal.

Now, we show that $U_j(t)$ is increasing. From above, $\Pr(U_j(t) > \max_{k \neq i} U_j(t_k))$ is the same for both agent types, we refer to it as $\Pr(W \mid U(t))$. First, note that $U_j(t)$ is continuous. If there was a discontinuity, then at least one of the principals could reduce the utility she gives to both agents by the same amount without affecting the probability of attracting the agent. Such a reduction in utility would not affect the incentive compatibility constraints. Suppose that $U_j(t)$ is not monotonic. Since it is continuous, and each agent type is attracted to the same principal, there exists two principals t_h and t_k, with $t_h > t_k$, and $U_j(t_h) = U_j(t_k)$, $j = L, H$. Consider a deviation from the equilibrium contract by principal $t \in \{t_h, t_k\}$ consisting of an increase by ε in the utility offered to both types of agent, without changing the effort levels; t's expected payoff becomes

$$\sum_{j=L,H} p_j [R(x_j(t); t) - C(x_j(t); \theta_j) - U_j(t) - \varepsilon] \Pr(W \mid U(t) + \varepsilon)$$

Differentiating with respect to ε and letting ε go to zero we obtain

$$\sum_{j=L,H} p_j [R(x_j(t); t) - C(x_j(t); \theta_j) - U_j(t)] \frac{d \Pr(W \mid U(t))}{dU} - \Pr(W \mid U(t))$$

The above expression must be zero for t_h and t_k. This implies

$$\sum_{j=L,H} p_j [R(x_j(t_h); t_h) - C(x_j(t_h); \theta_j) - U_j(t_h)] = \sum_{j=L,M} p_j [R(x_j(t_k); t_k) - C(x_j(t_k); \theta_j) - U_j(t_k)]$$

However, from the optimization program of principal t_h we have

$$\sum_{j=L,H} p_j [R(x_j(t_h); t_h) - C(x_j(t_h); \theta_j) - U_j(t_h)] \Pr(W \mid U(t_h)) \geq$$

$$\sum_{j=L,M} p_j[R(x_j(t_k),t_h) - C(x_j(t_k),\theta_j) - U_j(t_k)]\Pr(W \mid U(t_k)) >$$
$$\sum_{j=L,M} p_j[R(x_j(t_k),t_k) - C(x_j(t_k),\theta_j) - U_j(t_k)]\Pr(W \mid U(t_k))$$

Thus, there cannot exist two principals t_h and t_k, with $t_h > t_k$, and $U_j(t_h) = U_j(t_k)$, $j = L, H$, and hence $U_j(t)$ must be monotonic. Clearly, it cannot be decreasing, otherwise the highest type principal would never attract the agent and would get a zero payoff. Such a principal would then profit by mimicking the offer of any principal making a positive payoff. ■

Proof of Lemma 5. First, we show that $\phi^F_{H,L}(t) > 0$ for all t. Concavity of the revenue function, convexity of the effort function, and $x^F_L(t) > x^F_H(t)$ imply:

$$R(x^F_H(t),t) > R(x^F_L(t),t) - R_x(x^F_H(t),t)[x^F_L(t) - x^F_H(t)]$$

$$C(x^F_L(t),\theta_H) > C(x^F_H(t),\theta_H) + C_x(x^F_H(t),\theta_H)[x^F_L(t) - x^F_H(t)]$$

Hence,

$$\phi^F_{H,L}(t) > -R_x(x^F_H(t),t)[x^F_L(t) - x^F_H(t)] + C_x(x^F_H(t),\theta_H)[x^F_L(t) - x^F_H(t)]+$$
$$\frac{\int_a^t [R_t(x^F_L(\tau),\tau) - R_t(x^F_H(\tau),\tau)]G^{N-1}(\tau)d\tau}{G^{N-1}(t)}$$
$$= \frac{\int_a^t [R_t(x^F_L(\tau),\tau) - R_t(x^F_H(\tau),\tau)]G^{N-1}(\tau)d\tau}{G^{N-1}(t)} > 0$$

which completes part (i) of the lemma.

Suppose that $\phi^F_{L,H}(t) > 0$ for all $t \in [a,b]$. If principal t offers menus consisting of $x_j(t) = x^F_j(t)$ and $w_j(t)$ satisfying (6) for each agent j; then the incentive compatibility constraints do not bind, and therefore the standard auction equilibrium is an equilibrium of the incentive auction. ■

Proof of Lemma 6. (i) Consider equation (9), with $j = L$ and $k = H$. Note that $R_t(x_L^F(\tau);\tau) - R_t(x_H^F(\tau);\tau)$ is smaller than $\varepsilon[x_L^F(\tau) - x_H^F(\tau)]$. This implies that

$$\begin{aligned}\phi_{LH}^F(t) &> R(x_L^F(t);t) - C(x_L^F(t);\theta_L) - R(x_H^F(t);t) + C(x_H^F(t);\theta_L)\\ &\quad - \varepsilon\int_a^t [x_L^F(\tau) - x_H^F(\tau)]\frac{G^{N-1}(\tau)}{G^{N-1}(t)}d\tau\end{aligned}$$

Since $x_L^F(\tau)$ and $x_L^F(\tau)$ are bounded, by letting ε go to zero we conclude that

$$\phi_{LH}^F(t) > \left\{[R(x_L^F(t);t) - C(x_L^F(t);\theta_L)] - [R(x_H^F(t);t) - C(x_H^F(t);\theta_L)]\right\} > 0 \tag{28}$$

This holds because the first expression in square brackets is the first best total surplus when the agent is of type θ_L, while the expression in the second square brackets is total surplus when the agent is type θ_L, but effort is $x_H^F(t)$ rather than $x_L^F(t)$.

(ii) As b converges to a, the last two expressions on the r.h.s. of (9) converge to zero and thus $\phi_{LH}^F(t)$ converges to the expression in curly brackets in (28), which is positive. Thus, bids are as in a standard auction and by (5) the agent's utility converges to the first best surplus.

(iii) As n goes to infinity, the last two expressions on the r.h.s. of (9) converge to zero. Thus, $\phi_{LH}^F(t)$ converges to a positive number for all $t \in [a, b]$. Equation (5) shows that in the standard auction as n goes to infinity the agent gets the first best surplus level.

(iv) Equation (9) implies that $\lim_{\theta_H \to \theta_L} \phi_{LH}^F(a) = 0$. Next, divide both sides of equation (9),with $j = L$ and $k = H$, by $\theta_H - \theta_L$; then divide and multiply the first four terms on the l.h.s. by $x_L^F(t) - x_H^F(t)$. Finally, divide and multiply the two integrands on the l.h.s. by $x_L^F(\tau) - x_H^F(\tau)$. The limit as θ_H goes to θ_L is:

$$\lim_{\theta_H \to \theta_L} \frac{\phi_{LH}^F(t)}{\theta_H - \theta_L} =$$

$$\int_a^t R_{tx}(x_L^F(\tau);\tau)\frac{\partial x_L^F(\tau)}{\partial \theta}\frac{G^{N-1}(\tau)}{G^{N-1}(t)}d\tau - \left[R_x(x_L^F(t);t) - C_x(x_L^F(t);\theta_L)\right]\frac{\partial x_L^F(t)}{\partial \theta} =$$

$$\int_a^t R_{tx}(x_L^F(\tau);\tau)\frac{\partial x_L^F(\tau)}{\partial\theta}\frac{G^{N-1}(\tau)}{G^{N-1}(t)}d\tau < 0$$

where the second equality follows from $x_L^F(t)$ being the first best effort level, and the inequality follows from $x_L^F(t)$ being a decreasing function of θ. ■

Proof of Lemma 7. Equations (15) and (16) imply that $x_H(t) < x_L(t) = x_L^F(t)$: This in turn implies that if (IC_L) binds for principal t, then (IC_H) is slack. Thus, (IC_H) can be violated only when (IC_L) is slack. We know from Lemma 5 that if (IC_L) is slack for all principals in an interval $[a;t]$, then (IC_H) is also slack. Hence if (IC_H) is violated by our solution, it must be the case that (IC_L) binds in some interval $[t_*;t_L]$, that (IC_L) is slack and (IC_H) is not violated in an interval $(t_L;t_H)$, and that (IC_H) binds in an interval $[t_H;t_{**}]$ and is then violated in an interval $(t_{**};t_{***})$ (note that t_H could equal t_{**}).

Recall that (IC_H) binding at t_H is equivalent to

$$U_H(t_H) - U_L(t_H) + C(x_L(t_H);\theta_H) - C(x_L(t_H);\theta_L) = 0$$

We now show that (IC_H) cannot bind at t_H. First note that for $t \in [t_L;t_H]$, $e^{\varphi(t)-\varphi(t_H)} = G^{N-1}(t)/G^{N-1}(t_H)$. Integrating (20) and (21) between t_L and t_H yields

$$\begin{aligned} U_j(t_H) &= R(x_j(t_H);t_H) - C(x_j(t_H);\theta_j) - \frac{\int_{t_L}^{t_H} R_t(x_j(\tau);\tau)G^{N-1}(\tau)d\tau}{G^{N-1}(t_H)} \\ &\quad -[R(x_j(t_L);t_L) - C(x_j(t_L);\theta_j) - U_j(t_L)]\frac{G^{N-1}(t_L)}{G^{N-1}(t_H)} \end{aligned}$$

Thus,

$$\begin{gathered} U_H(t_H) - U_L(t_H) + C(x_L(t_H);\theta_H) - C(x_L(t_H);\theta_L) = \\ R(x_H(t_H);t_H) - C(x_H(t_H);\theta_H) - R(x_L(t_H);t_H) + C(x_L(t_H);\theta_H) \\ -[R(x_H(t_L);t_L) - C(x_H(t_L);\theta_H) - U_H(t_L)]\frac{G^{N-1}(t_L)}{G^{N-1}(t_H)} \\ +[R(x_L(t_L);t_L) - C(x_L(t_L);\theta_L) - U_L(t_L)]\frac{G^{N-1}(t_L)}{G^{N-1}(t_H)} \end{gathered}$$

$$+\frac{\int_{t_L}^{t_H}[R_t(x_L(\tau);\tau) - R_t(x_H(\tau);\tau)]G^{N-1}(\tau)d\tau}{G^{N-1}(t_H)} >$$

$$-[R(x_H(t_L);t_L) - C(x_H(t_L);\theta_H) - U_H(t_L)]\frac{G^{N-1}(t_L)}{G^{N-1}(t_H)}$$

$$+[R(x_L(t_L);t_L) - C(x_L(t_L);\theta_L) - U_L(t_L)]\frac{G^{N-1}(t_L)}{G^{N-1}(t_H)}$$

$$+\frac{\int_{t_L}^{t_H}[R_t(x_L(\tau);\tau) - R_t(x_H(\tau);\tau)]G^{N-1}(\tau)d\tau}{G^{N-1}(t_H)} >$$

$$\frac{\int_{t_L}^{t_H}[R_t(x_L(\tau);\tau) - R_t(x_H(\tau);\tau)]G^{N-1}(\tau)d\tau}{G^{N-1}(t_H)} > 0$$

The first inequality follows from $x_H(t_H) = x_H^F(t_H) < x_L^F(t_H)$, and the concavity of $R(\cdot) - C(\cdot)$ with respect to x. The second inequality follows from the fact that (IC_L) binds at t_L, $x_L(t_L) = x_L^F(t_L)$, and the concavity of $R(\cdot) - C(\cdot)$ with respect to x. The third inequality follows from $R_{tx} > 0$ and $x_L(t) > x_H(t)$. ■

Proof of Theorem 3. Consider the continuation games after messages have been sent. Let $p(\mu)$ be the posterior beliefs of all principals that the agent is θ_L after observing message μ. Clearly, if $p(\mu) \in (0,1)$, then in the unique symmetric equilibrium of the continuation game each principal offers a pair of contracts, one for each type, as specified by Theorem 1. If $p(\mu)$ is equal to 1, then each principal offers the standard auction contract to θ_L. There are many different equilibrium offers that principals could make to θ_H, since the probability of this agent type is zero. It is natural to assume that if $\phi^F_{L,H}(t) \geq 0$ for all principals in $[a,t]$, then t will also offer the standard auction contract to θ_H. If $\phi^F_{L,H}(t) < 0$ for some subinterval of principals in $[a,t]$, then all we assume is that t will offer θ_H a contract with the property that θ_L will not want to claim that he is θ_H; that is, the offer to θ_H must be such that (IC_L) is not violated.[10] Similarly, if $p(\mu) = 0$, so that the principals think that with probability one the agent is θ_H, then each principal offers θ_H the same contract as in the standard auction and offers θ_L a contract that satisfies (IC_H).

(i) Assume that both agents send message μ in equilibrium. Let the out of equilibrium beliefs attach probability one to the agent being type θ_L. Clearly, θ_L would not want to deviate, since he would lose information rents. Type θ_H would also not want to deviate. To see this, first observe that if $\phi^F_{L,H}(\cdot) \geq 0$ for all principals in $[a, b]$, then by deviating to a message different from μ type θ_H would end up with the same contract that he is currently receiving from principal t. If $\phi^F_{L,H}(\cdot) < 0$ for some subinterval of principals, by deviating θ_H can only receive the payoff that he would obtain if he claimed to be θ_L in the standard auction. This gives θ_H a lower payoff than he could obtain by sending the equilibrium message μ and then claiming that he is θ_L in the incentive auction.

We should note that in cheap talk games, equilibrium refinements that are derived from strategic stability, like the intuitive criterium and divinity, do not help to refine the set of sequential equilibria (see Kreps and Sobel (1994)). Furthermore, Grossman and Perry's (1986) perfect sequential equilibrium and Farrell's (1993) neologism proof equilibrium do not eliminate our pooling equilibrium, because if the agent makes the out of equilibrium statement that he is a type θ_H and the principals believe him, then there are best responses by the principals under which both agent types are better off than in the pooling equilbrium. This is because when the principals believe that the agent is type θ_H with probability 1, then they will offer θ_H the standard auction contract and can offer θ_L as much as they want as long as (IC_H) is not violated.

(ii) Type θ_j prefers to send his equilibrium message than θ_k's message if and only if

$$\int_a^b U_j^A(t)dG^N(t) \geq \int_a^b [U_k^A(t) - C(x_k^F(t), \theta_j) + C(x_k^F(t), \theta_k)]dG^N(t)$$

This condition is satisfied for all agent types if and only if (i) $E[\phi^F_{L,H}] \geq 0$ and (ii) $E[\phi^F_{H,L}] \geq 0$. By Lemma 5, $\phi^F_{H,L}(t) > 0$ and (ii) always holds. Specifying off the equilibrium path beliefs for all principals that assign probability one to θ_L completes the proof. ■

Endnotes

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2. Papers which model jurisdictions bidding for specific firms include Bond and Samuelson (1986), Black and Hoyt (1989), and Biglaiser and Mezzetti (1997). In each of these only one side of the market posseses private information.

3. This result rests on the agent's reservation utility being type independent. Several papers have examined type dependent reservation utility (e.g., Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995), Jullien (1997), and Rochet and Stole (1999)) and have shown that either upward or downward incentive constraints can bind and that bunching may occur.

4. These conditions are sufficient for deterministic incentive schemes to be optimal.

5. If the probability p that the agent's type is μ_L is very large, the principal prefers that the high type exert zero effort, so as to reduce the information rents of the low type agent; in such a case the individual rationality constraint for type μ_L binds.

6. Julien (1997) assumes that the agent's reservation utility is a function of his type, while Rochet and Stole (1999) assume that it is a random variable that is correlated with the agent's type. In both models the agent's reservation utility is exogenous; thus, they are not directly comparable to ours.

7. We should mention that verifying that (IC_H) is slack is more complicated than in the standard single-principal model.

8. Even if explicit transfers generally do not occur between states, implicit tranfers (e.g., quid pro quo agreements) could be used to enforce the moratorium.

9. Armstrong and Vickers (1998) and Rochet and Stole (1999) study models

with two horizontally differentiated principals with the same cost and no private information. They show that efficiency results when the principals offer two-part tariffs.

10. Our results do not depend on what offers the principals make for an agent when the probability that they are facing that agent is zero in equilibrium.

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