Simple strategy-proof approximately Walrasian mechanisms.[*]

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Abstract

This paper provides an indirect analysis of the incentive properties of the Walrasian mechanism. It presents mechanisms under which truth-telling is a dominant strategy in finite exchange economies (in contrast to the Walrasian mechanism) and whose outcomes (generically) approximate Walrasian ones for large economies. These mechanisms provide new insights on the well-known trade-off between efficiency and incentive compatibility in finite economies.

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1 Introduction.

An extensive literature has been developed in the last decades on the implementation of different social choice correspondences. Among these correspondences is the one associating to each exchange economy the set of its Walrasian equilibria, a concept that lies at the very heart of the economic theory. The results on the implementability of the Walrasian equilibria correspondence can be dramatically different, depending on our information assumptions. These results are quite positive in the case of complete information and quite negative if information is incomplete.

More specifically, it has been shown that if the members of an exchange economy know all the parameters of the economy, then there exist institutions which implement the Walrasian correspondence. Unfortunately, no hope exists to get a similar result if information is incomplete and the number of agents is finite.[1] First of all, it was proven in the seminal paper by Hurwicz (1972) that Walrasian allocations are not strategy-proof.[2] This insight was deepened by Palfrey and Srivastava (1987) who showed that the Walrasian correspondence fails to satisfy even Bayesian incentive compatibility. Moreover Palfrey and Srivastava (1987) demonstrated that Bayesian incentive compatibility is also violated by the rational expectations equilibrium correspondence. So one cannot overcome the difficulties faced by Walrasian equilibria simply considering this alternative solution concept.

The problem disappears completely for continuum economies having a non-atomic measure space of agents.[3] For such economies Walrasian equilibrium is strategy-proof. These economies can be viewed as limits of economies with large but finite numbers of agents. Continuum economies have been extensively used in the analysis of many economic questions, like the financing of public goods, market liberalization or redistributive welfare programs.[4] But in order to assess the relevance of the conclusions obtained for continuum economies, one should always provide some limit theorem proving that there exist a sequence of mechanisms (or solutions) for finite economies, which approximate the initial mechanism (or the solution) in the limit and possess similar properties, at least generically. To illustrate this point, let us recall the foundations for two classical "equivalence theorems" for continuum economies. Various core convergence results for finite economies have supported the "core-equilibrium equivalence" established by Aumann (1964). (See Hildenbrand (1975) or Anderson (1992) for surveys.) On the other side, a recent result of Anderson, Trockel and Zhou (1997) on the nonconvergence of the Mas-Colell bargaining set shows that at

[1] Positive results are possible in a very special circumstances, e.g. if any single agent's information can be verified by the remaining agents (so called case of the nonexclusive information). In many relevant situations conditions of that type are not satisfied. Generally, agents may know about their own preferences more than about the preferences of other agents.

[2] Strategy-proofness means dominant strategy incentive compatibility.

[3] Continuum economies were introduced by Aumann (1964, 1966) in general equilibrium theory.

[4] In the case of optimal redistributive income taxation Vickrey (1945) and Mirrlees (1971) are very early references.

least more sophisticated constructions are needed to support in …nite economies the "bargaining equivalence theorem" established by Mas-Colell (1989).

The present paper addresses a similar question with respect to the Walrasian correspondence. Recall that Walrasian equilibrium in continuum economies is strategy-proof, individually rational, and Pareto e¢cient. The Walrasian correspondence preserves individual rationality and Pareto e¢ciency in an economy of any size. But, as we have seen above, the Walrasian equilibrium fails to satisfy incentive-compatibility properties in …nite economies. One line of research has been devoted to test the intuitive idea that these incentive properties of the Walrasian equilibrium become stronger as economies grow larger. Before I present my contribution to this subject, let me review some relevant results.

One set of papers studies the robustness of price-taking equilibria to incentive constraints. This literature begins with the paper of Roberts and Postlewaite (1976). Roberts and Postlewaite proved that, given some regularity condition on the limit economy, the bene…ts from individual deviations from price-taking behavior became arbitrary small for a …xed agent in an economy of increasing size. This result was strengthened by Jackson (1992), who showed that individual behavior would also approximate competitive behavior, again under the regularity condition. However, these papers do not deal with the question whether the resulting allocations approximate the Walrasian ones.

Otani and Sicilian (1990) have shown that the set of Nash equilibria of the Walrasian strategic game[5] can be quite large and stay di¤erent from the competitive allocations, even as the number of agents increase. Otani and Sicilian have also stated conditions ensuring that Nash equilibrium allocations approximate Walrasian ones.[6] But the use of the Nash equilibrium concept requires the information to be complete.

In their recent paper Jackson and Manelli (1997) have identi…ed conditions on the beliefs of agents under which all the market-clearing prices and allocations of the reported economy approximate the competitive equilibria of the true economy. Another recent paper by Makowski, Ostroy and Segal (1995) has identi…ed conditions on the richness of the domains of preferences that allow to characterize e¢cient and incentive compatible mechanisms as perfectly competitive.[7] None of these two papers require incentive compatibility everywhere. Jackson and Manelli have restricted domains for incentive compatibility by beliefs. Makowski, Ostroy and Segal asymptotically

[5] That is, agents strategically submit demand functions to some Walrasian auctioneer.

[6] As usual, among these assumptions there is the regularity condition, but by the nature of the Nash equilibrium, the regularity is required on the limit of the reported economies. It is rather negative news for the justi…cation of Walrasian equilibrium from Nash equilibrium in large economies that in order to get a positive result the possible reports should be restricted not in a Cartesian way. An example provided by Jackson and Manelli (1997) shows that there is no hope to obtain a positive result about convergence without the regularity assumption on the limit of reported economies.

[7] In the special sense: each agent must be unable to in‡uence prices or anyone's wealth.

achieve the condition generically.

One can conclude that no paper in this line of the literature provides a clear general response to the following question, under incomplete information: How can one explain the strong incentive properties of Walrasian equilibrium in continuum economies, given that it has such weak incentive properties for …nite economies ? This is the question I address in this paper.

Not many papers, in fact, deal with implementation under incomplete information. Two of them should be mentioned since they provide partial responses to the question of our interest.

Gul and Postlewaite (1992) established that in large replica exchange economies with incomplete information, there exists an incentive compatible and individually rational allocation rule that is nearly e¢cient. This rule is the competitive rule applied to an arti…cial economy, which is near the actual one with high probability. But this result crucially depends on the …niteness of the type space.[8]

Mas-Colell and Vives (1993) demonstrates that for any continuous mechanism, which implements uniquely some Walrasian allocation in the continuum economy, the following robustness result holds. In the large …nite economies obtained as samples from the continuum economy, all Bayesian equilibria will (with probability close to one) yield an allocation which is almost Walrasian. However this result is proved only for so called parametric mechanisms. That is, it is assumed that the agents are independently drawn from a known distribution of possible agents' characteristics. Therefore the most challenging case of an unknown distribution is not considered.

One of the main problems of the Bayesian approach to implementation under incomplete information is that the results depend crucially on the priors. That is, essentially these results are not robust. One way to overcome this di¢culty is to provide results, as e.g. Mas-Colell and Vives (1993) does, for all Bayesian equilibria. Another way is to require a certain robustness to the change in the priors. In the weakest sense this means the continuity of the implemented allocations with respect to priors. In the strongest sense robustness would require the allocations resulting from implementation and even the equilibrium strategies to be the same for any priors. The latter is equivalent to implementation in dominant strategies. Since what is interesting in our context is the revelation of truth (for e¢ciency reasons), we restrict attention to strategy-proofness. An additional advantage of strategy-proof mechanisms is its simplicity. In a strategy-proof mechanism no agent should guess anything about the preferences or beliefs of other agents but can simply reveal his or her own preferences.

Unfortunately in …nite exchange economies no strategy-proof mechanism can be e¢cient and individually rational. This trade-o¤ between e¢ciency and incentive compatibility was discovered in the above mentioned pioneering work of Hurwicz

[8] See Jackson and Manelli (1997) for a discussion of this paper.

(1972). General and generic impossibility theorems for finite quasi-linear models with public and private goods were established, within the same spirit, by Hurwicz and Walker (1990), Beviá and Corchón (1995). Another rather negative result is provided by Barberà and Jackson (1995). Barberà and Jackson have shown that if strategy-proof behavior is required on the part of individuals and the possible mechanisms are restricted by two other properties called anonymity and non-bossiness, then the only achievable allocations are far from efficient even in large economies. Their result together with those mentioned above strongly suggest that probably there is no good limit theorem for the Walrasian correspondence in terms of strategy-proofness.[9]

The first attack on this problem was made recently by Córdoba and Hammond (1998), who present the following strategy-proof and individually rational mechanism. The population is divided into two groups for which the market-clearing prices are computed separately. Budget sets are based on one of these two prices, with some additional transaction fees for buying and selling all goods except the numéraire (commodity one). Then each agent on one part of the economy faces a budget set computed on the basis of the characteristics of agents in the other part of economy. The mechanism is clearly strategy-proof. To make it balanced, Córdoba and Hammond consider that agents are independently drawn from some distribution of possible agents' characteristics. The law of large numbers and a careful choice of transaction fees allows then to make this mechanism balanced with high probability in large economies. Clearly this mechanism is also approximately Walrasian for all agents except for the balancing agent. Córdoba and Hammond also present two balanced versions of this mechanism. One variation is strategy-proof but individually rational only in large economies with high probability. The other is individually rational but strategy-proof only in large economies with high probability.

Córdoba and Hammond's (1998) paper is subject to some criticism. First of all, it leaves it unclear how much the result is due to the probabilistic approach to the problem. In the Córdoba-Hammond mechanisms either strategy-proofness, individual rationality, or balancedness are satisfied in large economies only with high probability. Thus there is still room for large non-balancedness, manipulability, or non-rationality with some small probability, even in arbitrarily large finite economies. On the other side, due to the careful choice of transaction fees, the mechanisms that Córdoba and Hammond propose are rather complicated and their proofs (based on the probabilistic results) are far from being straightforward. In the present paper I show that simple mechanisms can be used to get similar results which are even certain in nature.[10]

The following general idea has been suggested by several people as the basis to

[9] Barberà and Jackson (1995) also implies that non-anonymous or bossy mechanisms are the only candidates for the limit theorem. The mechanisms considered in the present paper are anonymous but bossy.

[10] That is, the results are of generic nature for all sufficiently large economies. No probabilities are required to formulate the theorems.

construct a strategy-proof, individually rational and asymptotically Walrasian mechanism. Consumers in a given economy are asked to report their demands. Then for each consumer a market-clearing price for the economy formed by all consumers except this one is computed. This price is assigned to that (removed) consumer. Then agents are allocated the consumption bundle that results from their reported demands and their assigned prices.

Obviously, the price that each consumer faces is una¤ected by her reported demand and it is a dominant strategy to announce one's truthful (price-taking) demand. Thus the mechanism is clearly strategy-proof and individually rational. One drawback of the mechanism is its lack of balance. Some outside party will have to absorb excesses and fund shortfalls. But it has been conjectured that in large …nite economies two things will happen: (A) the aggregate imbalance will not be too large, and (B) the resulting allocation will be close to the Walrasian (competitive) one.

This mechanism, in fact, owes much to the spirit of the Vickrey-Clarke-Groves mechanisms (Vickrey (1961), Clarke (1971), Groves (1973)), which are well-known and apply to models with quasi-linear utilities. For classical exchange economies it was implicitly mentioned in Hammond (1979). This mechanism was called a "folk" mechanism in private communications and e.g. in Jackson and Manelli (1997). So there is a good reason to adopt this name.

The above description of the folk mechanism has avoided a very important issue. In general, an economy can have more than one market-clearing price. Therefore, a complete speci…cation of the mechanism must prescribe precisely one price selection under a multiplicity of market-clearing prices. Yet, to choose a price selection rule that always works is impossible.[11] In this paper we consider folk mechanisms speci…ed with some price selection rules which are generically continuous on the parameters describing the economy. The existence of such price selection rules is established in Mas-Colell (1985, Proposition 5.8.18). This makes the relevant mechanism to be rigorously speci…ed and we can address the conjecture about its behavior.

First we show that the aggregate imbalance generated by the mechanism can be arbitrarily large (relative to the individuals' endowments), even for large economies. In the example that we provide no multiplicity of market-clearing prices arises, and therefore this fact is independent of the price selection rule. Thus, part (A) of what we could call the "folk conjecture" can be supported only in per capita terms. Then we demonstrate (the Proposition) that, with the speci…cation of the price selection rule that we made, part (B) of the "folk conjecture," as well as the per capita version of part (A) hold generically. This result appears to be simple and straightforward.

[11] This is because such a rule should meet the following features. On the one side, the selection of the price assigned to each consumer should not be a¤ected by the reported demand of that consumer: otherwise the strategy-proofness of the mechanism will be lost. On the other side, the selection should be consistent across all the consumers, in order to guarantee that prices assigned to di¤erent individuals will be close to each other: otherwise, there would be no hope to control imbalances if the assigned prices were arbitrary di¤erent. Finally, if we want to state a general result, no speci…c properties can be asked from the limit economy.

All standard smoothness assumptions on the preferences are needed only to make it possible to appeal to the mentioned above result of the Mas-Colell. To complete the research on the "folk conjecture" we show that no extension of the Proposition can be obtained that holds not only generically but always. The proof is based on an example due to Roberts and Postlewaite (1976).

Obviously the main drawback of the folk mechanism is that it can be unbalanced. Because of that, we present a modified mechanism which is also strategy-proof and (generically) balanced for all sufficiently large economies.[12] We call it "ε-tax" mechanism because in a first stage all agents are taxed an ε proportion of the minimal endowment and then the folk mechanism is applied at a second stage. This first-stage taxation weakens some of the properties of the folk mechanism, but it forces balancedness in large economies. Moreover, the allocations prescribed by the ε-tax mechanism are individually ε-rational, ε-efficient, and ε-Walrasian. Therefore, this family of mechanisms illustrates how the strong incentive compatibility properties of the Walrasian mechanism appear in the continuum economies. The family of the ε-tax mechanisms provides a natural and simple answer to the question addressed in the present paper.

The next section states the definitions and describes the folk mechanism. Section 3 presents results on the "folk conjecture." In the section 4 the balanced version of the folk mechanism is described and the main result (the Theorem) is stated. The last section contains some final remarks.

2 Definitions.

There is finite set A of individuals and a finite number of goods, l. For simplicity, each agent $a \in A$ has a positive endowment of all goods $e(a) \in R^l_{++}$ and a consumption set R^l_{++}. Agent a's preferences are represented by reflexive, complete and transitive binary relation on R^l_{++} and denoted $\succsim_a$. Preferences are C^2, differentiably strictly convex,[13] strictly monotone,[14] and satisfy the standard boundary condition.[15] $\tilde{A}$ denotes the set of all conceivable agents whose preferences and endowments satisfy the above assumptions.

An agent a's competitive (excess) demand correspondence d_a is a map from Δ_+, the unit simplex of R^l_+, into R^l with the properties that if $x \in d_a(p)$, then $p \cdot x = 0$, $(x + e(a)) \in R^l_+$ and $(x + e(a)) \succsim_a (y + e(a))$ for any y, such that $p \cdot y \leq 0$. Given the properties of preferences, it follows that for any a $d_a(p)$ is single valued at $p \gg 0$ and

[12] The mechanism is balanced in the sense that no shortfall occurs. The mechanism still runs a surplus and requires the existence of an agent (who is not part of the economy) to accept the excesses.

[13] That is, Gaussian curvature is positive.

[14] $x \succsim_a y$ and not $y \succsim_a x$, whenever $x_k \geq y_k$ for all $1 \leq k \leq l$ and $x \neq y$.

[15] $\{z \in R^l_{++} : z \succsim x\}$ is closed in R^l for all x.

empty otherwise. Moreover, being defined only for $p \gg 0$; d_a is also C^1 function.[16] With some abuse of notations we will standardly use for d_a a word "function" instead of "correspondence."

An exchange economy $\mathcal{E} = \left\langle A; \{e(a); \nu_a\}_{a \in A} \right\rangle$ is given by a set of agents and a list of their endowments and preferences. An alternative description of the economy $\mathcal{E}$ is $\mathcal{E} = \left\langle A; \{d_a\}_{a \in A} \right\rangle$, i.e. as a set of agents and a list of their competitive demand functions. Although the second description is not so informative as the first one, for much of our purposes it is sufficient.

Let us consider $D := \bigcup_{a \in A} d_a$. Let us give to D the topology of the uniform convergence on compacta of the values of the function and the first derivative. Let δ be any metric that generates this topology. A convenient way to represent the economy $\mathcal{E}$ is by a simple probability measure μ defined on D. The interpretation is that a proportion $\mu(d)$ of agents in the economy have a competitive (excess) demand function d.

Let M denote the set of all Borel probability measures on D which have compact support. M is endowed with the following topology of convergence: we say that $\nu_n \to \nu$ if (i) $\operatorname{supp}(\nu_n) \to \operatorname{supp}(\nu)$ (in the Hausdorff distance h) and (ii) $\nu_n \to \nu$ weakly (that is, with respect to the Prohorov metric ρ.[17]). Let us endow M with the following metric $\bar{\rho}$: $\bar{\rho}(\gamma; \mu) = h(\operatorname{supp}(\gamma); \operatorname{supp}(\mu)) + \rho(\gamma; \mu)$:

The (Walrasian) equilibrium price correspondence Π from M to Δ_+ is defined by

$$\Pi(\mu) = \left\{ p \in \Delta_+ : 0 \in \int d(p) d\mu \right\}.$$

Notice, that given the structure of D, if $p \in \Pi(\mu)$ for some μ, then $p \gg 0$.

A selection from the equilibrium correspondence is a mapping $p : M \to \Delta_+$; such that $p(\xi) \in \Pi(\xi)$ for all $\xi \in M$: We will call some selection from the equilibrium price correspondence $p : M \to \Delta_+$ an appropriate selection if this selection is measurable on the entire M and is continuous on some dense and open subset of M. All our mechanisms will be defined with the use of an appropriate selection.

Existence of an appropriate selection. The measure $\mu \in M$ (associated with the economy $\mathcal{E}$) is regular if there exist a neighborhood B of μ and a finite number of continuous functions $p^i : M \to \Delta_+$, $1 \le i \le n$; such that, for each $\xi \in B$, $p^i(\xi) \neq p^j(\xi)$ if $i \neq j$ and $\Pi(\xi) = \bigcup_{i=1}^n \{p^i(\xi)\}$. Let us denote the set of all regular measures by M^*: It follows from the definition that M^* is open in M: Our previous assumptions on preferences assure that M^* is dense in M:[18]

Our assumptions do not guarantee the existence of a continuous selection defined on the whole domain M, or even on the whole sub-domain of regular measures M^*:

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[16] See Mas-Colell (1985, Prop. 2.7.2).

[17] $\rho(\gamma; \mu) = \inf\{\varepsilon > 0 : \gamma(E) \le \mu(B_\varepsilon(E))$ and $\mu(E) \le \gamma(B_\varepsilon(E))$ for every Borel set $E \subset D\}$, where $B_\varepsilon(E)$ denotes the ε-neighborhood of E.

[18] See Mas-Colell (1985, Prop. 5.8.14 and 5.8.16). Alternatively Dierker (1974) or K.Hildenbrand's appendix 2.3 in Hildenbrand (1974) provide the genericity analysis of regular economies (measures).

But, Proposition 5.8.18 in Mas-Colell (1985) establishes the existence of a continuous selection on some open and dense subset of M^* (and therefore also an open and dense subset of M).

Moreover, since the equilibrium price correspondence has a closed graph[19] there exists a selection that is measurable everywhere.[20] This makes possible to choose a particular selection $p : M \to \Delta_+$ from the equilibrium price correspondence, such that this selection is measurable on the whole M and continuous on some dense and open subset M_p of M. Of course, such a selection need not to be unique.

The description of the folk mechanism. Now we can define the folk mechanism: Let us fix some appropriate selection p^*:

1st Stage. Consumers are asked to report their excess demands.[21] Let a measure μ describe a reported economy, in the sense that $\mu(d)$ represents a proportion of agents that announce a demand d.

2nd Stage. For each consumer i consider an economy without that consumer μ_{-i} and compute a price $p^*(\mu_{-i})$. If the consumer i has reported demand $\hat{d}_i$ then she is allocated $\hat{d}_i(p^*(\mu_{-i}))$.

Notice that no consumer can influence the price assigned to herself. Therefore, it is a dominant strategy to report the true demand, and the folk mechanism is strategy-proof. It is also obvious that this mechanism is individually rational. The drawback of this mechanism is that it is not balanced, but we will clarify in the next section that the folk mechanism is approximately balanced and approximately Walrasian. A balanced version of the folk mechanism which we name ε-tax mechanism will be presented in a special section.

3 On the "folk conjecture."

The following simple Example illustrates that any specification of the folk mechanism can create an arbitrarily large aggregate imbalance (relatively to the uniform bound on endowments) even in large economies. Notice that no price selection is necessary in the Example, since under truth-telling reports there is always a unique market-clearing price. So, the result is independent of the possible variations of the mechanism.

Example. Consider an arbitrarily large integer m. We construct a replica sequence of economies with two goods. All agents have the same preferences represented by

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[19] See Hildenbrand (1974, Prop. 4, p.152).

[20] It follows from Hildenbrand (1974, Lemma 1, p.55 and Prop. 1, p.22).

[21] This is a common knowledge that consumers have excess demands from the set D. Consumers can report their excess demand only from the set D. They can not pretend to have e.g. non-smooth or non-convex preferences.

utility function $u(x_1, x_2) = x_1 x_2$. Agents differ only in their endowments, which are either of type $e(a) = (0, 2)$ or $e(b) = (2, 0)$. The economy k has k agents of the first type and mk agents of the second type. The price taking demands of agents are $d_a(p_1, p_2) = (\frac{p_2}{p_1}, 1)$ and $d_b(p_1, p_2) = (1, \frac{p_1}{p_2})$. The competitive equilibrium price in the economy k will be $\frac{p_2}{p_1} = m$ for any k. For the price p^a assigned by the mechanism to the agents of type a, the price ratio will be a solution of the market-clearing equation: $(k - 1)\frac{p_2^a}{p_1^a} + mk = 2mk$, that is $\frac{p_2^a}{p_1^a} = \frac{mk}{k-1} = m + \frac{m}{k-1}$. So, the imbalance created by the mechanism for good 1 will be $k(m + \frac{m}{k-1}) + mk - 2mk = m(\frac{k}{k-1}) > m$. Thus for any k the mechanism creates a shortfall of good 1 of size larger than m. Obviously one can change slightly the endowments to achieve $e(a), e(b) \in R^l_{++}$ but still facing for any k a shortfall of good 1 of size larger than $\frac{m}{2}$.

Therefore the Example illustrates that Part (A) of the "folk conjecture" must be qualified. No hope exists for the aggregate imbalance to become "small" in large economies. However, the following Proposition shows that, under the specification that we made for the price selection, Part (B) of the "folk conjecture," as well, as per capita version of Part (A) can be supported generically.

Proposition. Consider a folk mechanism based on some appropriate selection rule p^*. Let M_{p^*} denote the open and dense subset of M where p^* is continuous. Let this mechanism be applied to some sequence of finite exchange economies $\{E^k\}$, described by measures $\{\mu_k\}$, and such that $|A^k| \to \infty$ and $\mu_k \to \mu$, where $\mu \in M_{p^*}$. Let μ_k^{-i} describe the same economy as μ_k but without consumer i. Then truth-telling is a dominant strategy for any consumer, and under truth-telling the following holds:

$$(a): \lim_{k\to\infty} \max_{i\in A^k} \left\| p^*(\mu_k^{-i}) - p^*(\mu_k) \right\| = 0.$$

(That is, prices prescribed by the mechanism are approximately Walrasian.)

$$(b): \lim_{k\to\infty} \max_{i\in A^k} \left\| d_i(p^*(\mu_k^{-i})) - d_i(p^*(\mu_k)) \right\| = 0.$$

(That is, allocations prescribed by the mechanism are approximately Walrasian.)

$$(c): \lim_{k\to\infty} \frac{1}{|A^k|} \left\| \sum_{i\in A^k} d_i(p^*(\mu_k^{-i})) \right\| = 0.$$

(That is, the per capita imbalance of the mechanism vanishes in large economies)

Proof of the Proposition: 1). Let us begin with some auxiliary constructions. Let us define $\Delta_n = \left\{ x \in \Delta_+ : x_i \geq \frac{1}{n}, 1 \leq i \leq l \right\}$. There exist n^* such that $p^*(\mu) \in \mathrm{Int}\,\Delta_{n^*}$. Consider any $\eta > 0$. By the definition of the metric δ there should exist $\varepsilon > 0$ such that $\max_{p\in\Delta_{n^*}} \left\| d(p) - \tilde{d}(p) \right\| < \eta$ whenever $\delta(d, \tilde{d}) < \varepsilon$.

Consider now for any $d \in supp(\mu)$ an open ball $B(d;\frac{\varepsilon}{2}) = \left\{ d' : \delta(d;d') < \frac{\varepsilon}{2} \right\}$. Since $supp(\mu)$ is compact, there exists a finite set $\{d_j\}_{j\in J}$ such that $supp(\mu) \subset \bigcup_{j\in J} B(d_j;\frac{\varepsilon}{2})$ and $\mu(B(d_j;\frac{\varepsilon}{2})) > 0$ for any $j \in J$. Then, because $\mu_k \to \mu$ and $\left|A^k\right| \to \infty$, there exists k_ε, such that if $k \geq k_\varepsilon$: (i) $supp(\mu_k) \subset \bigcup_{j\in J} B(d_j;\varepsilon)$ and (ii) for any $j \in J$ there exist two consumers $l, m \in A^k$ such that $d_l, d_m \in B(d_j;\varepsilon)$.

2). Then (ii) implies that for $k \geq k_\varepsilon$, $i \in A^k$: $h(supp(\mu_k); supp(\mu_k^{-i})) < \varepsilon$. Thus we have that $\max_{i\in A^k} h(supp(\mu_k); supp(\mu_k^{-i})) \to 0$ (all our statements on convergence are for $k \to \infty$). Since $\rho(\mu_k;\mu_k^{-i}) \leq \frac{1}{|A^k|}$ for any $i \in A^k$ and $\left|A^k\right| \to \infty$, we have that $\max_{i\in A^k} \rho(\mu_k;\mu_k^{-i}) \to 0$. Hence $\max_{i\in A^k} \tilde\rho(\mu_k;\mu_k^{-i}) \to 0$.

Since $\mu_k \to \mu$, we have that $\tilde\rho(\mu_k;\mu) \to 0$. Therefore $\max_{i\in A^k} \tilde\rho(\mu_k^{-i};\mu) \to 0$. Then, because $\mu \in M_{p^*}$, M_{p^*} is open, and p^* is continuous on M_{p^*}, we get that

$$\max_{i\in A^k} \left\| p^*(\mu_k) - p^*(\mu_k^{-i}) \right\| \to 0.$$

3). Thus there exists k', such that if $k \geq k'$ then $p^*(\mu_k), p^*(\mu_k^{-i}) \in \Delta_{n^*}$ for any $i \in A^k$. Also for any $j \in J$, since d_j is continuous, there exists k^j such that for any $k \geq k^j$

$$\max_{i\in A^k} \left\| d_j(p^*(\mu_k)) - d_j(p^*(\mu_k^{-i})) \right\| < \eta.$$

Then for any $k \geq \max\left\{ k_\varepsilon, k', \max_{j\in J}(k^j) \right\}$ for any $d \in supp(\mu_k)$ there exists d_j, $j \in J$ such that $\delta(d;d_j) \leq \varepsilon$ (and therefore $\max_{p\in\Delta_{n^*}} \|d(p) - d_j(p)\| < \eta$) and we get $\left\| d(p^*(\mu_k^{-i})) - d(p^*(\mu_k)) \right\| \leq \left\| d(p^*(\mu_k^{-i})) - d_j(p^*(\mu_k^{-i})) \right\| + \left\| d_j(p^*(\mu_k^{-i})) - d_j(p^*(\mu_k)) \right\| + \|d_j(p^*(\mu_k)) - d(p^*(\mu_k))\| < \eta + \eta + \eta = 3\eta$. Thus we get that for any $d \in supp(\mu_k)$:

$$\max_{i\in A^k} \left\| d(p^*(\mu_k)) - d(p^*(\mu_k^{-i})) \right\| \to 0.$$

4). And finally $\frac{1}{|A^k|} \left\| \sum_{i\in A^k} d_i(p^*(\mu_k^{-i})) \right\| = \frac{1}{|A^k|} \left\| \sum_{i\in A^k} \left[d_i(p^*(\mu_k^{-i})) - d_i(p^*(\mu_k)) \right] \right\| \leq \frac{1}{|A^k|} \sum_{i\in A^k} \left\| d_i(p^*(\mu_k^{-i})) - d_i(p^*(\mu_k)) \right\| \leq \max_{i\in A^k} \left\| d(p^*(\mu_k)) - d(p^*(\mu_k^{-i})) \right\| \to 0.$ ■

Remark 3.1. Justification of the generic nature of the result. The famous example from Roberts and Postlewaite (1976) can be used to show that our Proposition cannot be generalized for all sequences of economies with any limit. Thus there is no version of the Proposition that would hold on the whole domain of the limiting measures. Notice that this applies independently of the price selection rule, since no multiplicity of market-clearing prices arises.

Without going to the very details of the example, let us just mention that Roberts and Postlewaite (1976) presented a sequence of economies (with nonregular limit) in which agent 1 can gain significantly by misrepresenting his competitive demand (say

d_a) with another demand (say d_b). In fact, his deviation also changes significantly the (unique) market-clearing price. (Let's say from $p(a)$ to $p(b)$.) What is important for us is that $\delta(d_a; d_b)$ can be taken very small in the example.

Now, let us consider the same sequence of economies that was described there, but with one more agent that has competitive demand d_b. Then the price prescribed by any variation of the folk mechanism to the agent with demand d_a, will be a market-clearing price computed without her (but with d_b, of course). So, this agent will be prescribed the price $p(b)$. On the other side, the agent with demand d_b will be prescribed the price $p(a)$. These prescribed prices will be unchanged even for large economies. And so, it can't be the case that $\max_i \|p^i(k) - p^*(k)\| \to 0$ for some prices $\{p^*(k)\}$, simply because $\|p(a) - p^*(k)\| + \|p(b) - p^*(k)\| \geq \|p(b) - p(a)\|$, that is unchanged and positive. In the same way the resulting allocations can't converge to some Walrasian ones, since allocations prescribed to agents with demands d_a and d_b are significantly different and unchanged even if the economy grows.

Remark 3.2. On the smoothness assumptions. We have seen in the previous remark that any analog of the Proposition can be only of a generic nature. To study generic properties we need to assume a sufficient level of smoothness on the preferences. However, it is clear from the proof of the Proposition that we use the smoothness assumptions only to establish an existence of an appropriate price selection. Thus the Proposition holds with any continuous, strictly monotone, and strictly convex preferences, given an existence of an appropriate selection.

The main drawback of the folk mechanism is it's lack of balancedness. In the next section we present a balanced version of the folk mechanism.

4 Balanced mechanisms.

Let us make the additional assumption that all endowments are uniformly bounded away from zero by some $\hat{e} \in R^l_{++}$, that is $\hat{e} \leq e(a)$ for any $a \in A$. Let us assume the existence of one more agent $g \notin A$; called the government or the principal, who will be willing to accept any excesses of goods. Now we can present a new mechanism.

The description of the ε-tax mechanism. Let us fix $\varepsilon \in R$; $0 < \varepsilon < 1$:

1st Stage. A stock of goods is collected in the following way: each agent $a \in A$ contributes to the stock a proportion ε of the minimal endowment $\hat{e}$ (that is $\varepsilon\hat{e}$) as an explicit participation cost.

2nd Stage. Consumers from A report their demands and the folk mechanism (based on some selection rule) is applied. The initially created stock is used only to cover possible shortfalls. The rest of the stock and possible excesses of goods go to the balancing agent g:

It turns out that, unlike the folk mechanism, the ε-tax mechanism is balanced in all sufficiently large economies. To show this and other nice properties let us first introduce some definitions for some fixed economy $E = \left(A; \{e(a); \succeq_a\}_{a\in A}\right)$.

Notations. An allocation x is a function from A to R^l_{++}: An allocation x is attainable if $\sum_{a\in A} x(a) \leq \sum_{a\in A} e(a)$: Let x_k denote the k^{th} coordinate of $x \in R^l_{++}$: We will say that allocation y is ε-close to x ($y \in B(x;\varepsilon)$) if $x_k - \varepsilon < y < x_k + \varepsilon$ for any $k = 1; \ldots; l$:

Individual ε-rationality. An allocation x is ε-rational for agent a if $(1-\varepsilon)e(a) \preceq_a x(a)$: An allocation x is individually ε-rational if it is ε-rational for any $a \in A$: (That is, each agent is not worse off than with a proportion $(1-\varepsilon)$ of her initial endowment.)

ε-efficiency. An attainable allocation x is ε-efficient if there exist an allocation y such that: (i) $y(a) \in B(x(a);\varepsilon)$ for any $a \in A$; (ii) y is Pareto efficient, and (iii) $\sum_{a\in A} y(a) \geq (1-\varepsilon)\sum_{a\in A} e(a)$: (That is, at least a proportion $(1-\varepsilon)$ of the total endowment is distributed ε-close to an efficient distribution.)

ε-Walrasian allocations. An attainable allocation x is ε-Walrasian if there exist an allocation y such that (i) $y(a) \in B(x(a);\varepsilon)$ for any $a \in A$ and (ii) y is a Walrasian equilibrium in the economy $E' = \left(A; \{w(a); \succeq_a\}_{a\in A}\right)$ where $(1-\varepsilon)e(a) \leq w(a) \leq e(a)$: (That is, an allocation is ε-close to some Walrasian equilibrium based on endowments that are smaller but within a proportion ε of the initial endowments.)

Notice that for $\varepsilon = 0$ our definitions correspond to the standard ones. Namely, an allocation x is 0-efficient (or individually 0-rational, 0-Walrasian) simply means that x is Pareto efficient (correspondingly, x is individually rational, x is a Walrasian allocation in the initial economy). Also one can check that if an allocation x is ε-Walrasian, then x is ε-efficient.

Now we can state a result that summarizes properties of the ε-tax mechanisms. Mathematically it is a simple consequence of the Proposition, so the word "theorem" is used only to stress the fact that this is the main result of the paper.

Theorem. Consider any ε-tax mechanism, $0 < \varepsilon < 1$: Then:

(I) (Generic properties) There exist a set M_ε; the open and dense subset of M; such that if this mechanism is applied to some sequence of finite exchange economies $\{E^k\}$; described by measures $\{\mu_k\}$; and $|A^k| \to \infty$, $\mu_k \to \mu \in M_\varepsilon$; then the following is true:

(A) truth-telling is a dominant strategy for any consumer, and

(B) under truth-telling there exist k_ε; such that for all $\{E^k\}$; $k \geq k_\varepsilon$ the allocations prescribed by the mechanism are (i) attainable (that is, the mechanism is balanced), (ii) ε-Walrasian (and thus ε-efficient), and (iii) individually ε-rational.

(II) Moreover if this mechanism is applied to any finite exchange economy then the following hold: (a) the balancing agent g is willing to participate, (b) truth-telling is a dominant strategy for any consumer, and (c) any agent gets an ε-rational allocation for her by telling the truth.

Proof of the Theorem: Part (II) (and therefore statements (A) and (iii) of (B) in Part (I)) directly follows from the construction of the mechanism. For the rest of the proof let us restrict our attention to the generic situation described in the Proposition:

1). Since imbalance of the folk mechanism vanishes in per capita terms, but each consumer contributes a fixed bundle $\varepsilon \hat{e}$ to the stock, for all sufficiently large economies the stock will be sufficient to cover possible shortfalls. Excesses will be accepted by the balancing agent. Thus the mechanism is balanced for all large economies.

2). Notice that at the first stage of the ε-tax mechanism endowments are reduced by a proportion no larger than ε: Then, since allocations prescribed by the folk mechanism uniformly approximate Walrasian ones, in all large economies allocations of the ε-tax mechanism will be ε-close to the Walrasian allocations of the economies with reduced endowments. Hence the allocations will be ε-Walrasian (and thus ε-efficient) for all large economies. ■

5 Final remarks.

Remark 5.1. On the Vickrey-Clarke-Groves mechanisms. As it was mentioned in the Introduction, the folk mechanism owes much of the spirit to the Vickrey-Clarke-Groves mechanisms. Increasingly more general theorems on the asymptotical efficiency of these mechanisms have been provided by Green and Laffont (1979), Rob (1982), Mitsui (1983). Our Proposition and Theorem can be considered as a counterpart of these results in the case of a more general economic environment - exchange economies.

Remark 5.2. On the Córdoba-Hammond conjecture. Córdoba and Hammond (1997) conjecture that there is no mechanism that is simultaneously individually rational, resource-balanced, strategy-proof, and asymptotically Walrasian. If this conjecture is true, then there would be no mechanism with significantly better performance in large economies than the ε-tax mechanism described in our paper. Thus, proving the above conjecture, or a similar one, would help to clarify the scope of our positive results.

Remark 5.3. On the instruments of the mechanisms. In the mechanisms considered in this paper, agents submit their excess demands. This is necessary in an incomplete information environment. Notice that under complete information agents do know Walrasian allocations and the task of the planner is limited to the decentralization of an outcome which is already known by the agents. In contrast, in a more general case, considered in this paper, neither the planner nor the agents know the Walrasian allocations. The only way to calculate such allocations is to have information about excess demands. Thus no essentially simpler instruments can be used. (Except, of course, if some restrictions on the preferences are known a priori.)

Remark 5.4. Possible variants of the results. It is possible to get a version of the results of the current paper for the topology of weak convergence on the set of measures describing economies. (Thus without the Hausdor¤ convergence of the supports. This would require substituting the Mas-Colell's result on the existence of an appropriate selection by the analogous one in Makowski, Ostroy and Segal (1995, Theorem 3).) However under the topology of weak convergence the genericity level of results will be essentially weaker and results will hold for some residual set, while the current ones hold for some open and dense set. We prefer the latter because of the more straightforward intuition behind.

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