Department of Economics
UNC-Chapel Hill

Spring 2007

Syllabus ECON 890
FOUNDATIONS FOR CONTINUOUS TIME ASSET PRICING

Meeting time and place:
Tuesday: 11:00am to 12:15pm, Gardner 009
Thursday: 11:00am to 12:15pm, Gardner 009

Professor: Eric RENAULT

How to reach me:
My office is located in Gardner Hall 300G, my office phone number is 966-5326.
I will hold office hours on Monday and Wednesday from 10:00am to 11:00 am.
My email address is renault@email.unc.edu

Prerequisites:
Some working knowledge of measure-theoretic probability theory and some familiarity with discrete time stochastic processes would be helpful.
To be comfortable with the relevant mathematical tools, the following sequence of fall/spring STAT courses is optimal:
STAT 154 "Measure and Integration"
STAT 155 "Probability Theory".
Otherwise, a recommended background reading is:
J. Jacod and P.E. Protter (2003) "Probability Essentials", 2nd edition, Universitext, Springer Verlag.
However, even with an elementary background in probability theory (see e.g. ECON 271 "Introduction to Econometric Theory"), the course is self-contained.

Goals:
This course is primarily conceived to give a firm understanding of why and how continuous time methods work for asset pricing. This is the reason why the course contains proofs or outlines the proofs of many assertions, focusing on the role played by the assumptions with economic content while ignoring some regularity conditions which have little bearing on applied work. Complementary readings will be proposed to students interested in doing research in asset pricing or financial econometrics. While the focus is on continuous time mathematical finance (stochastic calculus, martingale methods), we will also pay attention to underlying economic assumptions (no arbitrage and risk premium in market equilibrium) as well as to practical relevance (from discrete time data to continuous time models including stochastic volatility and jumps).

Evaluation:

The grade for the course will be based on three homework sets, each with weight of 10%, a midterm exam (30%) and a final exam (40%).
Homework sets: They require that you derive theoretical results from theorems proven in class. The focus will be not only on mathematical derivations but also on discussion of modeling and statistical issues.
Midterm: The midterm will have the same format than the homework. Please don't neglect questions about discussion or interpretation of results which can always be solved with non-technical, albeit rigorous arguments, even when associated mathematical derivations have been skipped.
Final: The final will be cumulative, i.e. cover all the chapters since the beginning of the semester. The format is similar to homework and midterm.
The grading will be numerical (the maximum grade being 20), which will then be converted to H, P, L or F.

Textbook and Readings:

Any classical course or textbook in modern asset pricing must make a balance between the mathematical and economic/financial sides of the subject. Since this course is about "foundations", the focus is more on general mathematical principles for asset pricing, without details on specific markets.

Required:

S. Shreve (2004) *"Stochastic Calculus for Finance II, Continuous-Time Models"*, Springer Verlag.
Covers all the important topics but is a bit succinct about jump processes.

Other books of interest are:

J. Cvitanic and F. Zapatero (2004) *"Introduction to the economics and mathematics of financial markets"*
Very elementary but nicely reviews the essentials, especially about economic interpretations.

I.Karatzas and S.Shreve (1991) *"Brownian Motion and Stochastic Calculus"*, 2nd edition, Springer Verlag.
More complete than Shreve (2004) in terms of mathematical tools. Follows a middle path between the original constructions of stochastic integral with respect to Brownian motion and the more recent theory of stochastic integration with respect to general semi-martingales. However, it is also avoiding discontinuous martingales.

R.Cont and P. Tankov (2004) "*Financial Modeling with Jump Processes*", Chapman and Hall.
Financial modeling beyond Brownian motion appears to be necessary due to the reality of jumps in prices and volatilities. During the last decade Levy process and other stochastic process with jumps have become increasingly popular for modeling market fluctuations, both for risk management and option pricing purposes. This book provides a self-contained overview of the theoretical, numerical and empirical aspects involved in using jumps process in financial modeling, and does so in terms within the grasp of non-specialists. The price to pay is some lack of closeness about the required mathematical theory of discontinuous martingales.

P.E. Protter (2004) "*Stochastic Integration and Differential Equations*", 2nd edition, Applications of Mathematics, 21. Springer Verlag.
Cumulates the advantages of the two previous books: stochastic integration with respect to general semimartingales and mathematical closeness. One of the best books in the domain, if you are ready to read a purely and quite difficult mathematical book, where applications to mathematical finance are not made explicit.

T. Bjork (2004) "*Arbitrage Theory in Continuous Time*", 2nd edition, Oxford University Press.
Makes a neat job of providing a decent introduction to arbitrage theory without going into abstract measure theory. In the second edition, additional appendices of measure and probability theory define the prerequisites for the more advanced parts of the main text that are marked with a star. However, while the non-starred material is too elementary for our purpose, this kind of binomial tree of chapters to read (starred and non-starred sections) is not pedagogically optimal.

Overview of the course:

Chapter 1:
Modeling Asset Prices in Continuous Time.
Stochastic Processes and σ-Fields.
Martingales and Stopping Times.
Stieltjes Integration and Change of Variables
Naïve Stochastic Integration is Impossible

Chapter 2: Poisson process, Brownian Motion and Levy processes.

Chapter 3: Semimartingales, Stochastic Integral and Ito Calculus.

Chapter 4: The Girsanov theorem and the mathematics of the martingale approach to arbitrage pricing.

Chapter 5: Differential Equations