The Grand Traverse:
The U.S. Domestic Economy, c. 1817 to c. 1897

Variables	Average Values for quasi-steady-state periods 1800 - 1835	1890 - 1905
$E_L^* = E^*/\theta_L$, labor efficiency g.r.	.0085	.0155
L^*, labor input (manhours) g.r.	.0342	.0243
θ_L, non-property share in gross product	.68	.54
θ_R, unimproved land share in gross product	.09	.09
R^*, unimproved land input g.r.	.0281	.0195
$G = [\theta_L\{L^* + E^*_L\} + \theta_R R^*] / (\theta_L + \theta_R)$, the "natural" rate of growth of output	.0410	.0369
Y^*, the actual rate of growth of output	.0421	.0378
s_G, the real gross savings rate	.11	.28
δ, depreciation rate on reproducible stock	.017	.036
${}_Bv = s_G / (G+\delta)$, real reproducible capital-output ratio required for steady-state growth	1.90	3.84
v, the actual real capital-output ratio[1]	1.80	3.62
$(v/{}_Bv)$, actual as fraction of equilibrium capital-output ratio	0.95	0.94

Adjustment Speed Parameters for Sub-Periods of the Traverse

	1835 - 1855	1855 - 1890
G, the "natural" rate	.0366	.0337
δ, the depreciation rate	.0209	.0287
$\lambda = (G + \delta)(1 - \theta_K)$	.0420	.0402

[1] Geometric average of end-point values of the quasi-steady state periods.

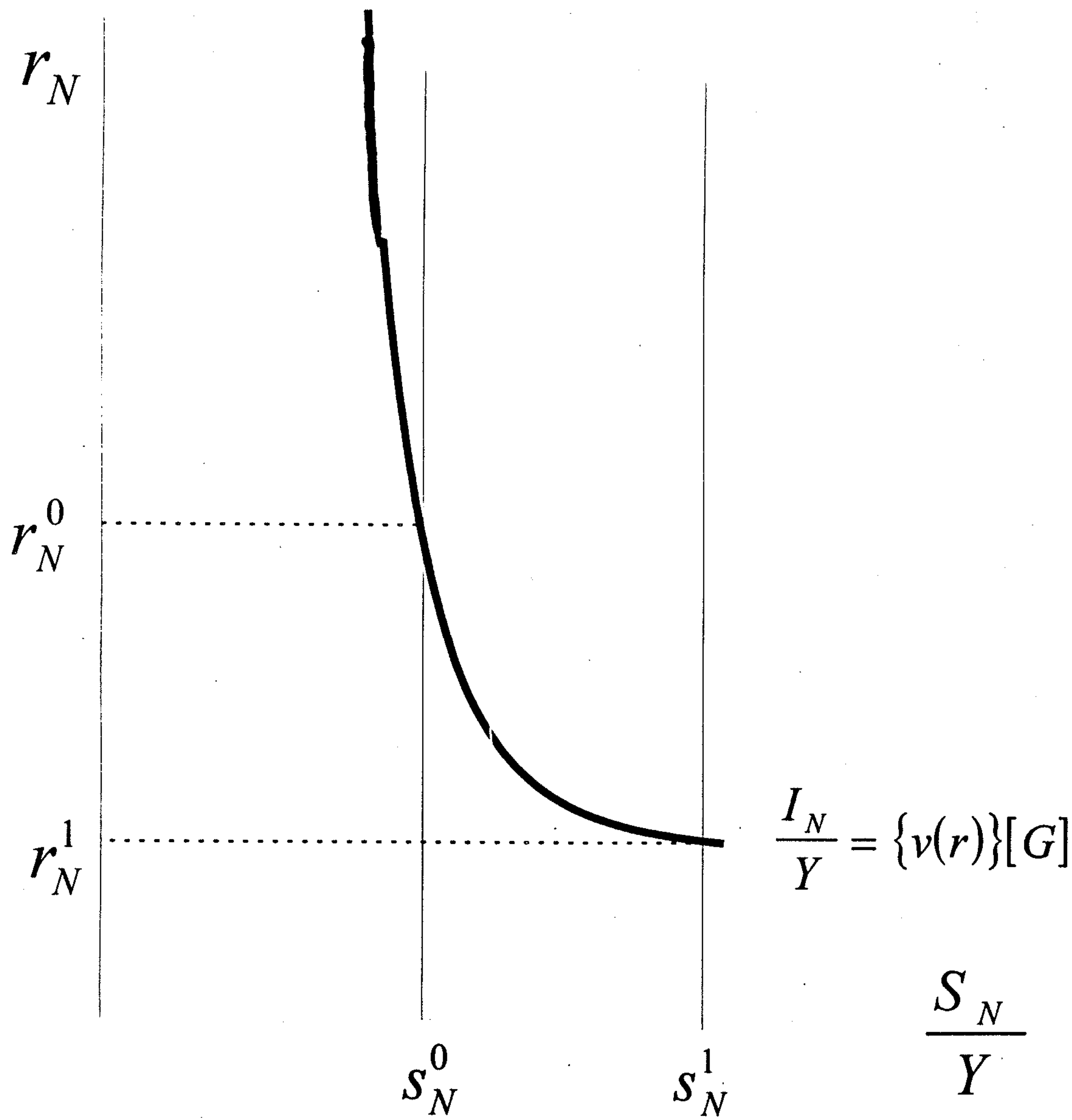
Dynamics of the U.S. "Grand Traverse" 1835 - 1890:
Adjustment Speed in the Standard NeoClassical Growth Model
(Calculations Assuming Discontinuous Savings Rate Shifts in 1835 and 1871)

Adjustment Duration Equation (after K. Sato, 1966):

$t(\varepsilon) = 1/\lambda\ [ln\{1 + (\varepsilon/1 - \varepsilon)][v(0) / {}_Bv(t)]$, where ε is the completion fraction.

Subperiods of "Traverse":	1835-1871	1871-1890
Actual Duration, t (years)	35	19
Estimated Average Adjustment Rate, λ	0.0420	0.0402
Proportional Savings Supply Shift, $s_G(t) / s_G(0)$	(.20/.11)=1.818	(.28/.20)=1.4
Proportional Capital-Output Ratio Adjustment Required, ${}_Bv(t) / v(0)$	(3.47/1.80) = 1.84	(3.84/3.12)=1.23
Subperiod Endpoint Magnitudes	1871	1890
Adjustment Ratio: $(\varepsilon / 1 - \varepsilon) = [\exp\{\lambda t\}-1][{}_Bv(t) / v(0)]$	6.162	1.410
Implied Fraction of Adjustment Completed, ε	0.86	0.585
Predicted $v(t) = \{[{}_Bv(t) - v(0)] / [v(0)]\}\varepsilon + 1\}v(0)$	3.10	3.54
Actual v(t)	3.12	3.62
Actual Fraction of Full Adjustment Completed, $\varepsilon = [v(t)] / [{}_Bv(t)]$	0.90	0.94

Figure 1



Specifications for "the Cantabridgian Synthesis"

I. Underlying Structural Equations

(1) Cambridge-Type Nominal Savings Function for Gross Savings in Current Prices

$$\left(\frac{S_G}{Y_T}\right)_t = s_L(\theta_L)_t + s_P\,(\theta_{TK})_t\,, \qquad \theta_L + \theta_{TK} = 1.$$

(2) Full Employment Condition in Investment Market

$$(S_G\ /\ Y_t)_t = [\ p_k(t)\ ]\ [I_G\ /\ Y]_t$$

(3) Neoclassical Desired Capital-Output Relationship (From CES Production Function)

$$v(t) = Z(t)\ [\ E_K(t)\ ]^{-(1-\sigma)}\ [r_G(t)]^{-\sigma}$$

where

$$v(t) = K(t)\ /\ Y(t)$$

$$Z(t) = [1 - \theta_R]^{\sigma}\ [\ E_R(t) \cdot R_t\ /\ Y_t]^{-(1-\sigma)(\theta_R)/(1-\theta_R)}.$$

(4) AcceleratorType Investment Demand Model (Jorgenson-Type Replacement Demand)

$$(I_G\ /\ Y)_t = v(t)\ [Y_t^*] + v(t)\ [\delta_t].$$

(5) Steady-State Growth Condition

$$Y_t^* = G_t.$$

Savings and Investment Flow Equations:

(6) Planned Rate of Real Gross Savings

$$s_G(t) = [P_k(t)]^{-1}\{[(s_P - s_L)\theta_R + s_L] + (s_P - s_L)Z(t)[E_k(k)]^{-(1-\sigma)}[r(t)]1-\sigma\}$$

(7) Planned Rate of Real Gross Investment

$$i_G(t) = I_G(t) / Y(t) = Z(t)\ [G_t + \delta_t][E_k(t)]^{-(1-\sigma)}[r_t(t)]^{-\sigma}$$

(8) Steady-State Full Employment Equilibrium

$$s_G(t) = i_G(t)$$

II. Model Parameter Estimates

Savings Equation (1), estimated from 1800 - 1900 Decadal Observations:

$$\left(\frac{S_G}{Y_T}\right)_t = -0.1140 + 0.7247\ \theta_{TK}\ , \qquad implies\ (\hat{s}_p) = 0.6107$$

$$(-.03680) \quad (0.0896) \qquad \overline{R^2} = 0.628; \quad (\hat{s}_t) = -.1140$$

Production and Investment Demand Function (Eq. 3) Parameters:

$$\text{From: } (K^{*} - L^{*})_t = \left(\frac{\sigma}{(1-\sigma)}\right) [\theta_L^{*} - \theta_K^{*}]_t + \lambda$$

From Regression Estimates from Abramovitz and David (1973), David (1977), based on Decadal Growth Rates for 1800 - 1900:

For $K^* = K^*$,	$\sigma = .08$,	$\lambda = .015$
For $K^* = K^*_T$,	$\sigma = .20$,	$\lambda = .017$

Implied Rate of Capital-UsingTechnological Change:

$$E_K^{*} = (E_L^{*} - \lambda) = -.0039$$

$$E_{KT}^{*} = (E_L^{*} - \lambda) = -.0055.$$

$$E_{KT}(1897)\ /\ E_{KT}(1817) = (\exp[E_{KT}^{*}(t)])\ [1897-1817] = (-.0055)(80) = 0.644.$$

III: Parameter Values in Underlying Structural System that are Subject to Shifts:

	1800 - 1835	1890 - 1905
$p_k(t)$	1.081	0.821
$G_t + \delta_t$	0.0580	0.0729
$E_k(t)$	1.00	0.644
$A(t)$	1.603	1.228

Dated Equations (for σ=.08):			**1800 - 1835**		**1890 - 1905**
(6)	$s_G(t)$	=	$-0.054 + 1.291(r)^{.92}$	;	$-0.061 + 2.896(r)^{.92}$
(7)	$i_G(t)$	=	$0.093(r)^{-0.08}$	;	$0.233(r)^{-0.08}$

Equilibrium Solutions for Equations (6) and (7) -- Approximated:

r_e	.11	.10
$s_G = i_G$	.11	.28

Hypothetical Solution for Case of Unshifted s_G Schedule, with Shift in i_G Schedule as Specified:

r_{eh}	.11	.22
$S_{gh} = i_{Gh}$	.11	.26
$r_{eh} - O_{kh}$	.32	.60 implied by (4)

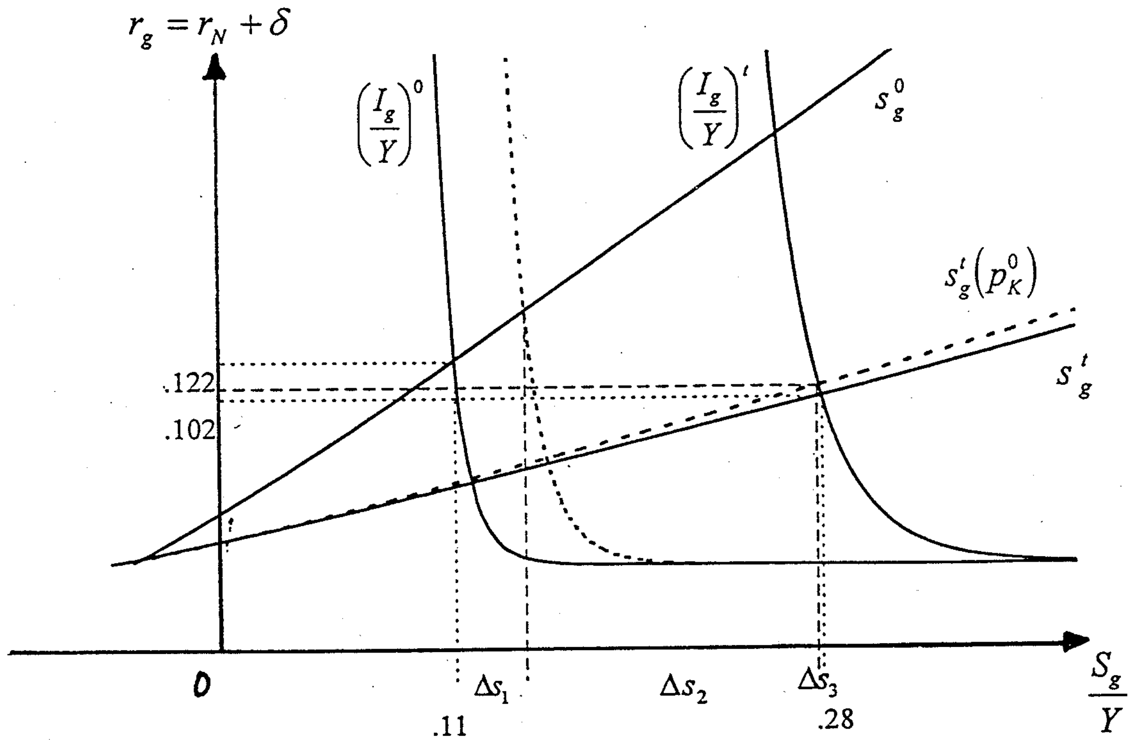


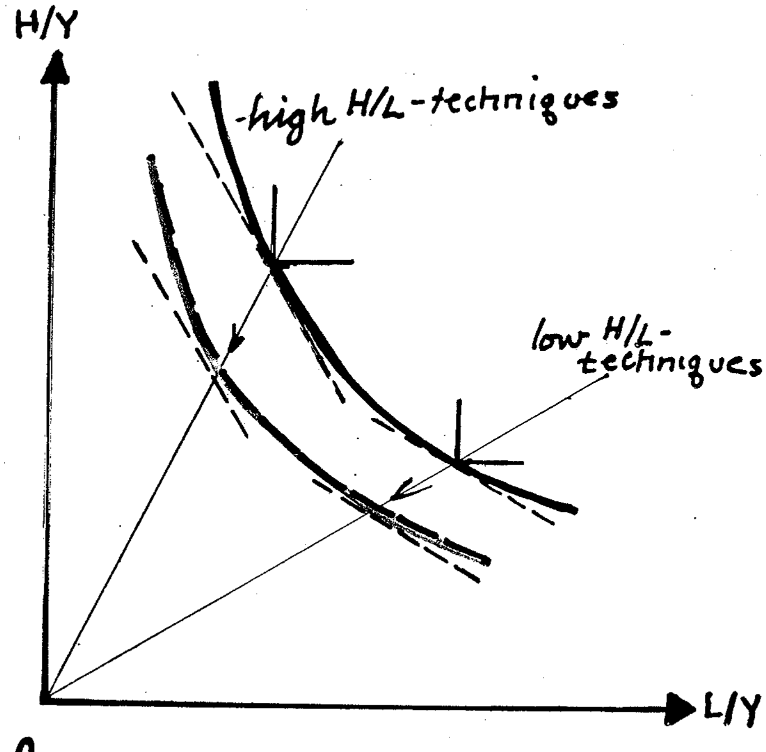
Figure 2: A Cantabridgian Synthesis: Components of the Shift in the U.S. Real Gross Savings Rate between the 1800-35 and 1890-1905 "Golden Ages" (for σ = .08)

Δs_1: Indirect effect of biased technological change upon the depreciation rate, caused by the price-induced expansion os share of investment in shorter-lived capital assets.

Δs_2: Direct effect of technological bias towards capital-deepening, i.e., Harrod labour-saving innovations.

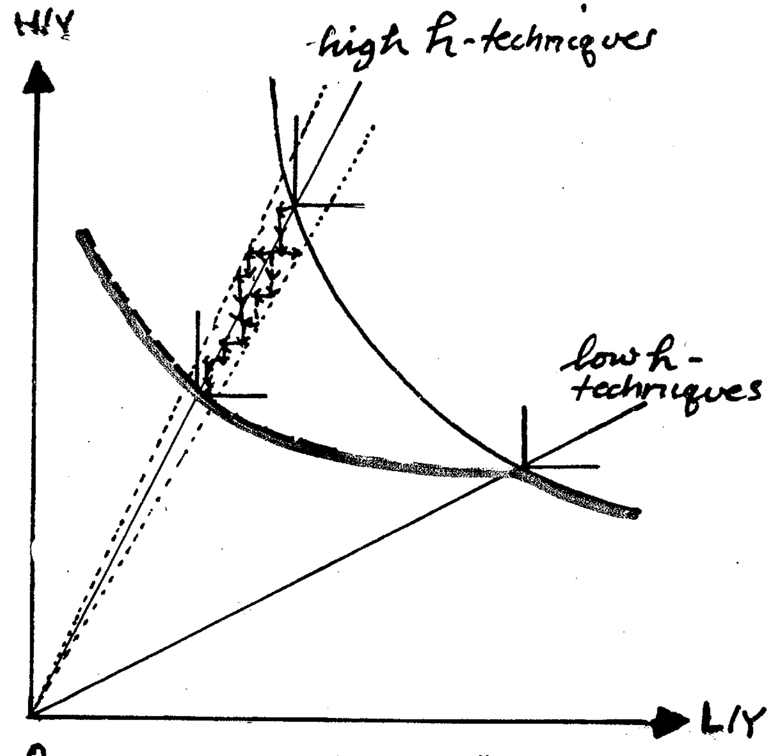
Δs_3: Indirect effect of biased technological change, in the form of differential productivity growth in producer durables industries leading to relative decline of investment goods prices.

Figure 3



Hicks-neutral Global Innovation preserves relative marginal productivities of inputs (H, L) along given factor-proportions rays:

$E_L^* = E_H^* > 0$



"Locally Neutral" Stochastic Learning results in "H-using" Global Bias of Technological Opportunity Set:

$(E_L^* - E_H^*) \gg 0$ and $E_H^* < 0$