Optimal R&D Subsidies under Incomplete Information

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Abstract

This paper analyses the optimal R&D subsidies in the context of signaling and screening models. A domestic government subsidizes R&D activity of a domestic firm in a sector in which the firm competes with a foreign firm in exporting products to a third country. I assume that R&D activity affects both marginal and fixed costs of production and that only its effect on the latter is a random variable. It will be shown that if a domestic government knows the exact effect of domestic R&D activities and a foreign firm does not (Section 3: *a signaling model*), then the government gives a *higher* subsidy to a domestic firm than under complete information. On the other hand, if the government is not well-informed on the effectiveness (Section 4: *a screening model*), then the government faces a tradeoff between shifting rents home from abroad by giving a strategically high subsidy to the domestic firm and preventing the domestic firm from reporting untrue effectiveness of its R&D activities. As a result, the government gives a *lower* subsidy to the domestic firm than under complete information. Importantly, these directions in which the incomplete information distorts the policy in signaling and screening models do not change irrespective to the mode of competition (output choice or price choice). This robustness sharply contrasts with the results of signaling and screening models developed in the so-called "third market" model.

1. Introduction

In an actual economy, many governments subsidize research and development (R&D) activities of domestic firms, especially in sectors in which domestic and foreign firms compete. These R&D subsidies can be viewed as more realistic policy tools than direct export subsidies, partly because developed countries gave up the right to subsidize exports under the Subsidy Code of the GATT (with the exception of some primary goods) but this ban did not extend to R&D subsidies.

Spencer and Brander (1983) developed a three stage R&D subsidy model, in which a domestic government commits to giving a subsidy to domestic R&D activities and sets its level in stage 1, a domestic and a foreign firm choose R&D levels in stage 2, and the firms determine export levels to the third market in stage 3. They found that if the government is the first mover and can make a credible commitment about the subsidy, then it chooses a positive R&D subsidy to influence equilibrium outcomes. In this setup, R&D activities by the firms reduce their production costs with certainty. In this sense, their model is deterministic. They focused only on the output competition case in stage 3.

Bagwell and Staiger (1994) made the model more realistic by introducing uncertainty associated with R&D investments. They assumed that R&D activities reduce the expected production costs on average but might increase the riskiness of the cost distribution as well. Nobody can tell the effect of the R&D projects at the time of the investment. They showed that when the R&D investment reduces the mean but does not change the variance of the cost distribution, then optimal subsidies are positive whether the firms choose quantities as Cournot duopolists or prices as Bertrand duopolists in a product market[1].

In this paper, I introduce uncertainty in a different way by assuming that information on the effectiveness of R&D activities is privately observed. The effect of R&D projects on production of goods depends upon a kind of infrastructure such as technology and educational levels the country already has. A higher level of technology and education allow the country to make better use of new technology developed by R&D investments. Information about this can be private across national borders (a signaling model). There can also be a communication gap between public and private agents, with this information held only by the private firms (a screening model). My first concern is on how the presence of asymmetric information alters the optimal R&D policy. The signaling model is analyzed in section 3 and the screening model is in section 4.

The mode of competition in the product market can affect the distortion in the optimal policy caused by the asymmetric information. Since the beginning of the research on strategic trade policy, it has been often pointed out that the outcome that the theory predicts is too sensitive to the assumption of the models. Especially, it has been thought to be problematic that whether the firms compete as quantity or price setters changes the optimal policy too drastically. For example, in the seminal "third-market" model developed by Brander and Spencer (1985) in which two firms compete as quantity setters

[1] In this paper I use the term "Cournot (Bertrand) competition" in the sense that two firms producing the same (differentiated) good(s) compete as quantity (price) setters, though the original Bertrand model was associated with a homogeneous good.

exporting their goods to a third country, the optimal policy is an export subsidy. However, Eaton and Grossman (1986) showed that if the firms are price setters then the optimal policy should be an export tax[2]. This sensitivity of the outcomes has been targeted by the critics of the strategic trade policy theory[3]. One focus of recent research in this area has been to pursue robustness in outcomes. As stated above, Bagwell and Staiger (1994) showed that the sign of the R&D policy is robust to the mode of competition in a product market. This robustness tends to emerge when there is a stage of competition in a non-product market inserted between the policy stage and the production competition stage[4]. Maggi (1996) developed a model in which capacity constraints affect the mode of competition endogenously and the firms compete in expanding their own capacities in the second stage, and proved a similar robustness by showing that the government achieves welfare-improving outcomes with a single rate capacity subsidy. My second concern in this paper is to analyze in the presence of asymmetric information whether the optimal R&D policy choice will be affected by altering the mode of competition. In the third market model without R&D, not only the sign of the optimal policy (subsidy or tax) but also the direction in which asymmetric information affects it are totally reversed by changing the mode of competition, as reviewed in appendix 1. Since in the R&D model the robustness of the sign is proved by Bagwell and Staiger (1994), my concern here is robustness of the effect of incomplete information on the optimal policy when the non-product market stage (the R&D stage) is inserted between the policy and production stage.

Besides the sensitivity, an element in the third market model that seems not to fit the actual economy is the optimal policy's slope with respect to production. In the third market model with linear demands and linear costs, the optimal positive export policy for

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In fact, Bagwell and Staiger (1994) uses a Hotelling model when the firms choose prices, but the same result is obtained in a Bertrand model.

[2] When the firms' response functions are downward (upward) sloping or their choice variables are strategic substitutes (complements), the optimal policy becomes a subsidy (a tax). Even under Cournot competition, if a very convex demand function is assumed, the choice variables might become complements and the optimal policy might become an export tax.

[3] The sensitivity of the theory is pointed out, for example, in Brander (1995). And a stronger skeptical view on the strategic trade policy theory in part due to this non-robustness is found in Krugman (1993) also cited in Maggi (1996).

[4] If the Spencer and Brander (1983) had investigated the Bertrand case, they would have reached a similar conclusion.

the domestic firm is decreasing in the marginal cost of the domestic firm. De Meza (1986) argues that this is counterfactual, since in the actual world, governments tend to give higher export subsidies to less competitive firms with higher cost. This point in the context of the R&D model will be revisited later[5].

In the following, I analyze the optimal R&D subsidies in the context of the signaling and screening models. Section 2 investigates a complete information case as a benchmark. The result in a Counot case is compared to a Bertrand case. Moreover I will introduce a costly budget and see that this modified assumption does not alter the optimal policy if non-linear policies are allowed. This costly budget case becomes a benchmark for a screening model. In section 3, I introduce asymmetric information across a national border in the R&D model and apply the signaling model to this model using the method of Collie and Hviid (1993). Section 4 deals with a situation in which information is asymmetric across the public and private sectors. I will see that the domestic firm is likely to underreport its R&D effectiveness when the government announces the complete information contract even though there is incomplete information. Then I develop a screening model for the Cournot case following Brainard and Martimort (1992). The result will be compared to that under Bertrand competition. Then I add an analysis on the case when the government announces a *fully* non-linear contract. Section 5 concludes.

2. An R&D model with complete information

2.1 Basic model under Cournot competition

There are two countries (domestic and foreign) with a single firm in each of them. These firms export products to a third country. Only the government in the domestic country is active and the foreign government is not. The domestic government subsidizes R&D activities of the domestic firm. Following the tradition since Spencer and Brander (1983), I consider a three stage R&D model: In stage 1, the domestic government chooses

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[5] If we allow the model to have varying marginal costs, the high-cost country could offer high subsidies. However, de Meza insists that a constant marginal cost seems empirically plausible.

an R&D subsidy. In stage 2, the domestic firm and the foreign firm decide R&D levels simultaneously. In stage 3, the firms compete in the third market. They are assumed to begin to choose export quantities as Cournot duopolists.

I give a specific form to each firm's profit function including linear (inverse) demands. Subscripts 1 and 2 denote domestic and foreign variables respectively.

The two firms' profit functions are given by [6]:

$$\mathbf{p}_1 = (\mathbf{a} - y_1 - y_2)y_1 - [(\mathbf{b} - x_1)y_1 + F_1 - f_1 x_1] - vx_1^2 + sx_1 \qquad (1)$$

$$\mathbf{p}_2 = (\mathbf{a} - y_1 - y_2)y_2 - [(\mathbf{b} - x_2)y_2 + F_2 - f_2 x_2] - vx_2^2 \qquad (2)$$

where y_i is firm i's export level to the third market

$\mathbf{a} - y_1 - y_2$ is an inverse demand function in the third market,

$\mathbf{b} - x_i$ is firm i's marginal cost of production,

$F_i - f_i x_i$ is firm i's fixed cost of production,

x_i is firm i's technology created by its R&D activities,

vx_i^2 represents R&D expenditure by firm i.,

and s is an R&D subsidy given by the domestic government to the domestic firm.

The R&D activity affects both marginal costs and fixed costs. The inherent uncertainty related to the R&D investment can be put into two categories. The first one is the uncertainty associated with the potential results of the R&D project at the time of investment. In this case, v might become stochastic. The second one is the uncertainty related to the effect of the R&D project on the production process. In this paper, I focus on the latter case and assume that f_1 is stochastic. The effects of R&D activity on the marginal costs are assumed to be certain and common knowledge. For simplicity, f_2 is also certain and publicly observable.

The stochastic nature of the model can be understood as following. Let x_i be the number of robots created by R&D activities. R&D subsidies are given to the firm not based on the R&D expenditure, but based on the number of robots created. R&D

investments of vx_i^2 create x_i robots. At this point there is no uncertainty. These robots reduce firm *i*'s marginal costs and fixed costs of production. The assumption that f_1 is private information implies that it is not possible for the foreign firm to predict exactly how much the robots will reduce the fixed cost in, say, constructing a factory plant for domestic goods production. The assumption that the R&D activity affects both marginal and fixed costs but that only the effects on the latter are stochastic is a technical requirement in taking expectations of the firms' first order conditions later. But a possible explanation is that, for example, construction of the plant requires highly sophisticated technique and the effect of the R&D projects on it is harder to predict than the effect on the marginal production costs. To focus on the effects of the R&D projects on fixed costs, the parameters $\boldsymbol{b}$ and v are assumed to be common to both economies and common knowledge for all agents.

For a benchmark case, I consider a game with complete information first. In this case, f_1 is common knowledge to every agent. I derive Subgame Perfect Equilibrium by backward induction.

In stage 3, the two firms choose output (export) levels simultaneously. The Nash equilibrium is characterized by first order conditions,

$$\frac{\partial \boldsymbol{p}_1}{\partial y_1} = \boldsymbol{a} - 2y_1 - y_2 - \boldsymbol{b} + x_1 = 0 \quad (3) \qquad \frac{\partial \boldsymbol{p}_2}{\partial y_2} = \boldsymbol{a} - y_1 - 2y_2 - \boldsymbol{b} + x_2 = 0 \quad (4)$$

and second order conditions,

$$\frac{\partial^2 \boldsymbol{p}_1}{\partial y_1^2} = -2 < 0 \quad (5) \qquad \frac{\partial^2 \boldsymbol{p}_2}{\partial y_2^2} = -2 < 0 \quad (6).$$

The condition,

$$A \equiv \frac{\partial^2 \boldsymbol{p}_1}{\partial y_1^2}\frac{\partial^2 \boldsymbol{p}_2}{\partial y_2^2} - \frac{\partial^2 \boldsymbol{p}_1}{\partial y_1 \partial y_2}\frac{\partial^2 \boldsymbol{p}_2}{\partial y_2 \partial y_1} = 3 > 0 \qquad (7)$$

ensures uniqueness and global stability of the equilibrium. Equations (3) and (4) imply that both firms' reaction functions are downward sloping and their outputs are strategic substitutes. The solutions for outputs are

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[6] Spencer and Brander (1983) and Bagwell and Staiger (1994) use more general setups. I adopt linear demands and other specific functional forms in order to take the expectations of the first order conditions later.

$$y_1 = \frac{\alpha - \beta + 2x_1 - x_2}{3} \qquad (8) \qquad\qquad y_2 = \frac{\alpha - \beta - x_1 + 2x_2}{3} \qquad (9)$$

In stage 2, the firms choose R&D levels. I rewrite their profits as functions of x_1 and x_2 by plugging (8) and (9) into (1) and (2). Let g (for gain) represents each firm's rewritten profit function.

$$g_1(x_1, x_2; s) = \left(\frac{\alpha - \beta + 2x_1 - x_2}{3}\right)^2 - F_1 + f_1 x_1 - v x_1^2 + s x_1 \qquad (10)$$

$$g_2(x_1, x_2) = \left(\frac{\alpha - \beta - x_1 + 2x_2}{3}\right)^2 - F_2 + f_2 x_2 - v x_2^2 \qquad (11)$$

The Nash equilibrium in R&D levels is characterized by the first order conditions

$$\frac{\partial g_1}{\partial x_1} = \frac{4(\alpha - \beta) + 2(4 - 9v)x_1 - 4x_2 + 9f_1 + 9s}{9} = 0 \qquad (12)$$

$$\frac{\partial g_2}{\partial x_2} = \frac{4(\alpha - \beta) - 4x_1 + 2(4 - 9v)x_2 + 9f_2}{9} = 0. \qquad (13)$$

Second order conditions must satisfy

$$\frac{\partial^2 g_1}{\partial x_1^2} = \frac{8 - 18v}{9} < 0 \qquad (14) \qquad\qquad \frac{\partial^2 g_2}{\partial x_2^2} = \frac{8 - 18v}{9} < 0 \qquad (15)$$

To ensure uniqueness and stability of the equilibrium, we need

$$B \equiv \frac{\partial^2 g_1}{\partial x_1^2}\frac{\partial^2 g_2}{\partial x_2^2} - \frac{\partial^2 g_1}{\partial x_1 \partial x_2}\frac{\partial^2 g_2}{\partial x_2 \partial x_1} = \frac{[2(9v - 4)]^2 - 16}{81} > 0 \qquad (16)$$

If v > 2/3, then (14) – (16) are all satisfied[7] and the firms' best response functions are downward sloping (or their choice variables are strategic substitutes). This ensures that the optimal policy is a positive subsidy in stage 1. For simplicity, I assume $v = 1$ in the rest of the paper. Even if I change this value, the following conclusions do not change, as long as it satisfies the second order conditions of the first and second stage. From (12) and (13), the Nash equilibrium in stage 2 is

[7] Similar argument is found in Spencer and Brander (1983) with more general functional forms. They argue "(the second order conditions and the condition for uniqueness and global stability) will hold…, if the marginal cost reducing effect of R&D declines relatively rapidly". In this model, if v>2/3 then the uniqueness and stability condition is satisfied, and it is also sufficient for the second order conditions to hold and for the R&D levels to be strategic substitutes.

$$x_1 = \frac{4(\alpha - \beta) + 15 f_1 - 6 f_2 + 15s}{14} \qquad (17)$$

$$x_2 = \frac{4(\alpha - \beta) - 6 f_1 + 15 f_2 - 6s}{14} \qquad (18)$$

Note at this point that f_1 affects the product market *indirectly*. Usually the fixed cost does not affect the optimality condition in the goods market. Indeed, in this R&D model, f_1 (or f_2) does not appear in the firms' first order conditions (3) and (4) in stage 3. In this model, however, f_1 affects the R&D levels in stage 2 through its quadratic R&D expenditure function. And through the fact that these R&D levels affect marginal costs and thus the production levels, f_1 affects the production decision indirectly.

In stage 1, the domestic government chooses the optimal R&D subsidy. The government's welfare function is $W_1 = \pi_1 - sx_1$. Using (10), (17), and (18), I rewrite the objective function as B (for benefit).

$$B_1(s) \equiv g_1(x_1(s), x_2(s); s) - sx_1(s) = \left(\frac{\alpha - \beta + 2x_1(s) - x_2(s)}{3} \right)^2 - F_1 + f_1 x_1(s) - x_1(s)^2 \qquad (19)$$

where x's are given by (17) and (18).

The first order condition of the domestic government's maximization problem

$\frac{\partial B_1}{\partial s} = 0$ yields the optimal subsidy level $s = \frac{4(\alpha - \beta) + 8 f_1 - 6 f_2}{27}$ (20)

Note that if the domestic R&D project is effective (f_1 is high), then the domestic government chooses a high R&D subsidy.

2.2 Under Bertrand competition

In a Bertrand duopoly case the firms choose prices at the third stage. My concern in this subsection is the robustness of the optimal policy. For a Bertrand duopoly in a two-stage third market model (without R&D), the optimal policy under complete information is an export tax rather than a subsidy (Eaton and Grossman (1986)). In this subsection, I will compare the optimal policy in the R&D model with complete information under Bertrand competition to that under Counot competition.

In a Bertrand duopoly with differentiated goods, demands for each firm's product can be expressed as $y_1 = a_1 - p_1 + b_1 p_2$ and $y_2 = a_2 - p_2 + b_2 p_1$. I assume that these two goods are substitutes, so $0 < b_i < 1$. For simplicity, I assume $a_1 = a_2 = a$ and $b_1 = b_2 = 1/2$. Besides the convention in stage 3, all the other structure of the model is kept the same as in the preceding subsection. Two firms' profit functions under Bertrand competition are

$$\pi_1 = (a - p_1 + 1/2\, p_2)[p_1 - (\beta - x_1)] - (F_1 - f_1 x_1) - x_1^2 + s x_1 \qquad (21)$$

$$\pi_2 = (a - p_2 + 1/2\, p_1)[p_2 - (\beta - x_2)] - (F_2 - f_2 x_2) - x_2^2 \qquad (22)$$

The assumption $v = 1$ (in (1) and (2)) is maintained.

As a benchmark, I consider a complete information case, so that f_1 is known to all agents. In stage 3, the firms choose prices simultaneously. First order conditions for profit maximization are

$$\frac{\partial \pi_1}{\partial p_1} = a - 2p_1 + 1/2\, p_2 + \beta - x_1 = 0 \qquad (23)$$

$$\frac{\partial \pi_2}{\partial p_2} = a - 2p_2 + 1/2\, p_1 + \beta - x_2 = 0 \qquad (24).$$

Second order conditions and the condition for uniqueness and global stability of equilibrium are satisfied. Note that the firms' best response functions are upward sloping and their prices are strategic complements. This is the reason why negative export subsidies would be obtained if the third market model without R&D were considered. The Nash equilibrium in stage 3 is

$$p_1 = \frac{10a + 10\beta - 8x_1 - 2x_2}{15} \qquad (25)$$

$$p_2 = \frac{10a + 10\beta - 2x_1 - 8x_2}{15} \qquad (26)$$

In stage 2, the firms choose R&D levels. I rewrite profit functions by substituting (25) and (26) to (21) and (22), and name them g.

$$g_1(x_1, x_2; s) = \left(\frac{10a - 5\boldsymbol{b} + 7x_1 - 2x_2}{15} \right)^2 - F_1 + f_1 x_1 - x_1^2 + s x_1 \qquad (27)$$

$$g_2(x_1, x_2) = \left(\frac{10a - 5\boldsymbol{b} - 2x_1 + 7x_2}{15} \right)^2 - F_2 + f_2 x_2 - x_2^2 \qquad (28)$$

First order conditions in stage 2 are

$$\frac{\partial g_1}{\partial x_1} = \frac{70(2a - \boldsymbol{b}) - 352x_1 - 28x_2 + 225 f_1 + 225s}{225} = 0 \qquad (29)$$

$$\frac{\partial g_2}{\partial x_2} = \frac{70(2a - \boldsymbol{b}) - 28x_1 - 352x_2 + 225 f_2}{225} = 0 \qquad (30)$$

Also in stage 2, the second order conditions and the condition for uniqueness and global stability of equilibrium are satisfied. Strikingly, the firms' best response functions are downward sloping in stage 2. This switch of choice variables from strategic complements in stage 3 to strategic substitutes in stage 2 is caused by the fact that the last several terms in (21) and (22) are the same as those in (1) and (2). (These terms represent cost reduction effects of the R&D projects.) Note that the coefficients on x_1 in (25) and (26) are both negative. This contrasts with (8) and (9) where it is positive in (8). In fact, a larger R&D expenditure improves the domestic firm's profit but worsens the foreign firm's profit at the third stage in the Cournot case. By contrast, a larger R&D expenditure is detrimental to both of the firms at the third stage in the Bertrand case. This reflects the fact that choice variables at the third stage are strategic substitutes in the Cournot case but strategic complements in the Bertrand case. In the Bertrand case, however, a larger R&D expenditure makes the domestic firm better off in stage 2 because it reduces marginal cost, while it keeps the foreign firms worse off. Thus the choice variables become strategic substitutes in stage 2 in the Bertrand case as in the Cournot case. The fact that choice variables have the same relationship in stage 2 irrespective to the mode of competition in stage 3 explains why we get robust results of positive subsidies in R&D models. Nash equilibrium in stage 2 is represented by:

$$x_1 = \frac{126(2a - \boldsymbol{b}) + 440 f_1 - 35 f_2 + 440s}{684} \qquad (31)$$

$$x_2 = \frac{126(2a - \boldsymbol{b}) - 35 f_1 + 440 f_2 - 35s}{684} \qquad (32)$$

In stage 1, the domestic government maximizes $W_1 = \boldsymbol{p}_1 - sx_1$ by choosing an R&D subsidy. Using (27), (31), and (32), I rewrite the welfare function as B

$$B_1(s) \equiv g_1(x_1(s), x_2(s); s) - sx_1(s) = \left(\frac{10a - 5\boldsymbol{b} + 7x_1(s) - 2x_2(s)}{15}\right)^2 - F_1 + f_1 x_1(s) - x_1(s)^2 \tag{33}$$

where x's are given by (31) and (32).

The first order condition for the maximization problem in stage 1 yields the optimal subsidy

$$s = \frac{126(2a - \boldsymbol{b}) + 98 f_1 - 35 f_2}{14950} \tag{34}$$

As in the Cournot competition, optimal policy in the Bertrand case is a positive subsidy. And the subsidy is an increasing function of the domestic R&D effectiveness as in (20) for the Cournot case: the more effective the domestic R&D, the higher the optimal R&D subsidy the domestic government chooses. In this regard, the optimal R&D policy is robust to the mode of competition is. This result is similar to Bagwell and Staiger (1994). The facts that the relationship between choice variables (strategic substitutes or complements) determines the sign of the optimal complete-information policy and the relationship is robust regardless of the mode of competition lead to the robustness of the sign of the policy in R&D models.

2.3 When budget is costly,…

The above results were derived under the assumption that budget is not costly. With costless budget, governments are indifferent if they leave some rents in private firms. When budget is costly, this does not hold anymore. The government tries to absorb as much rent as possible from the firm.

Solving by backward induction, the levels of the choice variables in Nash equilibrium at stage 2 are exactly the same as in (17) and (18). Plugging (17) and (18) into (10), the domestic firm's profit function *before* a lump sum transfer is:

$$\boldsymbol{p}_1(s(f_1), f_1) = \left(\frac{6(\boldsymbol{a} - \boldsymbol{b}) + 12 f_1 - 9 f_2 + 12 s(f_1)}{14}\right)^2 - F_1$$

$$+(f_1+s(f_1))\frac{4(\boldsymbol{a}-\boldsymbol{b})+15f_1-6f_2+15s(f_1)}{14}-\left(\frac{4(\boldsymbol{a}-\boldsymbol{b})+15f_1-6f_2+15s(f_1)}{14}\right)^2 \quad (35)$$

I let U denote the domestic firm's rent *after* the transfer has been completed:

$$U_1(f_1)=\boldsymbol{p}_1(s(f_1),f_1)-t(f_1) \quad (36)$$

Then, the government's objective function at stage 1 becomes:

$$U_1(f_1)-(1+c)[s(f_1)x_1(s(f_1),f_1)-t(f_1)] \quad (37)$$

where 1 + c represents social costs of public funds which exceeds 1[8]. A participation constraint requires

$$U_1 \geq 0 \quad (38)$$[9].

This constraint binds because public funds are costly: The government absorbs all the firms rent by the transfer:

$$U_1(f_1)=\boldsymbol{p}_1(s(f_1),f_1)-t(f_1)=0 \quad (39)$$

Then the government maximizes (37) with respect to $s(f_1)$ and $t(f_1)$ subject to (39). Rearranging the objective function (37) using (35) and (39), and differentiating it with respect to s, we get the following optimal subsidy under complete information:

$$s(f_1)=\frac{4(\boldsymbol{a}-\boldsymbol{b})+8f_1-6f_2}{27} \quad (40)$$

As indicated above, this is exactly the same as (20). When a non-linear policy is allowed, the introduction of costly public funds does not alter the optimal subsidy under complete information. And if the firms compete as Bertrand duopolists, the same result holds. Even with costly budget, the government gives the same subsidy as in (34), when it uses non-linear policies. This setup will be used as a benchmark for a screening model. The reason will be explained in subsection 4.1.

3. The signaling model

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[8] This c represents, for instance, the cost of a tax collecting institution and the distortionary effects in other sectors.

[9] This participation constraint is called "zero-profit participation constraint" (ZPC). Brainard and Martimort (1992) analyze also a "non-intervention participation constraint" (NPC) case. In this model, I will focus on a ZPC case.

In this section, I assume that a domestic government and a domestic firm know the effect of domestic R&D on domestic production, though a foreign firm does not know it. This is the case when information on the factors affecting the effectiveness of R&D activities is shared only within the country. A domestic government and a domestic firm can observe these factors and have a better idea about the effect of R&D than a foreign firm. For simplicity, the effects of the foreign R&D activity on foreign production are assumed to be common knowledge. My two concerns are the direction in which this private information distorts the optimal policy and its sensitivity with respect to the mode of competition.

3.1 R&D signaling model in a Cournot duopoly

The effect of domestic R&D activities on domestic fixed costs, f_1, is continuously distributed with support on $\left[f_1^L, f_1^H\right]$. A larger value implies "more effective". The value of f_1 is known to the domestic government and the domestic firm, but prior to stage 3 only the distribution of f_1 is known to the foreign firm.

In stage 1, the domestic government sets the R&D subsidy level. Then in stage 2, after the two firms have observed the subsidy and the foreign firm has updated its beliefs about the effects of domestic R&D on the domestic production using the observation, the two firms choose R&D levels. In the transitional period from stage 2 to 3, the robots created by domestic R&D activities help construct a factory and reduce the fixed production costs. In stage 3, firms decide export levels.

I derive a perfect Bayesian equilibrium by backward induction using the method of Collie and Hviid (1993). Leaving the rigorous definition of the PBE to some game theory textbooks (see Osborne and Rubinstein (1996) for example), I will point out here the three most important elements of PBE: (i) Not well-informed player(s) have some beliefs on stochastic variables, (ii) Players maximize their own payoffs based upon the beliefs (Rationality), and (iii) The beliefs are updated by actual strategies using Bayes' rule (Consistency).

The model developed in subsection 2.1 is used as a benchmark. In stage 3, (3) and (4) still hold. In stage 2, the foreign firm does not know the exact value of f_1. But having observed the domestic R&D subsidies and updated its beliefs about the effect of domestic R&D, the foreign firm maximizes its expected profit.

Now the first order conditions for a Bayesian Nash equilibrium in R&D levels at the second stage are

$$\frac{\partial g_1}{\partial x_1} = \frac{4(\boldsymbol{a}-\boldsymbol{b}) - 10x_1 - 4x_2 + 9f_1 + 9s}{9} = 0 \qquad (41)$$

$$\frac{\partial E_2\langle g_2 | s\rangle}{\partial x_2} = \frac{4(\boldsymbol{a}-\boldsymbol{b}) - 4E_2\langle x_1 | s\rangle - 10x_2 + 9f_2}{9} = 0 \qquad (42)$$

where E is the expectations operator given the beliefs of the foreign firm about f_1 after observing *s*. To solve for the equilibrium R&D levels, we take expectations of (41).

$$\frac{4(\boldsymbol{a}-\boldsymbol{b}) - 10E_2\langle x_1 | s\rangle - 4x_2 + 9\hat{f}_1(s) + 9s}{9} = 0 \qquad (43)$$

where $\hat{f}_1(s) = E_2\langle f_1 | s\rangle$. Substituting $E_2\langle x_1 | s\rangle$ of (42) into (43), we get x_2. Then substitute it into (41) to obtain x_1.

$$x_1 = \frac{20(\boldsymbol{a}-\boldsymbol{b}) + 12\hat{f}_1(s) + 63f_1 - 30f_2 + 75s}{70} \qquad (44)$$

$$x_2 = \frac{4(\boldsymbol{a}-\boldsymbol{b}) - 6\hat{f}_1(s) + 15f_2 - 6s}{14} \qquad (45)$$

Note that if $\hat{f}_1(s) = f_1$, then (44) and (45) are equivalent to (17) and (18).
Using (19), (44), and (45), the domestic government's objective function is

$$B_1(s, \hat{f}_1, f_1) = \left(\frac{30(\boldsymbol{a}-\boldsymbol{b}) + 18\hat{f}_1(s) + 42f_1 - 45f_2 + 60s}{70}\right)^2 - F_1 +$$

$$f_1 \times \frac{20(\boldsymbol{a}-\boldsymbol{b}) + 12\hat{f}_1(s) + 63f_1 - 30f_2 + 75s}{70} - \left(\frac{20(\boldsymbol{a}-\boldsymbol{b}) + 12\hat{f}_1(s) + 63f_1 - 30f_2 + 75s}{70}\right)^2 \qquad (46)$$

The game of incomplete information can have many perfect Bayesian equilibria (pooling, hybrid, and separating). But pooling and mixed equilibria are ruled out by most equilibrium refinements[10]. Thus I analyze only the separating equilibrium. According to Mailath (1987), sufficient conditions for the existence and uniqueness of a separating equilibrium are belief monotonicity, type monotonicity, and single crossing conditions. The belief monotonicity condition requires the sign of $\frac{\partial B_1}{\partial \hat{f}_1}$ to be kept unchanged over the support of f_1. The type monotonicity condition requires the sign of $\frac{\partial^2 B_1}{\partial f_1 \partial s}$ to be kept also. The single crossing condition requires that $\left(\frac{\partial B_1}{\partial s}\right) \Big/ \left(\frac{\partial B_1}{\partial \hat{f}_1}\right)$ be monotonic in f_1. In this model these sufficient conditions are all satisfied and, thus, this game has a unique separating equilibrium[11].

To derive the separating equilibrium, let the R&D subsidy be given by $s = \boldsymbol{f}(f_1)$, where $\boldsymbol{f}(\bullet)$ is a differentiable and one-to-one function. In the separating equilibrium, incentive compatibility requires two things: First, the domestic government maximizes domestic welfare given the beliefs of the foreign firm. Second, the beliefs of the foreign firm are consistent with the separating equilibrium strategy. This consistency is satisfied if the beliefs are formed by inverting the equilibrium subsidy so that $\hat{f}_1(s) = \boldsymbol{f}^{-1}(s)$; hence, in the separating equilibrium, the foreign firm can correctly infer the effectiveness of domestic R&D from the subsidy chosen by the domestic government.

Maximizing domestic welfare (46), where the beliefs of the foreign firm are given by $\hat{f}_1(s) = \boldsymbol{f}^{-1}(s)$, with respect to R&D subsidy yields the first order condition

$$\frac{dB_1}{ds} = \frac{\partial B_1}{\partial s} + \frac{\partial B_1}{\partial \hat{f}_1}\frac{d\boldsymbol{f}^{-1}}{ds} = 0 \qquad (47)$$

The first term represents the direct effect of subsidy, as under complete information. The second term represents the signaling effects of the subsidy. In the separating equilibrium,

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[10] See Collie and Hviid (1993). They argue that Rasmusen's (1989) passive conjecture which is the most plausible out-of-equilibrium belief cannot be supported by the pooling equilibrium. The same logic applies here.

[11] Of the three conditions, it is straightforward to show that the type monotonicity holds with a positive sign. This implies that the more effective the domestic R&D, then the larger the gains to the domestic country from using an R&D subsidy. The other two turn out to hold in the process of solving PBE.

the foreign firm correctly infers the effect of domestic R&D from the R&D subsidy set by the domestic government, so $\hat{f}_1(s) = f_1$; hence (47) can be rewritten as the differential equation

$$\frac{d\boldsymbol{f}}{df_1} = \left(-\frac{\partial B_1}{\partial \hat{f}_1}\right) \Big/ \left(\frac{\partial B_1}{\partial s}\right) = N/D = \frac{-20(\boldsymbol{a}-\boldsymbol{b}) - 40 f_1 + 30 f_2 - 12s}{20(\boldsymbol{a}-\boldsymbol{b}) + 40 f_1 - 30 f_2 - 135s} \qquad (48)$$

where N and D represent numerator and denominator respectively. Note that $s = \boldsymbol{f}(f_1)$[12]. The separating equilibrium subsidy function, $\boldsymbol{f}(f_1)$, is a particular solution of this differential equation that satisfies the relevant initial-value condition. This first order homogeneous differential equation cannot be solved explicitly; however, we can use a qualitative analysis by drawing a figure and following the method of Collie and Hviid (1993). In figure 2, the two loci N = 0 and D = 0 are shown. Above (below) the N = 0 locus, the numerator is negative (positive). Above (below) the D = 0 locus, the denominator is negative (positive). The D = 0 locus corresponds to the optimal R&D subsidy under complete information (20). These two lines intersect at $\left(f_1^0, s^0\right)$ where

$$f_1^0 = \frac{-2(\boldsymbol{a}-\boldsymbol{b}) + 3 f_2}{4} \quad , \qquad s^0 = 0 .$$

We can derive two linear solutions of (48) that pass through this intersection. By assuming $d\boldsymbol{f}/df_1 = k$ and plugging $s = k(f_1 - f_1^0) + s^0$ in the right hand side of (48), I get the quadratic $135k^2 - 52k - 40 = 0$ which can be solved for two roots k = 0.77, -0.38. Both linear solutions are shown in figure 1. Moreover, to ensure only interior solutions, I draw the loci $x_1 = 0$ and $x_2 = 0$ using (44) and (45).

The next step in this analysis is to determine the initial-value condition that gives the specific solution of (48). This is related to the above belief monotonicity condition. In the region above the N = 0 line, the numerator in (48) is negative, so $\frac{\partial B_1}{\partial \hat{f}_1} > 0$ (belief monotonicity), that is, domestic welfare is increasing in the beliefs of the foreign firm

[12] At this point, if $\boldsymbol{a}$ is sufficiently large (actually I assume it to make the firms' profits positive), we can see that belief monotonicity condition is satisfied with a positive sign. This implies that the domestic government's welfare increases if the foreign firm believes that the domestic R&D activities are more effective.

about the effectiveness of the domestic R&D. The worst belief from the viewpoint of the domestic government is if the foreign firm believes that f_1^L is the actual effectiveness. Hence, if the effectiveness of domestic R&D is the lowest, there is no incentive for the domestic government to use R&D subsidy to signal the effectiveness, since the true effect will be revealed in the separating equilibrium. Therefore, the initial-value condition is that the domestic government sets the subsidy equal to the complete information level when $f_1 = f_1^L$.

Starting from this condition, there are two possible solutions, one with a positive slope (L) and the other with a negative slope (M) (see figure 1). But the latter can be eliminated since it does not satisfy the second order condition for welfare maximization: According to Mailath (1987) or Collie and Hviid (1993), the second order condition can be expressed as

$$\frac{d^2 B_1}{ds^2} = -\frac{d\boldsymbol{f}^{-1}}{ds}\left[\frac{\partial^2 B_1}{\partial f_1 \partial s} - \frac{\partial^2 B_1}{\partial f_1 \partial \hat{f}_1}\frac{\partial B_1}{\partial s}\Big/\frac{\partial B_1}{\partial \hat{f}_1}\right] < 0 \qquad (49)$$

The second term in the bracket approaches zero as $f_1 \to f_1^L$, because

$\partial B_1 / \partial s \to 0$ and $\partial B_1 / \partial \hat{f}_1 > 0$, so $\dfrac{d^2 B_1}{ds^2} \approx -\dfrac{d\boldsymbol{f}^{-1}}{ds}\dfrac{\partial^2 B_1}{\partial f_1 \partial s}$.

Since $\partial^2 B_1 / \partial f_1 \partial s > 0$ from type monotonicity, $d\boldsymbol{f}^{-1}/ds$ must be positive to satisfy (49). Thus only the solution L survives. Intuitively, type monotonicity implies that the more effective the domestic R&D, then the larger benefit to the domestic country from using the subsidy, thus the government sets higher subsidies than under complete information. In fact, as shown in figure 1, the optimal R&D subsidy with incomplete information is higher than the subsidy with complete information, and it starts to diverge from it at the lowest value of R&D effect[13].

Proposition 1

When the product market is under Cournot competition, if the effect of domestic R&D on domestic fixed production costs is unknown to the foreign firm, the domestic government

[13] At this point, the single crossing condition turns out to be satisfied.

sets a higher R&D subsidy than the optimal subsidy under complete information and signals the effectiveness of the domestic R&D.

Comparison of the slope of the linear solution (k = 0.77) with that of the complete-information subsidy (8/27) reveals that if the support of f_1 has a wide range, then the optimal subsidy under incomplete information is approximately twice large as that under complete information around f_1^H. There are several factors that affect k. One of them is the coefficient on s in (45): (-3/7). If its absolute value is larger, it implies that the foreign firm reduces its R&D more facing the larger domestic R&D subsidy. This has two effects: First, it raises the slope of the complete-information subsidy. Second, it increases $\partial W_1 / \partial \hat{f}_1$, that is, the larger belief the foreign firm has on the domestic R&D effect improves the domestic welfare more, and this effect causes a larger divergence of the signaling subsidy from the complete-information subsidy.

3.2 R&D signaling model in a Bertrand duopoly

Now I assume that the effect of domestic R&D on fixed costs is private information again. Using (29) and (30), the first order conditions in stage 2 are

$$\frac{\partial g_1}{\partial x_1} = \frac{70(2a - b) - 352x_1 - 28x_2 + 225 f_1 + 225s}{225} = 0 \qquad (50)$$

$$\frac{\partial E_2 \langle g_2 | s \rangle}{\partial x_2} = \frac{70(2a - b) - 28E_2 \langle x_1 | s \rangle - 352x_2 + 225 f_2}{225} = 0 \qquad (51)$$

As in the previous section, taking expectations of (50) and substituting $E_2 \langle x_1 | s \rangle$ yield

$$x_1 = \frac{11088(2a - b) + 245 \hat{f}_1(s) + 38475 f_1 - 3080 f_2 + 38720s}{60192} \qquad (52)$$

$$x_2 = \frac{126(2a - b) - 35\hat{f}_1(s) + 440 f_2 - 35s}{684} \qquad (53)$$

Interestingly, (52) and (53) in this Bertrand case have very similar structures to (44) and (45) under Cournot competition. In fact, calculating domestic government's objective function $B_1(s, \hat{f}_1, f_1)$, we notice that all the sufficient conditions for existence and uniqueness of separating equilibrium are satisfied with the same signs as in the previous subsection (see appendix 2). Figure 2 implies that the optimal R&D subsidy under Bertrand competition has a similar structure to that under Cournot competition.

Proposition 2

When the product market is characterized by Bertrand competition, a similar subsidy policy to proposition 1 will be followed by the domestic government: If the effect of domestic R&D on domestic fixed costs is unknown to the foreign firm, then the domestic government sets a higher R&D subsidy than the optimal subsidy under complete information in order to signal the effectiveness of the domestic R&D activities.

3.3 Welfare Consideration

In this section I will consider the implications the above policies have on the domestic, foreign, and world welfare. I will concentrate on the Cournot case, but similar results are obtained also in the Bertrand case. I will compare the following four cases:

(Case 1: Complete information without a government) In this case, the welfare values are calculated using (10), (11), (17), and (18) setting s to zero.

(Case 2: Complete information with a domestic government) In this case, they are calculated using (10), (11), (17), (18), (19), and (20).

(Case 3: Signaling equilibrium) In this case, we cannot derive an explicit form of the optimal subsidy although the approximate shape is seen in figure 1. So, I assume that f_1^L coincides with $f_1^0 = \dfrac{-2(\boldsymbol{a}-\boldsymbol{b})+3f_2}{4}$. Then the optimal R&D subsidy in the signaling equilibrium is given by the linear solution with k=0.77. (For the sake of comparison, I will assume this for all the four cases.) The welfare values are calculated using (10), (11), (19), (44), (45), and the linear subsidy given below (48).

(Case 4: Enforced pooling equilibrium) As stated before, pooling equilibrium is ruled out in this model. However, if the domestic government cannot observe the exact value of the domestic R&D effectiveness, it has to set the subsidy based on the initial distribution of the effectiveness. The government is assumed to have information on such initial distribution. It is also assumed not to use any screening device to induce the domestic firm to report its true effectiveness, which I will analyze in the next section. Then the government sets the subsidy to the level given by (20) with f_1 replaced by $\overline{f}_1$ (mean). The welfare values are calculated using this *s*, (10), (11), (44) and (45) with $\hat{f}_1$ replaced by $\overline{f}_1$.

Due to the complexity of the functional forms, I had to depend on simulation, rather than derivation of general results (table 1). Comparison of (Case 2) to (Case 1) is straightforward. The domestic government shifts foreign rents to the domestic country by a strategic positive R&D subsidy in a Spencer-Brander sense. For every realized value of the R&D effectiveness except for the f_1^L, the domestic welfare increases, the foreign welfare decreases, and the world welfare decreases in (Case 2).

Comparison of (Case 3) to (Case 2) is intuitively interesting. In Case 3, the foreign firm does not have the information on the exact value of the realized domestic R&D effectiveness. To signal it, the domestic government will give excessive subsidies to the domestic firm. This is done in order to differentiate itself from a country with lower effectiveness. This *reduces* all the domestic, foreign, and world welfare compare to (Case 2). It is interesting that even the domestic welfare gets smaller in the presence of private information.

The result of the comparison of (Case 4) to (Case 3) depends on the distribution of f_1. If it is skewed to f_1^L, then the domestic expected welfare becomes larger, and both the foreign and world welfare values become smaller in (Case 4).

3.4 Concluding remarks in the signaling model

If a domestic government and a domestic firm know the effect of R&D projects on the domestic production, but a foreign firm does not know it, then the domestic government sets a higher subsidy than is optimal under complete information to signal the effectiveness of the R&D. This conclusion holds whether firms compete in Cournot or Bertrand mode.

The results are highly restricted by many specific assumptions such as linear demands, linear cost structures and so on. Especially, the assumption that R&D investments reduce both marginal and fixed cost but only the effect on the latter is private information was required for analytical convenience. Generalizing these assumptions makes the model highly intractable.

However, the proposition that the government sets a positive subsidy under complete information regardless of the mode of competition at third stage has been already shown in a more general setup by Bagwell & Staiger (1994). Moreover, also under incomplete information, if Mailath's conditions for existence and uniqueness of separating equilibrium are satisfied with the same sign, then the government will set a higher subsidy than under complete information in order to demonstrate the effectiveness of domestic R&D regardless the mode of downstream competition. This robustness of the optimal subsidies seems to fit the actual world compared to the sensitivity of export intervention in the third market model, in which not only the sign of the policy but also the distortional effect of incomplete information on it are totally reversed depending on the mode. (This third-market model is briefly reviewed in appendix 1.)

4. The screening model

In this section, I introduce a different informational structure from the preceding section: The domestic government is not well-informed on the effectiveness of domestic R&D activities any more. Its exact meaning is that it does not know the realized value of f_1 at the time when it announces its R&D policy, but it knows the distribution of it. This might be likely to happen when the government is subsidizing an infant industry. To analyze the opposite case to the preceding signaling model, I assume that the foreign firm is well-informed on the value. That is, asymmetry of information exists between the government and the private firms rather than across a national border. In this case, it will turn out that the domestic government faces a dilemma between giving a high subsidy to shift rents from abroad and preventing the domestic firm from lying about its true effectiveness of R&D. As a result, the government gives *smaller* subsidies to the domestic firm than under complete information. This result is similar to that in Brainard and Martimort (1992), who applied a screening to a third market export model without R&D. Contrary to their model, however, in my R&D model, this result is robust regardless of the mode of competition in the product market.

4.1 The R&D screening model under a Cournot duopoly

The basic structure of the model analyzed in this subsection is very similar to that in the preceding signaling model. Again, the effectiveness of domestic R&D, f_1, is assumed to be a random variable[14]. However, I introduce one significant change: public funds are assumed to be costly. This follows a convention in the previous screening models: for example, Baron and Myerson (1982), Laffont and Tirole (1986), and Brainard and Martimort (1992, 1996). This assumption is related to the essential structure of screening models. In the context of the R&D model, the domestic government offers a contract that has two parts: a per-unit R&D subsidy and a lump sum (negative) transfer. The latter plays an important role in inducing the firm to report its true R&D effectiveness. To prevent the firm from reporting its untrue R&D effectiveness, the government offers a contract that allows positive rent inside the firm with a reduced lump-sum tax if it reports its true value. The lump sum-transfer is used as a screening device. And this transfer exists only because public funds are costly. Thus the costly budget case explained in subsection 2.3 is used as a benchmark for this screening model.

I could have assumed costly public funds also in the complete-information model in subsection 2.1 or in the signaling model in section 3. Recall that the policy was linear in those models (there was no lump-sum transfer). In this case, if public funds were assumed to be costly, then the optimal subsidy would become smaller both under complete and incomplete information (a signaling model). This result is similar to those in Neary (1991) and Gruenspecht (1988), though they focus only on a complete information case. As shown in the subsection 2.3, if non-linear policies with lump-sum

[14] In Brainard and Martimort (1992), fully symmetric profit functions for domestic and foreign are assumed. In my R&D model, this is to assume $f_1 = f_2 = f$. In their paper, this is assumed to make the

transfers are adopted, then the per-unit subsidy under complete information is exactly the same as in the case when public funds are not costly. To put it another way, if the form of policies is not restricted to linear subsidies, the introduction of the assumption of costly budgets does not affect the optimal per-unit subsidies under complete information.

I extend this costly budget model in the following screening model. The timing of the game is as follows: At stage 1, the domestic government offers a contract that specifies $\{s(\hat{f}_1), t(\hat{f}_1)\}$, where s is a per-unit R&D subsidy and t is a lump-sum transfer from the domestic firm to the domestic government. They are functions of the effectiveness of domestic R&D activities reported by the domestic firm before stage 2: $\hat{f}_1$[15]. At stage 2, the domestic and foreign firms choose R&D levels simultaneously. At stage 3, they simultaneously choose production levels.

The domestic government does not know the realized value of f_1 while the firms know it. Then the domestic firm's rent after the transfer is done becomes:

$$U_1(\hat{f}_1, f_1) = \pi_1(s(\hat{f}_1), f_1) - t(\hat{f}_1) \qquad (54)$$

where the specific functional form of π_1 is given by (35). Note that the government offers a contract $\{s(\hat{f}_1), t(\hat{f}_1)\}$ based on the firm's report $\hat{f}_1$.

When information is incomplete, the government's policy must satisfy the firm's incentive compatibility constraint to induce its truthful report as well as the participation constraint.

Before investigating the incentive compatibility constraint, suppose that the government offers the complete-information contract ((39) and (40) with the firm's report $\hat{f}_1$ instead of its true value f_1) by even though information is incomplete, following the step taken by Brainard and Martimort (1992). In this case, the government offers a contract such that:

$$s(\hat{f}_1) = \frac{4(\alpha - \beta) + 8\hat{f}_1 - 6f_2}{27} \quad \text{and} \quad t(\hat{f}_1) = \pi_1(s(\hat{f}_1), \hat{f}_1).$$

model easy to handle when the countervailing intervention by a foreign government is introduced. But I do not assume this for sake of comparison with the previous signaling model.

[15] The story becomes more complex if the foreign firm is allowed to report the R&D effect of the domestic firm to the domestic government. In this paper I assume that the foreign firm cannot do it because it exists out of the domestic government's jurisdiction.

The second equation comes from the participation constraint: The government tries to absorb all the firm's rents based on the report of the R&D effectiveness revealed by the firm. Then, we can solve for the firm's optimal report by differentiating (54) with respect to $\hat{f}_1$:

$$\hat{f}_1 = \frac{-532(\alpha - \beta) + 344 f_1 + 798 f_2}{1849} \qquad (55)$$

Noting that α is assumed to be sufficiently large to make the two firms' profits positive, this value is less than f_1. The domestic firm has an incentive to underreport the effect of its R&D activities in order to reduce the lump-sum tax paid to the government.

I next solve for the optimal policy that satisfies the firm's incentive compatibility constraint. I let $U_1(f_1)$ denote the domestic firm's rent when its incentive compatibility constraint is satisfied:

$$U_1(f_1) \equiv U_1(f_1, f_1)$$

The first order condition for the domestic firm's optimal choice of report is:

$$\frac{\partial \pi_1(s(f_1), f_1)}{\partial s} \frac{\partial s(f_1)}{\partial f_1} - \frac{\partial t(f_1)}{\partial f_1} = 0 \qquad (56)$$

where the functional forms of π_1 is given in (35). Using (55) and the envelope theorem yields the following incentive compatibility constraint that the government faces:

$$\dot{U}_1(f_1) = [40(\alpha - \beta) + 129 f_1 - 60 f_2 + 129 s(f_1)]/98 \qquad (57)$$

where $\dot{}$ represents a derivative with respect to f_1. The domestic government maximizes the expected value of (37) with respect to its policy subject to the participation constraint (38) and the incentive compatibility constraint (57). Substituting for the transfer transforms the government's problem to:

$$Max_{s(f_1), U_1(f_1)} \int_{f_1^L}^{f_1^H} [\pi_1(s(f_1), f_1) - s(f_1) x_1(s(f_1), f_1) - \gamma U_1(f_1)] h(f_1) df_1$$

subject to (38) and (57), where $\gamma \equiv c/1+c$, $h()$ is a density function, and the functional forms of π_1 and x_1 are given by (35) and (17) respectively.

Then the Hamiltonian for this problem is:

$$H(s, U_1, \lambda, f_1) = [\pi_1(s(f_1), f_1) - s(f_1) x_1(s(f_1), f_1) - \gamma U_1(f_1)] h(f_1)$$

$$+\ \lambda(f_1)\left[40(\alpha-\beta)+129f_1-60f_2+129s(f_1)\right]/98 \qquad (58)$$

where λ is the multiplier on the constraint (57).

The optimality conditions in Pontryagin's Principle are:

$$\dot{\lambda}(f_1) = -\frac{\partial H}{\partial U_1} \qquad (59) \qquad \text{and}$$

$$\frac{\partial H}{\partial s} = 0 \qquad (60).$$

(59) yields $\dot{\lambda}(f_1) = \gamma h(f_1)$. Since the domestic firm has an incentive to underreport the effect of its R&D activities given the complete-information contract, as expressed in (55), the participation constraint (38) binds when R&D activities are the least effective: $U_1(f_1^L) = 0$, while there is no restriction on $U_1(f_1^H)$. Thus $\lambda(f_1^H) = 0$ and this solves for the multiplier: $\lambda(f_1) = \gamma(H(f_1) - 1)$, where $H(\,)$ is a cumulative distribution function[16].

Solving (60) and substituting the above λ into the solution yield the following optimal subsidy in this screening game:

$$s(f_1) = \frac{4(\alpha-\beta)+8f_1-6f_2}{27} - \frac{43}{27}\gamma\frac{1-H(f_1)}{h(f_1)} \qquad (61)$$

Note that the first term is the optimal subsidy under complete information. It is obvious that asymmetric information distorts the optimal subsidy downward.

Proposition 3

When the product market is under Cournot competition, if the effect of domestic R&D activities on domestic fixed production costs is unknown to the domestic government, then the government faces a tradeoff between shifting rents from abroad and inducing truthful revelation by the domestic firm, and sets smaller R&D subsidy than under complete information.

If I assume a uniform density as in Brainard and Martimort (1992), then the optimal subsidy becomes:

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[16] I assume the usual monotone hazard property.

$$s(f_1) = \frac{4(\alpha - \beta) + 8f_1 - 6f_2}{27} - \frac{43}{27}\gamma(f_1^H - f_1) \qquad (62)$$

The optimal subsidy is shown in Figure 3. First of all note that the subsidy under complete information in this figure is exactly the same as that in figure 1. Introduction of incomplete information distorts the subsidy downward in this screening model. The assumption of uniform density makes the subsidy schedule under incomplete information a straight line. It starts to diverge from the complete-information subsidy at the right end. The optimal subsidy might turn to a tax (negative subsidy) for some part of the R&D effect if the support of the distribution has a wide range.

4.2 The R&D screening model under a Bertrand duopoly

Our next concern is the robustness of the optimal policy with regard to the mode of competition at the third stage. The result turns out to be robust: the optimal policy under complete information is an R&D subsidy and incomplete information distorts this subsidy downward as in the Cournot case. If I return to symmetric linear demand functions as in (21) and (22) and set the coefficient of R&D expenditure to 1 (v =1)[17], then the optimal subsidy under incomplete information is:

$$s(f_1) = \frac{126(2\alpha - \beta) + 98f_1 - 35f_2}{14950} - \frac{7573}{7475}\gamma\frac{1 - H(f_1)}{h(f_1)} \qquad (63)$$

See appendix 3 for the process of calculation. Again, the first term corresponds to the optimal subsidy under complete information and is equivalent to (34). The second term distorts the optimal subsidy downward under incomplete information:

Proposition 4

When the product market is characterized by Bertrand competition, a similar subsidy policy to proposition 3 will be taken by the domestic government. If the effect of domestic R&D on domestic fixed cost is unknown to the government, then the government sets a smaller R&D subsidy than under complete information.

[17] As before, for simplicity of calculation, I assume these specific coefficients like ½ and 1. The result does not change if I change those parameters as long as they satisfy the second order conditions in stage 1 and 2.

Indeed, we could loosely predict this result before starting cumbersome calculations, since (29) and (30) imply that choice variables are strategic substitutes at stage 2 even in this Bertrand competition, and the coefficient on f_1 in (29) has the same sign as in (12). As shown in appendix 2, the firm's incentive compatibility constraint in this Bertrand cse has a similar structure to that in the Cournot case. This yields the robustness: Not only the sign of the policy (subsidy or tax) but also the distortionary effects of incomplete information on the policy is robust regardless of the mode of competition.

4.3 The R&D screening model with a fully non-linear policy

In the above screening model, the policy was non-linear in the sense that it contained a lump-sum transfer. However, the policy schedule was not *fully* non-linear but *affine* in the sense that a subsidy s is given on a per-unit basis (Tirole (1988), pp.136). In this subsection, I analyze the case in which the policy maker uses a *fully* non-linear policy schedule following Brainard and Martimort (1996). As in their paper, the Revelation Principle applies without loss of generality: The domestic government offers a contract that contains two parts: a non-linear transfer schedule to the domestic firm, $\boldsymbol{t}$, and its R&D activity level, x_1. They are both functions of the firm's report on the effectiveness of its R&D activities, $\hat{f}_1$.

The timing of the game turns into as following: At stage 1, the domestic government offers the above direct revelation mechanism that specifies $\{\boldsymbol{t}(\hat{f}_1), x_1(\hat{f}_1)\}$. At stage 2, the domestic firm chooses its report on the effect of R&D, $\hat{f}_1$, and the foreign firm simultaneously chooses its R&D level, x_2. At stage 3, the firms simultaneously decide production levels (in the case of Cournot competition) or prices (in Bertrand).

The equilibrium production levels at the third stage are the same as (8) and (9). At the second stage, the domestic firm's rent is similar to (10):

$$U_1 = \left(\frac{\boldsymbol{a} - \boldsymbol{b} + 2x_1 - x_2}{3}\right)^2 - F_1 + f_1 x_1 - x_1^2 + \boldsymbol{t} \qquad (64)$$

Note that sx_1 in (10) is replaced by $\boldsymbol{t}$ in (64). The foreign firm's rent is the same as (11), but for convenience of notation we redefine it as U_2:

$$U_2 = \left(\frac{\boldsymbol{a} - \boldsymbol{b} - x_1 + 2x_2}{3} \right)^2 - F_2 + f_2 x_2 - x_2^2 \qquad (65)$$

Noting that x_1 and $\boldsymbol{t}$ are the functions of the domestic firm's report, we can express the first order condition for the firm's optimization problem at stage 2 evaluated at equilibrium as:

$$(4/9)[\boldsymbol{a} - \boldsymbol{b} + 2x_1(f_1) - x_2]\dot{x}_1(f_1) + f_1 \dot{x}_1(f_1) - 2x_1(f_1)\dot{x}_1(f_1) + \dot{t}(f_1) = 0 \qquad (66)$$

Since the foreign firm maximizes its rent with respect to its R&D level, the first order condition for its optimization problem at stage 2 is the same as (13):

$$[4(\boldsymbol{a} - \boldsymbol{b}) - 4x_1 - 10x_2 + 9f_2]/9 = 0 \qquad (67)$$

(66) and the envelope theorem yield the incentive compatibility constraint that the government faces:

$$\dot{U}_1(f_1) = (-2/9)[\boldsymbol{a} - \boldsymbol{b} + 2x_1(f_1) - x_2(f_1)]\dot{x}_2(f_1) + x_1(f_1) \qquad (68)$$

Note that since the government offers a contract before the foreign firm chooses its R&D level, the government anticipates the effect of its policy on the foreign firm's decision. That is why x_2 is a function of f_1 this time. Its form is given by differentiating the foreign firm's implicit best response function (67):

$$\dot{x}_2(f_1) = -2/5\,\dot{x}_1(f_1) \qquad (69)$$

Substituting (67) and (69) into (68) rewrites the compatibility constraint as:

$$\dot{U}_1(f_1) = (1/75)[4(\boldsymbol{a} - \boldsymbol{b}) + 16x_1(f_1) - 6f_2]\dot{x}_1(f_1) + x_1(f_1) \qquad (70)$$

The government maximizes the expected value of $U_1(f_1) - (1+c)\boldsymbol{t}(f_1)$. Using (64) and (67) transforms the government's objective function to:

$$Max_{x_1(f_1), U_1(f_1)} \int_{f_1^L}^{f_1^H} \left[\left(\frac{2(\boldsymbol{a} - \boldsymbol{b}) + 8x_1(f_1) - 3f_2}{10} \right)^2 - F_1 + f_1 x_1(f_1) - x_1(f_1)^2 - gU_1(f_1) \right] h(f_1) df_1 \qquad (71)$$

where the definitions of c, g, and $h(\,)$ are the same as before. The constraints of this maximization problem is the participation constraint (the same form as (38)) and the incentive compatibility constraint (70). To solve it, we introduce an auxiliary choice variable following Brainard and Martimort (1996):

$$z_1(f_1) \equiv \dot{x}_1(f_1) \qquad (72)$$

Then, the Hamiltonian becomes:

$$H(z_1, U_1, x_1, \lambda, \mu, f_1) = \left[\left(\frac{2(\alpha-\beta)+8x_1-3f_2}{10}\right)^2 - F_1 + f_1 x_1 - x_1^2 - \gamma U_1\right] h(f_1)$$

$$+\ \lambda\left[\frac{4(\alpha-\beta)+16x_1-6f_2}{75} z_1 + x_1\right] + \mu z_1 \qquad (73)$$

where λ and μ are the multipliers on the constraint (70) and (72) respectively.

The conditions of Pontryagin Principles are:

$$\frac{\partial H}{\partial U_1} = -\dot{\lambda}(f_1) \qquad (74)$$

$$\frac{\partial H}{\partial x_1} = -\dot{\mu}(f_1) \qquad (75)$$

$$\frac{\partial H}{\partial z_1} = 0 \qquad (76)$$

(74) yields $\dot{\lambda}(f_1) = \gamma h(f_1)$. Since the domestic firm's rent is an increasing function as implied by (70), the participation constraint binds for the firm whose R&D activity is the least effective: $U_1(f_1^L) = 0$, while there is no restriction on $U_1(f_1^H)$. Thus $\lambda(f_1^H) = 0$ and this solves for the multiplier: $\lambda(f_1) = \gamma(H(f_1) - 1)$, where $H(\ )$ is a cumulative distribution function. Using this, (75), and (76) yields the optimal contract:

$$x_1(f_1) = \frac{4(6-\gamma)(\alpha-\beta) + 75 f_1 - 6(6-\gamma) f_2 - 75\gamma \dfrac{1-H(f_1)}{h(f_1)}}{2(27+8\gamma)} \qquad (77)$$

Noting that α is large enough to make the firms' rents positive, this output is *smaller* than the complete-information Stackelberg output (which is equivalent to the above value when $\gamma = 0$). Thus, the same conclusion as proposition 3 is obtained: Incomplete information decreases the government's precommitment ability. And the similar result holds even when the mode of competition in stage 3 is Bertrand.

4.4 Concluding remarks in the screening model

In this section I assumed a different informational structure from the last section: The government does not observe the realized value of the domestic R&D effectiveness when it offers a contract to the firm. The domestic and foreign firms observe it. In this screening model, I obtained a similar conclusion to Brainard and Martimort (1992 and 1996) briefly reviewed in appendix 1: The government faces a dilemma between inducing the domestic firm to engage in more R&D activities and preventing its untruthful revelation about its R&D effectiveness. The latter undermines the government's precommitment ability. Contrary to Brainard and Martimort, however, this conclusion is robust irrespective to the mode of competition in the product market. This robustness comes from the fact that choice variables at stage 2 are strategic substitutes no matter what the choice variables are at stage 3 and that the firm's incentive compatibility constraint has the same structure irrespective to the mode.

5. Concluding Remarks

This paper analyzes the optimal R&D subsidies under various circumstances. I assume that R&D activities affect both marginal and fixed costs of production and that only their effects on the latter are random. This is the case in which, for example, the construction of the firm's plant requires a highly sophisticated technology. Under complete information, the government gives positive R&D subsidies to the firm and this subsidy is an increasing function of the R&D effectiveness. I introduce asymmetric information in this model. When the government observes the exact effect of domestic R&D activities and a foreign firm does not, then the government gives a higher subsidy to the domestic firm than under complete information in order to signal the effectiveness. On the other hand, if the government does not observe the effectiveness at the time of contract, then it faces a tradeoff between giving high subsidies and inducing truthful revelation of the effectiveness. Then, it gives a smaller subsidy than under complete information.

My second concern stated in introduction was sensitivity of those results to the mode of competition. This R&D model showed that those directions in which the incomplete information distorts the policy are robust regardless of the mode of competition at the third stage. This result sharply contrasts with that in the third-market model without R&D. In the third-market model, the sign of the policy depends on the mode of competition in a product market: a subsidy under Cournot, but a tax under Bertrand. Moreover, the range of possible outcomes are doubled by introduction of different information structure: In the signaling model, the subsidy is distorted upward under Cournot, but the tax is distorted downward under Bertrand. In the screening model, the subsidy is distorted downward under Cournot, but the tax is distorted upward under Bertrand. In the R&D model, however, these kinds of complexity the government faces are relaxed: The optimal R&D policy under complete information is a subsidy regardless of the mode of competition as proved in the past literature. As shown in this paper, with incomplete information, the well-informed government should distort the subsidy upward no matter which competition is going on. On the other hand, the not well-informed government should distort it downward regardless of the competition mode. The results depend on the specific assumption I used in the model, of course. However, as long as the choice variables in stage 2 are strategic substitutes, the sufficient conditions in signaling models hold with the same sign, and the firm's incentive compatibility condition holds with the same sign in the screening model, this robustness is going to be kept.

Besides the above robustness, there is another reason for the feeling that this R&D model fits the actual policies: This is related to the fact that *s* is an increasing function of f_1. In the R&D model under complete information the government gives a higher R&D subsidy to the domestic firm when R&D projects are more effective. This conclusion is strengthened under incomplete information. On the other hand, in the Cournot version of the third market model, the domestic government gives a higher export subsidy to the domestic firm when the marginal cost of production is lower (In figure 4, the optimal subsidy has a negative slope). These two results are saying the same thing in the sense that “the more competitive firm should be more subsidized”. However, as pointed out at the end of the introduction, the third market model seems to fail to fit what is actually going on, because there seems to be a historical experience that direct intervention on trade have been implemented in the less competitive area. As long as R&D are considered, however, it seems natural for the government to increase a subsidy to the more effective R&D activities.

References

Baron, D. and R. Myerson (1982), “Regulating a Monopolist with Unknown Costs”, Econometrica, 50, pp.911-930

Bagwell, K. and R.W. Staiger (1994), “The Sensitivity of Strategic and Corrective R&D Policy in Oligopolistic Industries”, Journal of International Economics 36, pp.133-150

Brainard, L. and D. Martimort (1992), “Strategic Trade Policy with Incompletely Informed Policy Makers”, National Bureau of Economic Research Working Paper no.4069

Brainard, L. and D. Martimort (1996), “Strategic Trade Policy Design with Asymmetric Information and Public Contracts”, Review of Economic Studies 63, pp.81-105

Brander, J.A. (1995), “Strategic Trade Policy” in Grossman and Rogoff (eds), Handbook of International Economics vol.III, Amsterdam, North-Holland

Brander, J.A. and B.J. Spencer (1985), “Export Subsidies and International Market Share Rivalry”, Journal of International Economics 18, pp.83-100

Collie, D. and M. Hviid (1993), “Export Subsidies as Signals of Competitiveness”, Scandinavian Journal of Economics 95, pp.327-339

Collie, D. and M. Hviid (1994), “Tariffs for a Foreign Monopolist under Incomplete Information”, Journal of International Economics 37, pp.249-264

Collie, D. and M. Hviid (1999), “Tariffs as Signals of Uncompetitiveness”, Review of International Economics, 7(4), pp.571-579

Eaton, J. and G.M. Grossman (1986), “Optimal Trade and Industrial Policy under Oligopoly”, Quarterly Journal of Economics 101, pp.383-406

Gruenspecht, H.K. (1988), “Export Subsidies for Differentiated Products”, Journal of International Economics, 24, pp.331-344

Krugman, P. R. “The Narrow and Broad Arguments for Free Trade”, American Economic Review 83(2), pp.362-366

Laffont, J.J. and J. Tirole (1986), “Using Cost Observation to Regulate Firms”, Journal of Political Economy 94, pp.614-641

Maggi, G. (1996), “Strategic Trade Policies with Endogenous Mode of Competition”, American Economic Review, vol 86, pp.237-258

Mailath, G.J. (1987), “Incentive Compatibility in Signaling Games with a Continuum of Types”, Econometrica 55, pp.1349-1365

de Meza, D. (1986), “Export Subsidies and High Productivity: Cause or Effect”, Canadian Journal of Economics 19, pp.347-350

Neary, J.P. (1991), “Export Subsidies and Price Competition” in E. Helpman and A. Razin (eds.), International Trade Policy, MIT press, Cambridge

Osborne, M.J., and A. Rubinstein (1996), “A Course in Game Theory”, MIT press, Cambridge

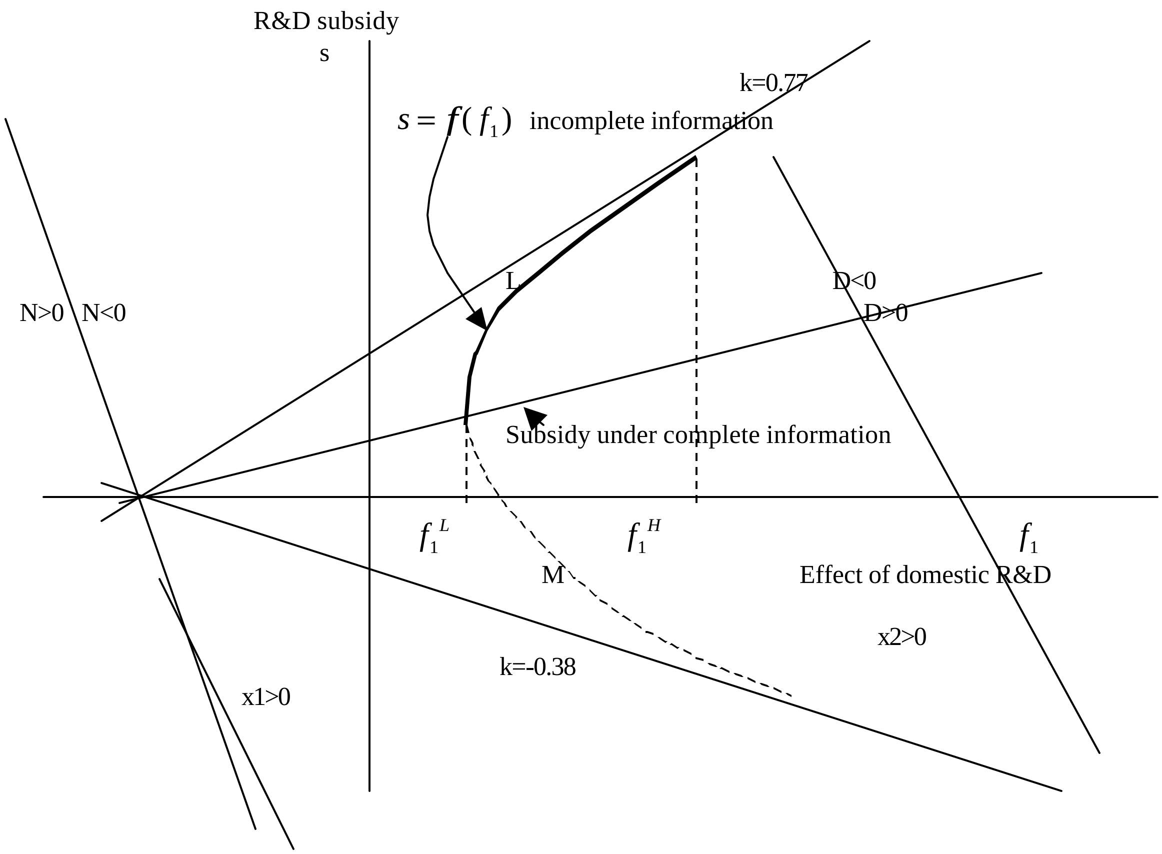
Qiu, L.D. (1994), “Optimal Strategic Trade Policy under Asymmetric Information”, Journal of International Economics 36, pp.333-354

Rasmusen, E. (1989), “Games and Information: An Introduction to Game Theory”, Basil Blackwell, Oxford

Spencer, B.J. and J.A. Brander (1983), “International R&D Rivalry and Industrial Strategy”, Review of Economic Studies 50, pp.707-722

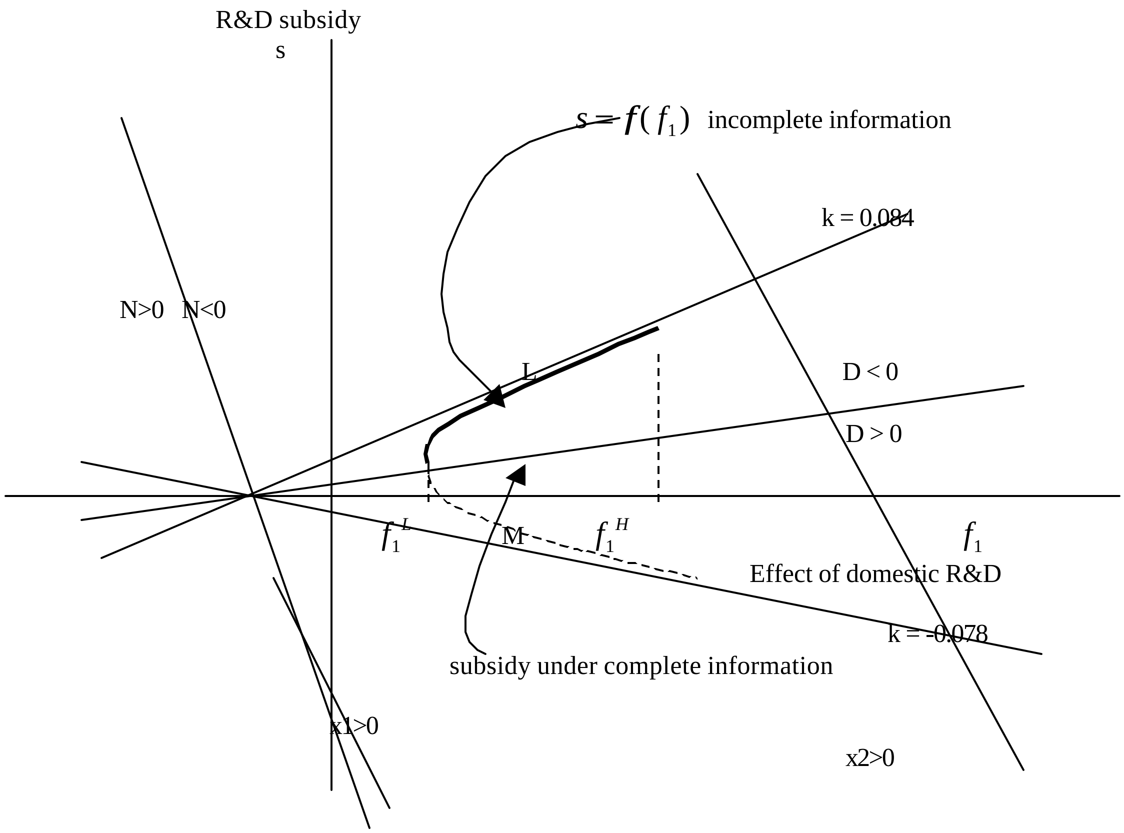
Tirole, J. (1988), The Theory of Industrial Organization, MIT Press, Cambridge

Figure 1 Optimal R&D Subsidy under Cournot Competition



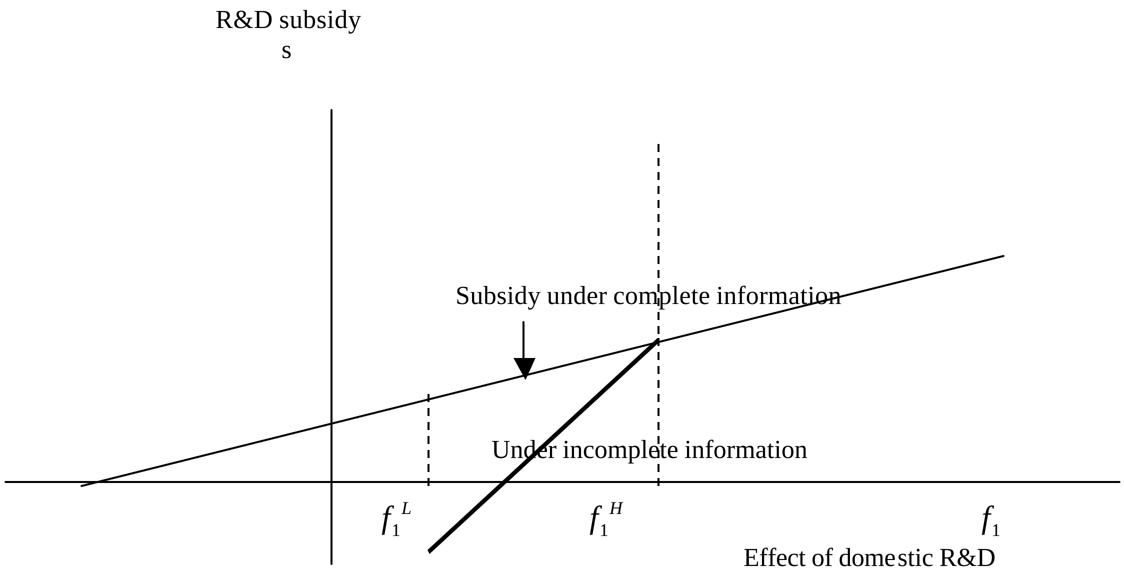
* Scales are not necessarily accurate.

Figure 2 Optimal R&D Subsidy under Bertrand Competition



* Scales are not necessarily correct.

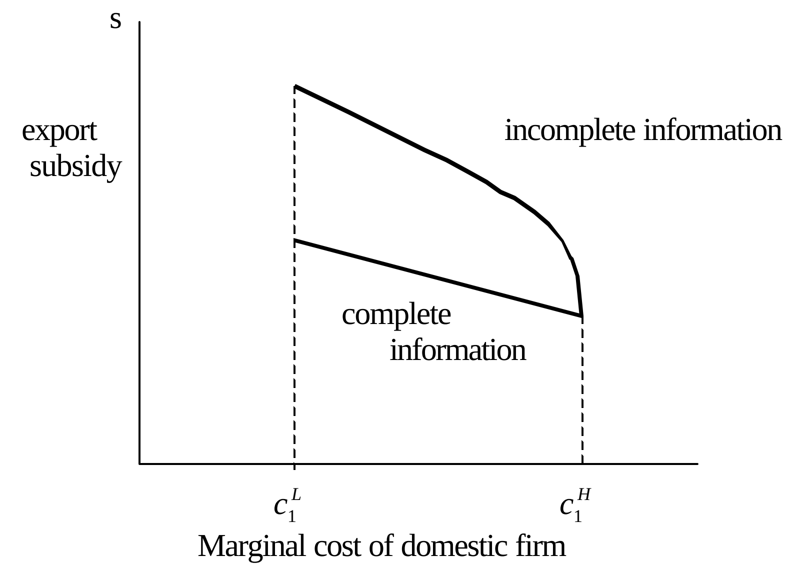
Figure 3 Optimal R&D Subsidy in the Screening Model under Cournot Competition



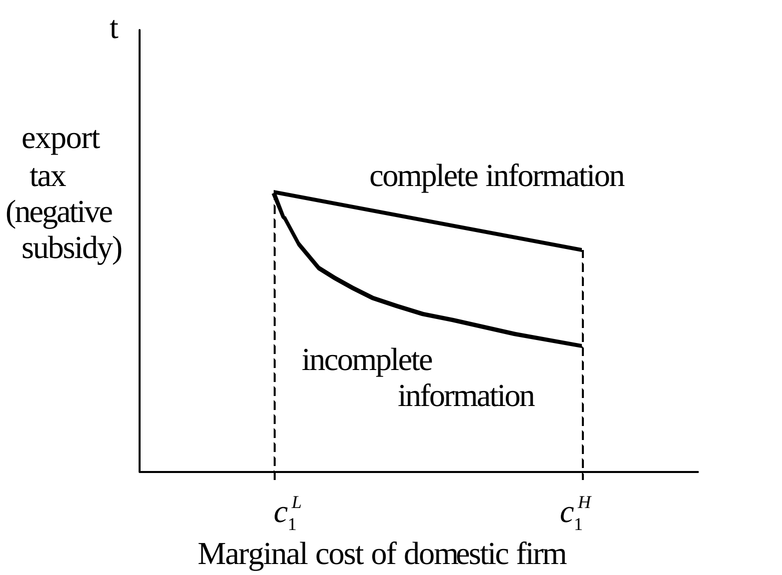
* Scales are not necessarily accurate.

Figure 4 Separating Equilibrim in the Third Market Model (Collie and Hviid (1993))

Under a Cournot duopoly



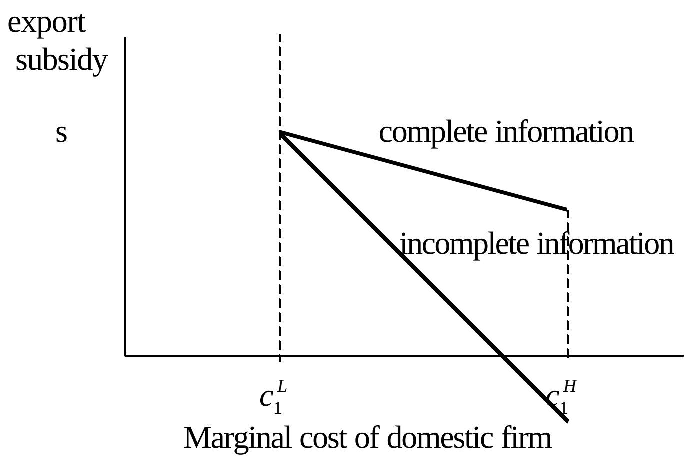
Under a Bertrand duopoly



(Note)1. The above figure is equivalent to figure 1 in Collie and Hviid (1993) with the horizontal axis replaced with the cost.
2. The below figure is drawn by the author based on the discussion in Collie and Hviid (1993), pp.338.

Figure 5 Optimal Subsidy in the Third Market Screening Model (Brainard and Martimort (1992))

Under a Cournot duopoly



(Note) In Brainard and Martimort (1992), marginal costs of domestic and foreign firms are assumed to be always equivalent.

Appendix 1: The past literature on strategic trade models under asymmetric information

I briefly review the past strategic trade models under incomplete information for sake comparison with the R&D model.

In international economics, players often reside in areas divided by national borders and some of them are likely to be less well-informed about the other players' preferences than about their own, though the possibility of asymmetric information will not be limited to such cross-border cases. Several international trade papers deal with the problem of such incomplete information, some of which are Brainard and Martimort (1992), and Collie and Hviid (1993), whose methods I used in this paper. Models are categorized based on the assumption on who own private information and who do not. Many models belong to the strands of screening or signaling models[18], most of which use the third market model as their setups[19].

A.1.1 Signaling Models

With complete information in a Cournot duopoly, the optimal policy is an export subsidy in the third market model. Collie and Hviid (1993) introduce incomplete information by assuming that the domestic government and the domestic firm know the marginal cost of the domestic production, but the foreign firm does not know it. They derive a separating equilibrium[20] and show that, with incomplete information in a Cournot duopoly, the domestic government will use a *higher* export subsidy to signal the competitiveness of the domestic firm (figure 4).

This result is reversed in Bertrand competition. With complete information in a Bertrand duopoly, as in Eaton and Grossman (1986), the optimal policy is an export tax (negative subsidy). Collie and Hviid (1993) showed that with incomplete information in a Bertrand duopoly, the domestic government sets a *lower* export tax to signal the domestic firm's "uncompetitiveness" (figure 4). Thus, not only the sign of the optimal

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[18] Some of these incomplete information models are briefly reviewed in Brander (1995).

[19] A few exceptions are Collie and Hviid (1994) which introduce incomplete information to the model in which a domestic country imports a product from a foreign monopolist, and Collie and Hviid (1999) which analyze a reciprocal markets model with incomplete information.

[20] There is also a model which focus on a pooling equilibrium (see Kolev and Prusa (1999)) in a different setup.

policy but also the distortionary effect of incomplete information on the policy is sensitive to the mode of competition.

A.1.2 Screening Models

Brainard and Martimort (1992) study a screening in the third market model. In their model, the domestic production cost is private information and the domestic government announces its trade policy without observing its realized value. The domestic and foreign firms know it[21]. This policy announcement comprises two parts: a per-unit export subsidy and a lump-sum transfer from the firm to the government. They are functions of the production cost reported by the domestic firm. Then the domestic firm reports its production cost. And the two firms produce goods and export them to the third country. In their model, the domestic government gives *smaller* subsidies than under complete information in order to induce the domestic firm to report its true cost (figure 5). As shown in this figure, if the distribution of the cost has a large support, then the optimal policy might become even an export tax (negative subsidy) for some range of the cost. In their third-market screening model, the conclusion switches when the firms compete as Bertrand duopolists. In the Bertrand case, the optimal export policy is a tax under complete information as mentioned before, and the optimal tax is distorted *upwards* under incomplete information, contrary to the Cournot case. Again, the direction in which incomplete information affects the policy is totally reversed by changing the mode of competition. The results of the R&D model developed in text contrast with the sensitivity in the past literature.

[21] Qiu (1994) assumes a different informational structure from those in the signaling and screening models. His model is a third market model without R&D, in which the domestic government and the foreign firm are not well-informed on the domestic production cost, but the domestic firm knows it. It is a two-cost-type model that combines screening and signaling. I did not adopt this approach in this R&D paper.

Appendix 2: Separating equilibrium when the firms are Bertrand duopolists

Calculation gets messy, partly because (21) and (22) contain parameters that are less than one.

Government's welfare function $W_1 = \boldsymbol{p}_1 - sx_1$ is rewritten using (33), (52), and (53):

$$B_1(s, \hat{f}_1, f_1) = \left(\frac{23760(2a - \boldsymbol{b}) + 525\hat{f}_1(s) + 17455 f_1 - 6600 f_2 + 18480s}{60192} \right)^2 - F_1$$
$$+ f_1 \times \left(\frac{11088(2a - \boldsymbol{b}) + 245\hat{f}_1(s) + 38475 f_1 - 6600 f_2 + 18480s}{60192} \right)$$
$$- \left(\frac{11088(2a - \boldsymbol{b}) + 245\hat{f}_1(s) + 38475 f_1 - 6600 f_2 + 18480s}{60192} \right)^2 \qquad \text{(A1)}$$

Mailath's three conditions are satisfied, since

$\frac{\partial B_1}{\partial f_1} > 0$, $\frac{\partial^2 B_1}{\partial f_1 \partial s} > 0$, and $\left(\frac{\partial B_1}{\partial s} \right) \Big/ \left(\frac{B_1}{\partial \hat{f}_1} \right)$ is monotonic in f_1.

The differential equation derived from the government's welfare maximization is

$$\frac{d\boldsymbol{f}}{df_1} = \left(-\frac{\partial B_1}{\partial \hat{f}_1} \right) \Big/ \left(\frac{\partial B_1}{\partial s} \right) = N/D = \frac{7[-1584(2a - \boldsymbol{b}) - 1232 f_1 + 440 f_2 - 35s]}{88[126(2a - \boldsymbol{b}) + 98 f_1 - 35 f_2 - 14950s]} \qquad \text{(A2)}$$

N = 0 and D = 0 lines intersect at (f_1^0, s^0) where

$$f_1^0 = \frac{-6498(2a - \boldsymbol{b}) + 1805 f_2}{5054}, \qquad s^0 = 0.$$

To derive two linear solutions of (A2) that pass through this intersection, we solve the quadratic $474931600k^2 - 3201709k - 3113264 = 0$, which have two roots k = 0.084, -0.078.

The loci $x_1 = 0$ and $x_2 = 0$ are calculated from (52) and (53).

The above results are shown in figure 2.

Appendix 3: Optimal subsidy in the R&D screening model when the firms are Bertrand duopolists

Plugging (31) and (32) into (27), the domestic firm's profit function before a lump sum transfer is:

$$\pi_1(s(f_1), f_1) = \left(\frac{270(2a-\beta)+210f_1-75f_2+210s(f_1)}{684}\right)^2 - F_1$$

$$+(f_1+s(f_1))\frac{126(2a-\beta)+440f_1-35f_2+440s(f_1)}{684}-\left(\frac{126(2a-\beta)+440f_1-35f_2+440s(f_1)}{684}\right)^2$$

This corresponds to (35) in Cournot case.

I assume that public funds are costly just as in the screening model under Cournot competition and follow exactly the same step as before.

The incentive compatibility constraint corresponding to (57) in the Cournot case is:

$$\dot{U}_1(f_1) = [11088(2a-\beta)+37865f_1-3080f_2+37865s(f_1)]/58482$$

The Hamiltonian becomes:

$$H(s,U_1,\lambda,f_1) = [\pi_1(s(f_1),f_1)-s(f_1)x_1(s(f_1),f_1)-\gamma U_1(f_1)]h(f_1)$$
$$+\ \lambda(f_1)[11088(2a-\beta)+37865f_1-3080f_2+37865s(f_1)]/58482$$

If we solve this Hamiltonian using the Pontryagin's Principle, we obtain (63).

Table 1 Comparison of Welfare

domestic welfare

f1	1.25	2	3	4	5	6
(case 1)	0.391	1.230	3.500	7.087	11.990	18.209
(case 2)	0.391	1.250	3.611	7.361	12.500	19.028
(case 3)	0.391	1.198	3.327	6.660	11.196	16.936
(case 4)	0.516	1.634	3.911	7.089	11.167	16.145

foreign welfare

f1	1.25	2	3	4	5	6
(case 1)	18.828	15.944	12.500	9.515	6.990	4.923
(case 2)	18.828	15.139	10.895	7.423	4.722	2.793
(case 3)	18.828	13.899	8.586	4.711	2.275	1.277
(case 4)	14.337	12.427	10.037	7.827	5.797	3.947

world welfare

f1	1.25	2	3	4	5	6
(case 1)	19.219	17.173	16.000	16.602	18.980	23.133
(case 2)	19.219	16.389	14.506	14.784	17.222	21.821
(case 3)	19.219	15.097	11.913	11.371	13.471	18.212
(case 4)	14.853	14.061	13.948	14.916	16.964	20.092

Assumptions:
alpha= 15, beta = 10, F1 = 0, F2 = 5, f2 = 5

If the support of f1 is taken wide enough, modifications of parameter values do not affect the relationship between values of each case.