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## Convertibility Risk: The Precautionary Demand for Foreign Currency in a Crisis

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and Alex Mourmouras*

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in a Crisis**

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and Alex Mourmouras<sup>1</sup>

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**Abstract**

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This paper presents theoretical work linking money demand to the perceptions of households about the risk that domestic currency may become inconvertible or that it may be devalued. An empirical investigation of the size of this effect is carried out using both cross section data and then monthly data for Korea to estimate an augmented demand for money equation. It is found that the fear of inconvertibility arising from the 1997 Korean currency crisis may have caused broad money demand to fall by 4-5 percentage points, equivalent to the loss of reserves of \$6-7½ billion (or about 30 percent of reserves as measured at end-November 1997).

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## I. INTRODUCTION

The understanding of the nature of foreign exchange crises has deepened significantly following several such episodes affecting emerging markets during the second half of the 1990s. Countries vulnerable to external shocks are now identified using well established vulnerability indicators, such as a large current account deficit, sustained loss of competitiveness, and low external reserves (measured for example relative to imports, to short-term external debt measured by residual maturity, and to money). Work on Early Warning Systems (EWS) has produced further econometric evidence regarding the usefulness of such indicators in predicting foreign exchange crises (see Berg et al, 2000).

However, the understanding of the specific channels through which a crisis can propagate remains less complete, weakening the ability to design appropriate preventive policies. In particular, it is often difficult to judge which set of vulnerability indicators is the most relevant for a particular country, and the work on EWS, although useful, is based much more on statistical concepts than on a well understood theoretical framework. Hence, even *ex post*, it is often difficult to assign a quantitative weight on the importance of various factors in explaining the incidence and severity of a crisis.

New theoretical work investigates a specific channel of vulnerability, the drop in the demand for money that arises from a fear of a crisis-induced inconvertibility of domestic currency. According to this argument, if domestic residents require access to foreign exchange for transactions purposes and come to believe there is a significant risk of interference with normal market access, they will attempt to protect themselves by a precautionary demand for foreign currency at the expense of their holdings of domestic currency. This will manifest itself in a simultaneous drop in the demand for domestic money and shrinkage of reserve asset holdings by the central bank. This result is demonstrated in a simple two-period optimizing model. In addition, the effectiveness of an interest rate defense against reserve loss is compared with devaluation as an option. It is shown that the currency substitution induced by inconvertibility risk is much stronger than that caused by devaluation risk.

The empirical relevance of this argument is demonstrated by including variables reflecting inconvertibility or fear of inconvertibility in the demand for money equation. The paper first estimates a cross section demand for money function in the form of a velocity equation, augmented to include a dummy variable reflecting presence or absence of current account convertibility, measuring the long-run negative effect of inconvertibility on the demand for money.

Next a conventional time series money demand equation for Korea is modified to include variables that are observable and that are linked to the probability that domestic currency becomes inconvertible following a crisis. The paper finds that such variables are helpful in explaining money demand, and especially the large drop around the time when the foreign exchange crisis afflicted Korea (toward the end of 1997). A counterfactual simulation is conducted to measure the impact of these "crisis" variables on money demand. These results are

then contrasted with empirical findings on the effects of inconvertibility from a cross-sectional data based approach.

Finally, conclusions are drawn regarding the quantitative importance of inconvertibility in crises, as well as more tentative conclusions on how to design policies to minimize external vulnerability given the findings in this paper.

## II. A MODEL OF CURRENCY INCONVERTIBILITY

Consider a small open Fisherian economy that lasts for two periods,  $t$  and  $t+1$ . A large number of representative agents consume two nonstorable goods at  $t+1$ , an internationally tradable domestic good  $x$  and an imported good  $y$ . The utility function  $u(x_{t+1}, y_{t+1})$  exhibits positive and diminishing marginal utilities and satisfies the Inada conditions for both goods. Each agent is endowed with  $W$  units of good  $x$  in period  $t$ . Two assets are available to households:  $h$  denotes domestic currency-denominated assets paying a real gross return  $R > 1$ . These assets are available in the date- $t$  home-country currency market at unit price  $1/p_t$  in terms of goods, that is determined by domestic market conditions. Agents may also access the foreign exchange market at date  $t$  to purchase foreign hard currency  $f$  at an exchange rate that is pegged to unity. Accumulating foreign currency assets, which pay a gross return of unity in foreign currency terms, is beneficial because there is a risk that the convertibility of the domestic currency will be suspended at  $t+1$ . We let  $\mathbf{a}_{t+1} \in [0, 1]$  denote the probability that households will be unable to convert their holdings of domestic currency into foreign currency in period  $t+1$ . This could arise because the monetary authority runs short of foreign exchange and rations its availability to the market rather than allowing its price to rise to clear the market.

The decision problem facing a young household entering period  $t$  is to maximize

$$E_t \{u(x_{t+1}, y_{t+1})\} \equiv (1 - \mathbf{a}_{t+1})u(x_{t+1}^1, y_{t+1}^1) + \mathbf{a}_{t+1}u(x_{t+1}^2, y_{t+1}^2) \quad (1)$$

subject to the following constraints: At date  $t$ ,

$$\frac{h_t}{p_t} + f_t = W; \quad (2)$$

At date  $t+1$ , with probability  $1 - \mathbf{a}_{t+1}$ ,

$$x_{t+1}^1 \leq \frac{h_t(1+i_t)}{p_{t+1}} - b_{t+1}; \quad y_{t+1}^1 \leq f_t + b_{t+1} \quad (3)$$

At date  $t+1$ , with probability  $\mathbf{a}_{t+1}$ ,

$$x_{t+1}^2 \leq \frac{h_t(1+i_t)}{p_{t+1}}; \quad y_{t+1}^2 \leq f_t \quad (4)$$

We let  $m_t \equiv h_t / p_t$  denote real domestic currency balances,  $\mathbf{p}_t \equiv (p_{t+1} - p_t) / p_t$  home-country inflation, and  $R_t \equiv (1+i_t)/(1+\mathbf{p}_t)$  the gross return of domestic currency, where  $i_t$  is the net nominal net rate of interest on domestic asset holdings. The quantity  $b_{t+1}$  is the planned

purchases (sales if negative) of foreign currency in the spot foreign exchange market in period  $t+1$  if convertibility is maintained.

Using the budget constraints, utility can be written in terms of the asset demands as follows.

$$(1 - \mathbf{a}_{t+1})u[(W - f_t)R_t - b_{t+1}, f_t + b_{t+1}] + \mathbf{a}_{t+1}u[(W - f_t)R_t, f_t]$$

As a preliminary, consider the situation in which the risk of inconvertibility is zero,  $\mathbf{a}_{t+1} = 0$ . In this case, the problem is simply to maximize  $u[(W - f_t)R_t - b_{t+1}, f_t + b_{t+1}]$ . If  $R_t > 1$  and the agent cannot borrow foreign-currency assets at  $t$ , the agent will select  $f_t = 0$ ,  $m_t = W$  and  $b_{t+1}$  solves  $\max u[WR_t - b_{t+1}, b_{t+1}]$ .<sup>2</sup> Stated differently, if the domestic real interest rate is positive, precautionary demand for foreign currency will be zero and foreign currency balances needed for imports will be met from the spot foreign exchange market.

To give a specific example, if utility is logarithmic,  $u(x, y) = \mathbf{g} \ln(x) + (1 - \mathbf{g}) \ln(y)$ , demand schedules for domestic and foreign goods are  $x = \mathbf{g}[f + R(W - f)]$  and  $y = (1 - \mathbf{g})[f + R(W - f)]$ . Utility is  $U(f) = \mathbf{g} \log(\mathbf{g}) + (1 - \mathbf{g}) \log(1 - \mathbf{g}) + \log[f + R(W - f)]$  and  $U'(f) = (1 - R)[f + R(W - f)]^{-1} < 0$  if  $R > 1$ . The case  $\mathbf{a}_{t+1} = 0$  leads to a corner solution for  $f=0$  and a plan for spot demand for foreign currency at  $t+1$  of  $b_{t+1} = (1 - \mathbf{g})R_t W$ . At the other extreme, if inconvertibility is deemed inevitable ( $\mathbf{a}_{t+1} = 1$ ), domestic households must plan to pay for their imports exclusively by building up precautionary balances of foreign currency, i.e.,  $b_{t+1} = 0$  and  $f_t$  maximizes  $u[(W - f_t)R_t, f_t]$ . From the Inada conditions, the solution for  $f_t$  is interior and satisfies the usual intertemporal condition  $\frac{u_2}{u_1}[(W - f_t)R_t, f_t] = R_t$  which, in the case of logarithmic utility, gives  $f_t = (1 - \mathbf{g})W$ .

More generally, if  $\mathbf{a}$  is positive but less than unity,  $f_t$  will be positive because of the Inada condition on good  $y$ , which requires that  $\partial u / \partial y \rightarrow \infty$  as  $y \rightarrow 0$  so we are justified in looking at interior solutions involving  $f_t > 0$ . The first order conditions for an interior optimum are

$$b_{t+1}: (1 - \mathbf{a}_{t+1})[-u_1(x^1, y^1) + u_2(x^1, y^1)] + \mathbf{a}_{t+1}[0 \cdot u_1(x^2, y^2) + 0 \cdot u_2(x^2, y^2)] = 0 \quad (5)$$

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<sup>2</sup> If households had access to the international capital market at a gross interest rate of unity, they would have incentives to borrow foreign currency and invest domestically.

$$f_t : (1 - \mathbf{a}_{t+1})[-R_t u_1(x^1, y^1) + u_2(x^1, y^1)] + \mathbf{a}_{t+1}[-R_t u_1(x^2, y^2) + u_2(x^2, y^2)] = 0 \quad (6)$$

The first condition is simply

$$u_1(x^1, y^1) = u_2(x^1, y^1) \quad (7)$$

Substituting (7) into (6) and simplifying yields

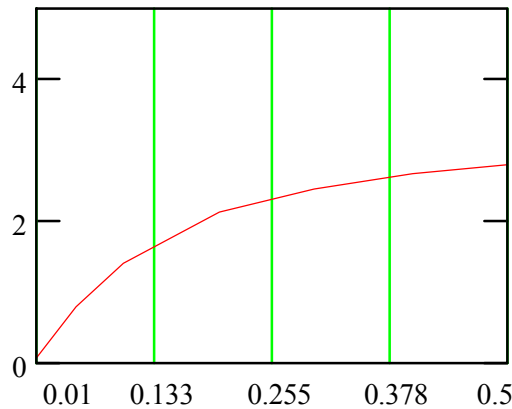
$$(1 - \mathbf{a}_{t+1})(R_t - 1)u_1(x^1, y^1) = \mathbf{a}_{t+1}[-R_t u_1(x^2, y^2) + u_2(x^2, y^2)] = 0. \quad (8)$$

Total differentiation with respect to  $\mathbf{a}_{t+1}$  and using the notation  $u^1 = u(x^1, y^1)$ , etc., yields

$$\frac{df_t}{d\mathbf{a}_{t+1}} = \frac{1 - R_t}{\mathbf{a}_{t+1}} \times \frac{u_2^1}{\mathbf{a}_{t+1}[R_t^2 u_{11}^2 - R_t(u_{12}^2 + u_{21}^2) + u_{22}^2]} > 0, \quad (9)$$

since the denominator is negative. Thus an increased risk of currency inconvertibility leads to increased precautionary holdings of foreign currency assets at the expense of domestic money. This result can be illustrated with a CES utility function with elasticity of substitution between home and foreign goods  $\hat{\epsilon} = 0.8$ ,  $R = 1.2$ ,  $W = 10$ , and the share of home goods in the utility function  $\tilde{a} = 0.7$ . As the risk of inconvertibility rises, the share of assets held in foreign currency rises from near zero to almost 30 percent.

Figure 1. Inconvertibility Risk: Share of Assets Held in Foreign Currency



Probability  $\alpha_{t+1}$

Several remarks are in order. First, as  $\mathbf{a}$  rises,  $b$  drops and  $f$  increases. This suggests that the volume of transactions in the spot market could contain useful information for the probability of crisis, which could be useful in designing early warning systems. Second,  $f$  rises sharply with the probability of crisis. These results hold for a general CES utility function but can be nicely illustrated in the case of log preferences,  $u(x, y) = \ln(x) + \mathbf{d} \ln(y)$ , where  $\mathbf{d} > 0$  is the weight of imported goods in preferences. Asset demands in this case are:

$$b_{t+1} = \frac{\mathbf{d}}{1+\mathbf{d}} R_t W - \frac{1+\mathbf{d}R_t}{1+\mathbf{d}} f_t \quad (10)$$

$$\left(\frac{f_t}{W}\right)^2 - \left(1 + \frac{\mathbf{a}\mathbf{d}}{1+\mathbf{d}} + \frac{\mathbf{a}}{R-1}\right) \left(\frac{f_t}{W}\right) + \frac{\mathbf{a}\mathbf{d}R}{(R-1)(1+\mathbf{d})} = 0 \quad (11)$$

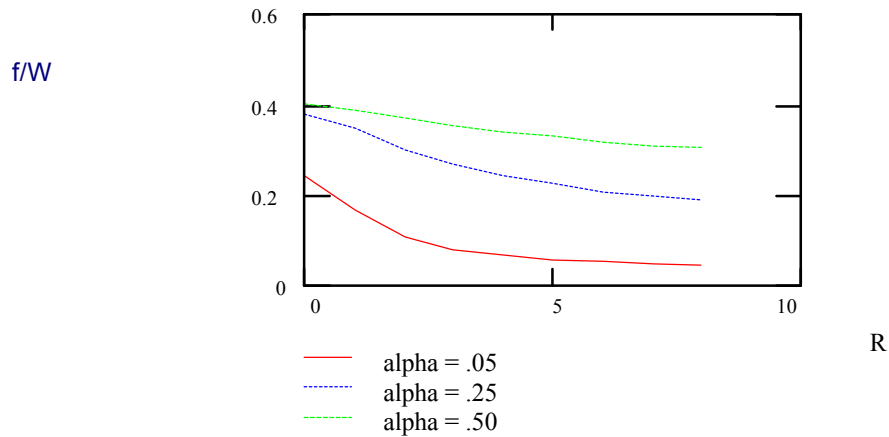
In equation (11), we choose the smaller root of the quadratic, which is less than unity. This example highlights, first, the inverse relationship between  $b$  and  $f$  (see equation (10)). In response to an increase in the likelihood of crisis, and the attendant rise in precautionary balances of foreign currency, the volume of transactions in the spot foreign currency market declines. This suggests monitoring changes in the volume of transactions in the foreign exchange market to infer the probability of crisis—an insight no doubt used in the design of early warning systems. Secondly, precautionary demand for foreign currency rises sharply with the probability of crisis. At a 5 percent real interest rate, agents with log preferences facing a five percent probability of crisis would hold a 25 percent of their income in precautionary foreign currency balances. When the probability of crisis rises to about 50 percent, the optimal share of foreign currency rises to 40 percent. While these numbers refer to the Cobb-Douglas case, the main result—that the share of foreign currency assets rises steeply as the probability of crisis rises above zero—is robust so long as an Inada condition is imposed on the consumption of imported goods, which is a realistic one to impose in most developing country contexts subject to crises. Third, the precautionary demand for foreign currency is higher the larger is the weight of imports in preferences. This suggests that the vulnerability due to inconvertibility risk is greater the more dependent on imports an economy is. Fourth, the opportunity cost of holding barren (but safe) foreign currency depends on the real interest rate paid on domestic currency assets. We now turn to examine this issue in detail.

### **An Interest Rate Defense of the Domestic Currency**

A central bank's main weapon during a currency crisis is to raise the short-term interest rate it controls, which raises demand for domestic assets and tends to avert or at least delay the crisis. The feasibility and optimality of raising interest rates to counter a currency crisis is the subject of a growing literature. Krugman (1979) assumed that the central bank is passive. Flood and Jeanne (2000) analyzed an interest rate defense in the Krugman-Flood-Garber model and concluded that it is never effective. In their model, an interest rate defense worsens the fiscal situation and helps bring about the crisis forward. Drazen (1999) and Végh and Lahiri (2000) argue that the relationship between the exchange rate and the interest rate is nonlinear. In Drazen's model, in which higher interest rates have signaling effects, higher interest rates may signal either that the authorities are more or less able to defend a peg depending on speculators'

information sets. Végh and Lahiri focus on the output and budgetary costs of an interest rate defense in a general equilibrium shopping-time monetary model in which non-cash, interest-bearing financial assets are part of money demand. They find that raising interest rates beyond a certain point raises public debt service or lower output and could bring forward the crisis.

Figure 2. Interest Rate Defense: The Share of Assets Held in Foreign Currency \*



\*Cobb-Douglas preferences  $u(x, y) = \ln(x) + \alpha \ln(y)$ ,  $\delta = 0.7$ ,  $R$  varies from 1.05 to 10.

In our model, an interest rate defense of the domestic currency could be effective in offsetting the increase in precautionary demand for foreign currency associated with an increase in the probability of a crisis. Unfortunately, however, this strategy is most effective when the probability of crisis is low, which is when it is least needed. For example, when the probability of a crisis is 5 percent, a tripling of real interest rates from 10 to 30 percent is sufficient to halve  $f$  from 16 to 8 percent of income. When the crisis probability is high, on the other hand, even massive hikes in real interest rates cannot fully offset the resulting increase in the precautionary demand for foreign currency. If  $\alpha$  rises to 50 percent, raising interest rates to 1000 percent only succeeds in lowering  $f$  from nearly 40 percent to nearly 30 percent of income. Things are nearly impossible for the monetary authority if the convertibility crisis is viewed as nearly inevitable, in which case  $\alpha$  approaches unity. In such cases, even massive increases in real interest rates would be incapable of reversing the currency shift and would likely involve large economic and budgetary costs. Again, while the specific numbers depend on the particular utility function, the general result—the ineffectiveness of an interest rate defense to stem flight from the domestic currency when inconvertibility risk is high—is due to the Inada condition on imported goods. This result is therefore robust to the specification of the utility function.

The main message of this analysis is that it is absolutely crucial for the monetary authorities to deal with the root causes of currency flight by credibly committing to maintain convertibility. Convertibility could be maintained by abandoning the currency peg (as discussed in Section III), in which case the required depreciation of the currency would be dictated by the elasticity of supply of foreign currency with respect to the price. However, if the central bank is not willing to allow the exchange rate to fluctuate enough to elicit the needed supply (or dampen the demand sufficiently) then the remaining options are for the central bank to (a) initiate

adjustment via some combination of interest rates, expenditure restriction, or devaluation; or (b) try to arrange financing via the IMF or other borrowing.

### III. DEVALUATION RISK

In this section we consider devaluation as an alternative to imposing inconvertibility. While in principle devaluation risk could exist in parallel (and be positively or negatively correlated) with convertibility risk, it is of some interest to compare the impact on the demand for domestic assets of a devaluation as an alternative to declaring the currency inconvertible. We show that while devaluation risk also generates a precautionary demand for currency, the strength of the asset substitution induced by devaluation risk is weaker than that induced by inconvertibility risk. Two other main results emerge. First, an interest rate defense is more potent in reversing asset substitution due to devaluation risk than it is in reversing asset substitution due to convertibility risk. Secondly, there is a tradeoff between an interest rate defense and the degree of devaluation.

The model is largely as described in the inconvertibility case. The value of the domestic currency in terms of U.S. dollars at the planning stage is still fixed at unity:  $e_t = 1$ ; domestic agents expect the currency peg to be maintained with some probability:  $e_{t+1} = 1$  with probability  $1 - \mathbf{a}_{t+1}$ , and  $0 < e_{t+1} < 1$  with probability  $\mathbf{a}_{t+1}$ . Agents are competitive in the spot market for foreign exchange, so they can buy or sell unlimited amounts of domestic for foreign currency. They are also able to borrow unlimited amounts at the prevailing interest rate, subject only to their wealth constraint. Households maximize (1) subject to (2) and

$$x_{t+1}^1 \leq \frac{h_t(1+i_t)}{p_{t+1}} - b_{t+1}^1 \quad (12)$$

$$y_{t+1}^1 \leq f_t + b_{t+1}^1 \quad (13)$$

$$x_{t+1}^2 \leq \frac{h_t(1+i_t)}{p_{t+1}} - b_{t+1}^2 \quad (14)$$

$$y_{t+1}^2 \leq f_t + e_{t+1}^2 b_{t+1}^2. \quad (15)$$

To solve this problem, we again proceed in two steps. First, second-stage household decision rules are derived (the  $b$ 's), taking as given the first-stage choices ( $f$  and  $m$ ). Then the first stage decision rules are derived taking into consideration agents' second-stage best responses. In the second stage agents treat  $m_t$ ,  $f_t$  and  $e_{t+1}^i$  as given and select  $b_{t+1}^i$  to maximize

$$u[R_t m_t - b_{t+1}^i, f_t + e_{t+1}^i b_{t+1}^i]. \quad (16)$$

The solution  $b_{t+1}^i = b_{t+1}^i(f_t, e_{t+1}^i)$  satisfies

$$-u_1[x_{t+1}^i, y_{t+1}^i] + e_{t+1}^i \cdot u_2[x_{t+1}^i, y_{t+1}^i] = 0 \quad (17)$$

In the first stage, agents choose  $m_t$  and  $f_t$  to maximize expected utility taking into account their choices of  $b_{t+1}^i$ . Expected utility as a function of  $f_t$  is:

$$(1 - \mathbf{a}_{t+1})u\left[(W - f_t)R_t - b_{t+1}^1(f_t), f_t + b_{t+1}^1(f_t)\right] + \mathbf{a}_{t+1}u\left[(W - f_t)R_t - b_{t+1}^2(f_t), f_t + eb_{t+1}^2(f_t)\right]. \quad (18)$$

In this problem,  $f$  is the safe asset, earning a gross dollar return of 1, while  $m$  is the risky asset, earning a dollar return  $R > 1$  with probability  $1 - \mathbf{a}$  and  $R \cdot e$  with probability  $\mathbf{a}$ . An interior solution exists if foreign currency is not a dominated asset: if  $e \cdot R$  were greater than or equal to 1, agents would borrow arbitrarily large amounts of foreign currency during the planning period, invest the proceeds in domestic assets and enjoy arbitrarily large expected utility:

$eR_t \geq 1 \Rightarrow f_t = -\infty$ ,  $m_t = +\infty$ . On the other hand, demand for domestic assets vanishes if the expected return of  $m$  is equal to the return of the safe asset:  $(1 - \mathbf{a})R_t + \mathbf{a}eR_t = 1 \Rightarrow m_t = 0$ .

Moreover,  $R_t < (1 - \mathbf{a} + \mathbf{a}e)^{-1}$  implies  $m_t < 0$  and  $f_t > W$ . In this last case, agents would sell their entire endowment plus borrowed funds in the foreign exchange market and hoard the foreign exchange proceeds. In light of these considerations, an interior solution in which  $eR_t < 1$  is characterized by

$$(1 - \mathbf{a}_{t+1}) \left[ u_1^1 \cdot \left( -R_t - \frac{\partial b_{t+1}^1}{\partial f_t} \right) + u_2^1 \cdot \left( 1 + \frac{\partial b_{t+1}^1}{\partial f_t} \right) \right] + \mathbf{a}_{t+1} \left[ u_1^2 \cdot \left( -R_t - \frac{\partial b_{t+1}^2}{\partial f_t} \right) + u_2^2 \cdot \left( 1 + e \frac{\partial b_{t+1}^2}{\partial f_t} \right) \right] = 0 \quad (19)$$

Using the FONC for the second-stage problem to simplify this expression leads to the following tangency condition, which can be used to derive the precautionary demand for  $f$ :

$$\frac{1 - \mathbf{a}_{t+1}}{\mathbf{a}_{t+1}} \cdot \frac{u_1^1}{u_1^2} = \frac{e^{-1} - R_t}{R_t - 1}. \quad (20)$$

From equation (20) it is evident that the demand for  $f$  will depend on the difference between the expected returns on foreign and domestic currency assets:

$$\mathbf{a}_{t+1}(e^{-1} - R_t) + (1 - \mathbf{a}_{t+1})(1 - R_t). \quad (21)$$

#### IV. EXAMPLE: CRRA PREFERENCES

Analytical solutions to the portfolio problem are possible for the CRRA class of preferences.

Let  $u(x, y) = \frac{x^{1-s}}{1-s} + \mathbf{d} \frac{y^{1-s}}{1-s}$ ,  $s \neq 1$ ; and  $u(x, y) = \log(x) + \mathbf{d} \log(y)$ ,  $s = 1$ . If  $s > 0$ , these preferences feature risk aversion and lead to smooth asset demands that correspond to the ones described in Sargent (1987), Chapter 8. Consumer demands in state  $i = 1, 2$  are

$$x^i = \frac{1}{e^i + (de^i)^{\frac{1}{s}}} [f + e^i(W - f)R]; \quad y^i = \frac{(de^i)^{1/s}}{e^i + (de^i)^{1/s}} [f + e^i(W - f)R]. \quad (22)$$

These schedules are positive in the  $f$ -interval  $f_0 \leq f \leq f_1$ , where  $f_0 \equiv \frac{-eR}{1-eR}W$ ,  $f_1 \equiv \frac{R}{R-1}W$ .

Assuming  $f_0 \leq f \leq f_1$ , Expected Utility is a strictly concave function of  $f$  if and only if  $s > 0$ .

In this case the unique maximum  $f^*$  is given by

$$f^* = RW \frac{r \left[ \frac{a(e^{-1} - R)}{(1-a)(R-1)} \right]^{\frac{1}{s}} - 1}{r \left[ \frac{a(e^{-1} - R)}{(1-a)(R-1)} \right]^{\frac{1}{s}} (R-1) + [e^{-1} - R]}, \quad (23)$$

where the parameter  $r > 0$  is defined by

$$r = \frac{1 + d^{\frac{1}{s}} e^{\frac{1-s}{s}}}{1 + d^{\frac{1}{s}}}. \quad (24)$$

The value of  $r$  depends on the preference parameter  $s$  and the extent of devaluation  $e$ .  $r = 1$  if  $s = 1$ , while  $r > 1$  if  $s > 1$  and  $e < 1$ .

As can be seen from (23),  $f$  depends on the difference (actually the ratio of) the expected returns on foreign and domestic currency assets  $a(e^{-1} - R) + (1-a)(1 - R)$ .

In the log case  $s = r = 1$ , and (34) reduces to  $f^* \equiv WR \left[ \frac{a}{R-1} - \frac{1-a}{e^{-1} - R} \right]$ .

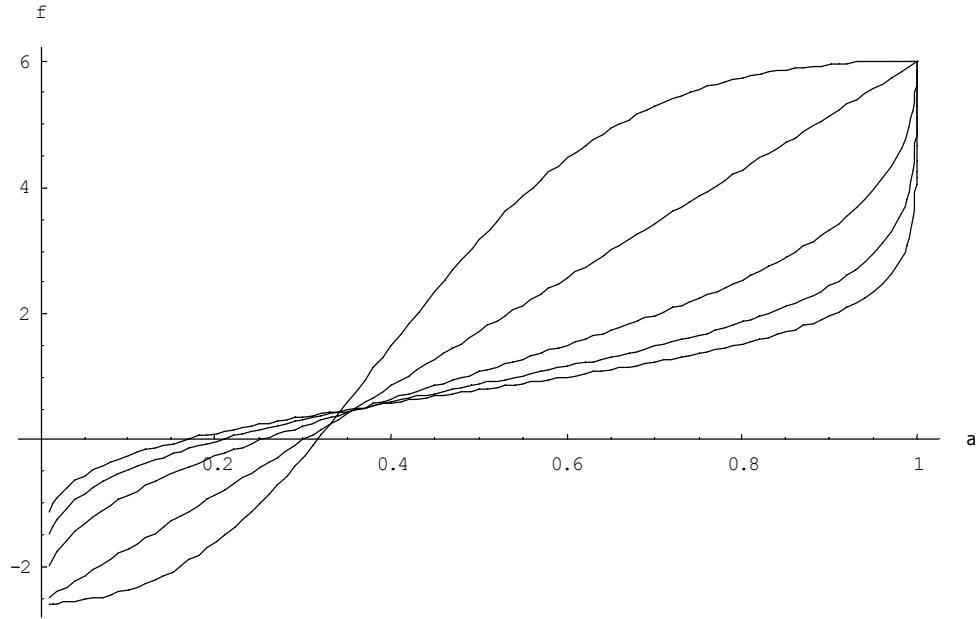
### Comparative Statics

Comparative static exercises were conducted by varying the parameters  $(a, e, R, s)$ : Figure 3 draws a family of demand schedules for foreign currency, one for each value of  $s$ , as functions of the probability of crisis  $a$ . The calculations assume a 40 percent devaluation ( $e=0.6$ ) and a 20 percent domestic real interest rate ( $R=1.2$ ).

Several properties of the  $f$  schedules stand out: (1) As in the inconvertibility case, the share of foreign currency in the optimal portfolio rises with the probability of devaluation. (2) However, unlike the inconvertibility case, where  $f > 0$  for all values of  $a$ , in the devaluation case  $f < 0$  for “low” values of the crisis probability. (3) Higher values of  $s$  reduce the amount of foreign currency desired at each value of  $a$  such that  $f$  is positive (in our example the threshold value of  $a$  is about 35 percent); on the other hand, for values of  $a$  below the threshold, increasing  $s$  raises the amount of foreign currency desired; (4) The  $f$ -schedules become flatter as  $s$  increases

and then rise rapidly to approach  $f_1$  as the probability of crisis approaches unity; (5) An increase in the magnitude of the devaluation (a lower value of  $e$ ) raises  $f$  at each probability and interest rate combination.

Figure 3. Devaluation Risk: Share of Assets Held In Foreign Currency



### Interest Rate Defense

Demand for foreign currency in the presence of devaluation risk is positive if domestic real interest rates are “low”. As was seen in the logarithmic case, demand for foreign currency is positive only if the domestic real interest rate is below a threshold that depends on the parameters  $(e, \mathbf{a}, \mathbf{s})$ . This is also the case for the asset demand schedule (23) generated by the general CRRA utility function in which the denominator of  $f^*$  is positive while the numerator is positive if and only if

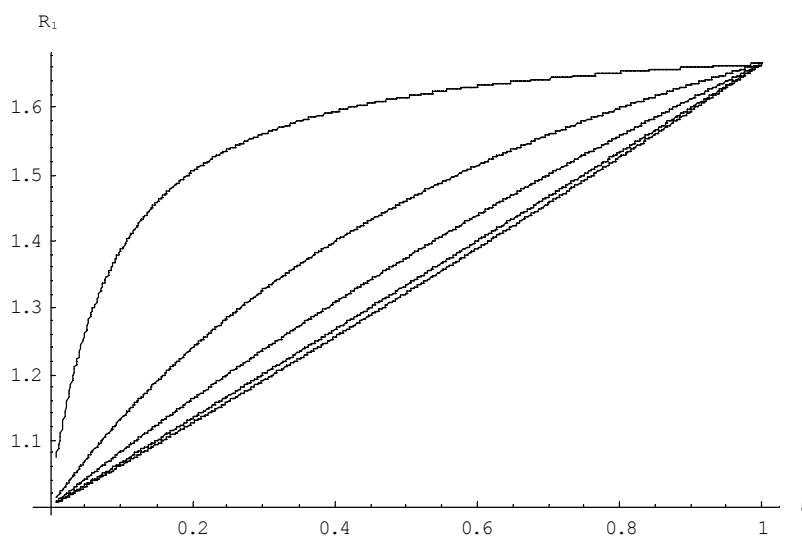
$$\left[ \frac{\mathbf{a}(e^{-1} - R)}{(1 - \mathbf{a})(R - 1)} \right]^{\frac{1}{\mathbf{s}}} > \mathbf{r}^{-\mathbf{s}} \Leftrightarrow$$

$$R < \bar{R} \equiv \frac{\mathbf{a}e^{-1} + (1 - \mathbf{a})\mathbf{r}^{-\mathbf{s}}}{\mathbf{a} + (1 - \mathbf{a})\mathbf{r}^{-\mathbf{s}}} > 1 \quad (25)$$

Equation (25) illustrates an important difference between inconvertibility and devaluation risk. We saw earlier that the asset substitution due to inconvertibility risk could not be successfully countered at high crisis probabilities. By contrast, the asset substitution due to devaluation risk

can be successfully countered by raising domestic real interest rates to the threshold given in (25) which depends on the characteristics of the crisis and of preferences ( $e, \mathbf{a}, \mathbf{s}$ ). If crisis conditions worsen, a higher interest rate is required (at each value of  $\mathbf{s}$ ) to dampen currency flight. In Figure 4 the threshold interest rate is plotted against the crisis probability  $\mathbf{a}$  for various values of  $\mathbf{s}$  (these calculations also assume  $e=0.6$ ). It is also clear that, given  $\mathbf{a}$ , higher values of  $\mathbf{s}$  raise the threshold interest rate.

Figure 4. Threshold Interest Rate Required for Interest Rate Defense



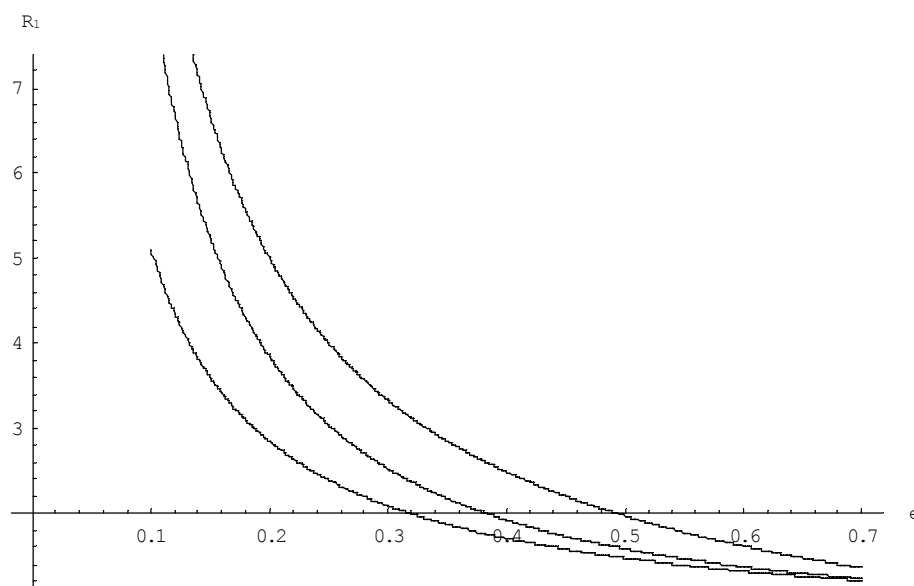
To summarize, in the face of inconvertibility risk demand for foreign currency assets remains positive regardless of the level of real domestic interest rates. This is due to the presence of the Inada condition on good  $y$ : agents would suffer large utility losses if they found themselves in the inconvertibility state with no precautionary foreign currency balances and no access to the spot foreign exchange market. Under devaluation risk, on the other hand, an interest rate defense is much more potent because access to the foreign exchange market is not an issue and, in light of assured access to the market  $t+1$ , agents' calculations are driven entirely by rate-of-return considerations.

### Interest Rate Defense and the Magnitude of Devaluation

Changes in  $e$ , which could be viewed as shifts in the public's expectation of the magnitude of devaluation or even a policy instrument of the authorities, are all-important in deciding how large a hike in interest rates is required to defend the currency. Expression (25) shows that there is a tradeoff between  $e$  and  $\bar{R}$ : a larger devaluation (a lower  $e$ ) requires a higher threshold interest rate to contain pressures to shift domestic assets into dollars (Figure 5). Assuming a 50 percent crisis probability, a 30 percent devaluation requires interest rates between 21 and 27

percent to induce domestic asset holders not to switch into dollars (Given  $e$ , the required interest rate hike is steeper if  $s$  is larger). If the magnitude of the expected devaluation increases to 60 percent, interest rates between 71 percent and 124 percent are required to keep asset substitution at bay. If an 80 percent devaluation is expected, real interest rates of between 185 and 380 percent are required.

Figure 5. The Tradeoff Between Interest Rate Defense and Devaluation



### V. CROSS SECTION DEMAND FOR MONEY<sup>3</sup>

In order to estimate the effect of permanent lack of convertibility on the demand for money, a dummy variable reflecting restrictions on current account foreign exchange transactions was calculated from the IMF's *Annual Report on Exchange Arrangements & Exchange Restrictions* for 135 countries. The velocity of broad money (M2) was then calculated from *International Financial Statistics*, as the logarithm of the ratio of nominal GDP to M2, averaged over the five-year period 1991-1995. In addition, consumer price inflation was averaged over the same five-year period. The following regression equation was specified, where  $V_i$  is velocity in country  $i$ ,  $\pi_i$  is inflation, and  $D_i$  is the dummy for lack of convertibility.

$$V_i = a + b\pi_i + gD_i + u_i \quad (26)$$

<sup>3</sup> This empirical work was ably performed by Sergiy Peredriy.

The results of estimating this equation are as follows, with standard errors below the coefficients.

$$V_i = 0.290 + 0.432 \pi_i + 0.156 D_i, \quad R^2 = 0.74 \quad (27)$$

(0.031) (0.116) (0.045)

According to these results, permanent loss of convertibility in the form of restrictions on current account payments is associated with a 15.6% increase in the velocity of broad money. This equation provides a simple description of the effect of inconvertibility on the quality of money and the demand for money. Since a cross section relationship can be expected to reflect long run behavioral responses, this should be considered an upper bound for the short run response of agents to a surprise risk of inconvertibility associated with a currency crisis.

## VI. DEMAND FOR MONEY ALLOWING FOR INCONVERTIBILITY

### Estimation Strategy

The estimation strategy is to extend the usual equation for money demand to encompass variables that are linked to the likelihood of crisis. The modern approach to the estimation of money demand functions is to use cointegration methods, which take into account the stationarity properties of the variables and which separate out the long-run from the short-run effects. This strategy has the important additional advantage that the effect of crises is most likely to be quantitatively important over the short-run, since a country (at least, a country like Korea, which has enjoyed relatively good macroeconomic performance over the past 30 years) will generally not be in a crisis or close to a crisis. Hence, the estimation strategy needs to focus separately over the short-run if it is to have any chance to separate out the impact of the crisis on money demand.

Equation (15) below focuses on the long-run relationship, as it estimates the link between the *levels* of money demand and the *levels* of the explanatory variables. In this setup, the coefficients give the estimated long-run impact of a change in the explanatory variables on money demand. Equation (13) focuses on the short-run relationship, as it estimates the link between the *changes* of money demand and the *changes* of the explanatory variables. In this setup, the coefficients give the estimated short-run impact. The link between the short-run and the long-run is captured by the inclusion of the “cointegration” variable in the short-run equation, which allows for a feedback between a disequilibrium today on short-run money demand.

The starting point of the empirical investigation is to select the candidate explanatory variables and to subject them to stationarity tests. The variable to be explained is real money demand, and the potential explanatory variables comprise real output, interest rates and inflation (these variables are standard), plus the exchange rate and reserves (the new variables that are thought

to be linked to the probability that domestic currency will become inconvertible).<sup>4 5</sup> Standard augmented Dickey-Fuller (ADF) tests are performed, confirming that the variables are non-stationary, and that they are integrated of order one (their differences are stationary).<sup>6</sup> Hence, a cointegration framework is the statistically appropriate way to estimate the money demand equation. The Annex presents more detail on the variable definitions and on the stationarity tests.

Given the focus of the study on the short-run, a two-step estimation procedure is selected, although the longest possible sample is included in the estimation (monthly data starting in February 1970 and ending in September 2000). The two-step Engle-Granger estimation procedure is followed here, as it allows us to focus on the short-run, and because of its simplicity, allowing for instance for an easy implementation of an instrumental variables estimation technique.<sup>7</sup> One important complication that needs to be dealt with is the possible endogeneity of exchange rates (they would in general be affected by money demand) and the link of reserves to money through the central bank balance sheet identity (the sum of reserves and net domestic assets equals money). To ameliorate the impact of endogeneity on the

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<sup>4</sup> One issue which is not entirely clear from theoretical framework is the monetary aggregate that would be appropriate to use in the empirical tests. Clearly, the fear of inconvertibility could in principle spill over to longer-maturity components of money, if they were liquid enough, which would argue for using a broad monetary aggregate. We ran our regressions both on narrow and broad definitions of money. While both definitions were influenced by the “crisis” variables, the residuals for narrow money were larger, presumably because that equation was not able to account satisfactorily for shifts between components of money. As these shifts are not the main objective of this paper, the analysis is focused on broad money (the sum of money and quasi money, as defined in the IFS database, excluding foreign currency deposits—series BBAA2113, obtained from the Bank of Korea web site, <http://www.bok.or.kr>). The national (Bank of Korea) definition of broad money is consistent with the IFS definition.

<sup>5</sup> Another qualification concerns the interpretation of inconvertibility. If the exchange rate is allowed to float, movements in the exchange rate capture return (akin to the interest rate) rather than inconvertibility considerations. The two versions of the theoretical model earlier presented capture these two different possibilities. The empirical equations here do not distinguish cleanly between the two complementary motivations.

<sup>6</sup> Noting that the fact that inflation is found to be integrated of order one (a result also sometimes found for other countries) implies that the CPI price index is integrated of order two. To keep the estimated equations balanced in terms of the order of integration between the left-hand and the right-hand sides implies that we use inflation in the long-run equation, and the change in inflation in the short-run equation.

<sup>7</sup> It remains for future work to implement more sophisticated estimation methods, such as the method due to Johansen.

estimated coefficients, an instrumental variables estimation technique was used (2SLS), using as instruments for the exchange rate and for reserves lagged exchange rate and lagged reserves.<sup>8</sup>

After some experimentation, the following long-run and short-run specifications were respectively selected:

$$\log\left(\frac{h_t}{p_t}\right) = 7.45 + 0.87 \log(y_t) - 0.01i_t + 0.007p_t + 0.02 \log(e_t) + 0.10 \frac{R_t}{M_t} + u_t \quad (28)$$

$$\begin{aligned} \Delta \log\left(\frac{h_t}{p_t}\right) = & 0.02 + \textit{seasonals} - 0.018u_{t-1} + 0.1\Delta \log\left(\frac{h_{t-1}}{p_{t-1}}\right) + \\ & 0.09\Delta \log(y_t) - 0.0003\Delta i_t - 0.004\Delta p_t - 0.11\Delta \log(e_t) + 0.01\Delta\left(\frac{R_t}{M_t}\right) + v_t \end{aligned} \quad (29)$$

where in addition to symbols already defined,  $y_t$  denotes real output,  $i_t$  the interest rate,  $e_t$  the (average period) exchange rate in won per dollar,  $\frac{R_t}{M_t}$  reserves divided by imports,  $u_t$  the estimated residuals from the long-run equation,  $\Delta$  the difference operator, and *seasonals* a set of monthly seasonal dummies (not shown here to conserve space). In terms of statistical significance, all variables except inflation and the exchange rate are found to be statistically significant at the 5 percent level in the long-run equation, and all variables except interest rates are found to be statistically significant at the 5 percent level in the short-run equation.

The estimated equations perform reasonably well when examined against standard benchmarks, which is striking considering that no allowance was made for possible breaks in the relationship and that different components of money are not modeled separately.<sup>9</sup> The long-run equation has an  $\bar{R}^2$  close to 0.99, and the short-run equation has an  $\bar{R}^2$  of 0.45, with a D.W. statistic of 2.05. Perhaps more impressive is the ability of the estimated equation to produce forecasts of real

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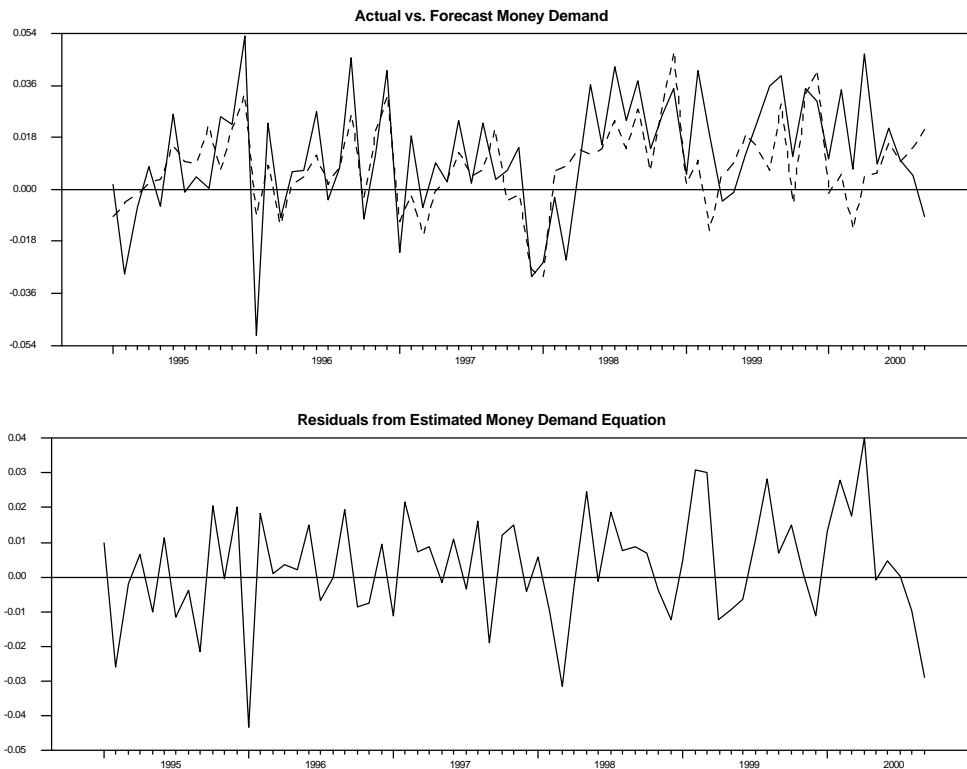
<sup>8</sup> This solution has its costs, mainly that one uses proxies that by necessity are not as informative as the actual variables themselves. To see how significant this loss of information is, the regressions were run both with and without the use of instruments. In the event, the regression coefficients for the exchange rate and reserves were fairly close between the two sets of regressions, and the main results from the counterfactual simulation were not significantly affected (although, as expected, the t-statistics are generally lower for the 2SLS estimates). For concreteness, the results reported in the paper focus on the 2SLS estimated long-run equation and the OLS estimated short-run equation.

<sup>9</sup> E.g., the macroeconomic model used by the Bank of Korea has several different equations separately explaining various components of broad money (Yang Woo Kim *et al* (1998).

money that track actual real money even during December 1997, the month when the crisis hit in earnest (see Figure 6). Finally, the “cointegration” residual  $u_t$  is found using the ADF test to be stationary, which further validates the use of the cointegration framework.<sup>10</sup>

**Figure 6. Money Demand Equation**

*(in changes; logarithmic scale)*



The estimated coefficients are in accord with theoretical expectations, including for the new theoretical framework that incorporates the effect of inconvertibility. Over the long-run, the demand for real money is driven positively by real output and negatively by nominal interest rates, while inflation turns out to be insignificant. Since the inconvertibility framework does not provide clear predictions over the long-run relationship between risk variables (here, proxied by

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<sup>10</sup> The ADF statistic (using the same assumptions as described in the Annex for the other variables) is  $-3.1$ , with a critical value for a test at a 5 percent significance level of  $-2.88$ .

exchange rate and reserves) and real money, we do not attempt to interpret their coefficients in the long-run equation.

Over the short run, real money demand adjusts sluggishly toward its long-run equilibrium. The coefficient on the  $u_{t-1}$  term in equation (8) shows that money demand adjusts by 1.8 percent per month in response to a disequilibrium (about 24 percent of the disequilibrium disappears over the course of one year, and 50 percent after 23 months). From the point of view of this paper, the key implication is that one can focus mainly on the short-run impact multipliers from the exchange rate and reserve variables onto money demand (which is very helpful for the counterfactual simulations), as any long-run disequilibrium effects would be negligibly small at the one-month frequency.

The main finding is that the “crisis” variables have the predicted impact on real money demand. As the short-run equation shows, a depreciation of the exchange rate and a loss of reserves both act to depress the demand for money, and the result is statistically significant. The next section examines the quantitative significance of this effect more closely, and provides a comparison with an alternative estimate based on the cross-sectional approach in the previous section.

## **VII. A COUNTERFACTUAL SIMULATION OF THE IMPACT OF INCONVERTIBILITY AND CRISIS ON THE DEMAND FOR MONEY AND ON RESERVES**

Armed with the estimated money demand equations, it is possible to produce an *ex post* estimate of the impact of the crisis. The money stock fell by about 3 percent in December 1997 and a further 2.5 percent in January 1998. The model predicted a fall of 2.0 and 2.6 percent respectively, a good prediction for what was an unusual period. One implication is that the model did not explain about 1 percentage point of the decline in money demand (this could have happened for various reasons, including the possibility that the proxy variables used to capture the risk of inconvertibility were less than perfectly correlated with the information set available to households at that time). Nevertheless, the key question is how much of what the model explains is due to the “crisis” variables.

A counterfactual simulation reveals that the “crisis” variables (exchange rates and interest rates) helped explain an important part of the reduction in money demand during December 1997. As already mentioned, the key question is how much of the model-predicted fall in demand can be explained by the “crisis” variables. This calculation can be done by (counterfactually) setting the change in the exchange rate and in reserves to zero during December 1997, and recomputing the model forecasts. The difference between these new, conditional, forecasts and the earlier forecasts is the model’s estimated impact of the crisis variables onto money demand. It is found that the model counterfactually forecasts that the demand for money would actually have *increased* (which is reasonable, as this would have been expected in line with the usual seasonal pattern) by 2.0 percent, a difference of 4.0 percentage points versus the model’s unconditional forecast (and 5.0 percentage points versus the growth in the actual money stock during December 1997).

Using real money at end-November 1997 as a base, the counterfactual simulation implies that the inconvertibility channel from the crisis to the demand for money translated into a loss of reserves of between \$6-7.5 billion.<sup>11</sup> The calculation is performed by using the end-of-period November 1997 exchange rate (1,163.8 won per dollar) to calculate broad money in dollars (\$150 billion). Then, one applies the estimated effect (4-5 percent) of the crisis variables on money demand to this number. To put the estimated impact on reserves in perspective, it represented almost 30 percent of foreign exchange reserves (excluding gold) measured as of end-November 1997 (\$24.4 billion).

One important problem discovered after the initial impact of the crisis is that reserves were mismeasured, with truly useable reserves being much lower than those reported publicly at that time. This has two distinct sets of implications for the analysis in this paper. One is whether the mismeasurement would influence the estimated coefficients, and the other is in assessing the importance of the inconvertibility effect on money demand. As to the former, it is in all likelihood not an important problem, because people would be reacting to the data that they had available at the time of the crisis. It therefore makes sense to use data that is closest to what would have been observable at the time. As to the latter implication, however, it is important to have an understanding of the true extent of the deterioration in reserves.

Given that useable reserves, excluding the emergency Fund assistance, declined to practically zero at end-1997, it is concluded that the impact of the possible loss of convertibility on reserves was a significant, but not the most important factor. Estimating the decline in reserves in the last few months of 1997 to be approximately \$24 billion, of which 30 percent was due to the inconvertibility effect on money demand, it is concluded that 70 percent of the loss of reserves was due to other factors. Without performing a detailed analysis of those other factors, it is not possible to conclude with precision how important these other factors might be. This could be a topic of future research endeavoring to isolate further the other channels by which crises influence reserves. However, it is known that much of the outflow from Korea during the crisis was linked to the inability to roll over short term debt, as creditors became unwilling to refinance their exposure. Tentatively, therefore, one can conclude that the short-term debt vulnerability indicator may have been of more significance than the inconvertibility channel.

In terms of the external vulnerability indicators, the analysis in this paper implies that Korean reserves are now at levels that are adequate to cover plausible shocks to money demand. Assuming that the shock from the 1997 crisis to money demand was at the high end (representing a kind of natural “stress test”), reserves should cover at least 5 percent of broad money. In fact, the reserve level of \$96 billion at end-2000 covers 11 percent. This should be contrasted with the inadequate level at end-1997, when useable reserves, inclusive of the emergency support from the Fund, were at 2.2 percent of broad money.

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<sup>11</sup> The effect could be somewhat larger if the value of real money then were translated into today’s money (between end-November 1997 and the end of the sample period in September 2000 the CPI index rose by about 10 percent).

The alternative approach using cross-sectional data finds a larger impact on money demand and reserves from a potential lack of convertibility. The main finding in the preceding section was that long run money demand was 15.6 percent lower for countries with permanently inconvertible currencies. This effect is between 3.1-3.9 times larger than what was estimated using the Korean-specific time series money demand function. The finding in the cross section analysis would have meant that reserves would have fallen to zero (or to below zero) on the strength of the inconvertibility channel alone.

It seems unlikely that the higher estimate for the impact on reserves is applicable in the majority of the cases. Given the method of estimation, it seems that only full and permanent inconvertibility would result in a reduction in money demand of the full 15.6 percent (so that amount can be viewed, in general, as an overestimate). More likely, households ascribe some probability to the possibility of inconvertibility that is less than 100 percent. Even given the severity of the crisis in Korea, it is not plausible that households came to expect full (not to mention permanent) inconvertibility. Under these circumstances, the estimate that money demand fell by between 4-5 percentage points seems a more reasonable one.

### **VIII. CONCLUDING REMARKS**

This paper presented a theoretical and empirical analysis of the impact of anticipated and actual inconvertibility on domestic money demand and reserves. Using a simple model featuring multiple currencies we showed that the prospect of inconvertibility gives rise to a precautionary motive for foreign exchange. Whereas in normal times the domestic currency is freely convertible at a fixed exchange rate, the threat of loss of convertibility raises demand for foreign currency assets at the expense of the demand for domestic money. Naturally, an increase in the probability that the currency will become inconvertible—as would occur during a crisis—leads to a decline in demand for domestic real currency balances. The central bank could respond by raising domestic interest rates, which would raise domestic money demand through a direct substitution effect. However, an interest rate defense of the currency is offset by budgetary and other economic costs. Instead, convertibility could be maintained by abandoning the currency peg, in which case the required depreciation of the currency would be dictated by the elasticity of supply of foreign currency with respect to the price. However, if the central bank is not willing to allow the exchange rate to fluctuate enough to elicit the needed supply (or dampen the demand sufficiently) then the only options left are initiating adjustment via some combination of interest rates, expenditure restriction, or devaluation or trying to arrange financing via the IMF or other borrowing.

In the empirical part of the paper, we use time series cointegration techniques to estimate the impact of the prospect of inconvertibility on money demand during several recent currency crises on Korea. Our preliminary estimates of the effect of the foreign crisis on money demand and reserves for Korea show that the impact of inconvertibility is significant. The loss of reserves attributed to inconvertibility was of the order of \$7 billion, or about one-third the reserves available at the time. This effect is separate from the reserve loss attributed to the non-rolling over of short term capital driven by the over-leveraging of Korean corporations.

This study offers some insights for policies designed to contain external vulnerability and particularly for reserves. Some of the conclusions, however, are tentative pending further research.

- At times of crisis, money demand will generally fall as households perceive the risk of domestic currency becoming inconvertible as rising. This intuitive theoretical prediction is born out by the empirical analysis carried out for the case of Korea in this paper.
- The negative impact of a crisis on money demand, and the attendant negative spillovers to international reserve holdings, can be significant but, for countries whose problems are perceived to be temporary, and that have a reasonable amount of initial reserves, manageable. Korea's current reserves, when compared with broad money, would more than cover a shock to money demand equivalent to that sustained during the 1997 crisis.
- From the usual indicators of external vulnerability, the analysis in this paper finds that the reserves-to-broad money ratio is an important indicator. At least for the case of Korea, one can tentatively say that the reserves-to-short term debt (measured by residual maturity) variable may have been quantitatively more important. Further theoretical and empirical work needs to be done, however, to conclusively rank the importance of alternative indicators of vulnerability.

## DATA

The data used are all collected from the Fund's International Financial Statistics (IFS) database, and are monthly series for the period February 1970 to September 2000.

The key definitions used include:

- Real broad money is the end-of-period sum of money and quasi money (minus foreign currency deposits) deflated by the CPI index, seasonally unadjusted.
- Real output is the industrial production index, seasonally adjusted.
- The interest rate is the monthly yield on deposits longer than 1 year.
- Reserves is end-of-period foreign exchange reserves excluding gold measured in dollars, divided by imports also measured in dollars.
- The exchange rate is the monthly average market rate, measured in won per US dollar. For converting money stocks to dollars the end-of-period rate is used instead.

To extend the sample back to 1970, it was necessary to use the aforementioned interest rate (some alternative series started much later). This entailed some loss of information as there were periods over which that rate did not change (there were periods over which it apparently was changed only infrequently). There was no definition of quasi money in the IFS database that was seasonally adjusted, and there was no definition of industrial production that was seasonally unadjusted. To account for this discrepancy, the short-run regression equation includes seasonal dummies.

The tests of variable stationarity yielded the following results:

**Table 1. ADF Tests<sup>1</sup>**

Variable	Levels	Difference	Conclusion
Broad money divided by CPI	0.24	-3.57	The variable is integrated of order 1
Output	-1.62	-4.18	The variable is integrated of order 1
Interest Rate	-2.73	-4.86	The variable is integrated of order 1
Inflation Rate	-2.68	-7.34	The variable is integrated of order 1
Exchange Rate	-1.65	-5.13	The variable is integrated of order 1
Reserves-to-Imports Ratio	-0.75	-3.87	The variable is integrated of order 1

<sup>1</sup>Performed assuming 12 lags and including a constant and no trend, and assuming that the constant equals zero under the null hypothesis.

Note: The 5 percent critical value for the ADF test is  $-2.88$  (James Hamilton **Time Series Econometrics**, 1994, Table B.6, case 2).

One notable result, often found also in other country cases, is that inflation is integrated of order one (hence, the difference of inflation is stationary). This implies that the CPI is integrated of order two. This explains why in the long run equation it is inflation that is used (to keep the orders of integration commensurate between the left-hand and the right-hand sides of the equation) and why it is that in the short-run equation it is the difference in inflation that is used

### CALCULATING THE OPTIMAL PORTFOLIO WITH DEVALUATION RISK

This Annex derives the optimal foreign-currency portfolio in the presence of devaluation risk for the CRRA class of preferences. Consumer demands in state  $i = 1, 2$  are

$$x^i = \frac{1}{e^i + (\mathbf{d}e^i)^{\frac{1}{\mathbf{s}}}} [f + e^i(W - f)R]; \quad y^i = \frac{(\mathbf{d}e^i)^{\frac{1}{\mathbf{s}}}}{e^i + (\mathbf{d}e^i)^{\frac{1}{\mathbf{s}}}} [f + e^i(W - f)R]. \quad (30)$$

These schedules are positive in the  $f$ -interval  $f_0 \leq f \leq f_1$ , where  $f_0 \equiv \frac{-eR}{1-eR}W$ ,  $f_1 \equiv \frac{R}{R-1}W$ .

Assuming  $f_0 \leq f \leq f_1$ , indirect utility in state  $i$ ,  $U^i$ , is proportional to real wealth in that state raised to the power  $1 - \mathbf{s}$ :

$$U^i = \frac{1}{1-\mathbf{s}} \cdot \frac{1}{e^i} \cdot [e^i + (\mathbf{d}e^i)^{\frac{1}{\mathbf{s}}}]^{\mathbf{s}} \cdot [f + e^i(W - f)R]^{1-\mathbf{s}}. \quad (1.31)$$

Expected utility and its first and second derivatives may be written as function of  $f$  as follows:

$$EU = (1-\mathbf{a}) \frac{1}{1-\mathbf{s}} [1 + \mathbf{d}^{\frac{1}{\mathbf{s}}}]^{\mathbf{s}} [f + R(W - f)]^{1-\mathbf{s}} + \mathbf{a} \frac{1}{1-\mathbf{s}} [e + (\mathbf{d}e)^{\frac{1}{\mathbf{s}}}]^{\mathbf{s}} e^{-1} [f + eR(W - f)]^{1-\mathbf{s}};$$

$$EU'(f) = (1-\mathbf{a}) [1 + \mathbf{d}^{\frac{1}{\mathbf{s}}}]^{\mathbf{s}} [f + R(W - f)]^{-\mathbf{s}} [1 - R] \\ + \mathbf{a} [e + (\mathbf{d}e)^{\frac{1}{\mathbf{s}}}]^{\mathbf{s}} e^{-1} [f + eR(W - f)]^{-\mathbf{s}} [1 - eR];$$

and

$$EU''(f) = (1-\mathbf{a}) [1 + \mathbf{d}^{\frac{1}{\mathbf{s}}}]^{\mathbf{s}} (-\mathbf{s}) [f + R(W - f)]^{-\mathbf{s}-1} [1 - R]^2 \\ + \mathbf{a} [e + (\mathbf{d}e)^{\frac{1}{\mathbf{s}}}]^{\mathbf{s}} (-\mathbf{s}) [f + eR(W - f)]^{-\mathbf{s}-1} e^{-1} [1 - eR]^2.$$

$EU''(f) < 0$  if and only if  $\mathbf{s} > 0$ . In this case there is a unique maximum  $f^*$  given by

$$(1-\mathbf{a}) [1 + \mathbf{d}^{\frac{1}{\mathbf{s}}}]^{\mathbf{s}} [f + R(W - f)]^{-\mathbf{s}} [R - 1] = \mathbf{a} [e + (\mathbf{d}e)^{\frac{1}{\mathbf{s}}}]^{\mathbf{s}} [f + eR(W - f)]^{-\mathbf{s}} e^{-1} [1 - eR].$$

Rearranging terms and simplifying yields the following:

$$\left[ \frac{f + R(W - f)}{f + eR(W - f)} \right]^{-\mathbf{s}} = \frac{\mathbf{a}}{1-\mathbf{a}} \left[ \frac{e + (\mathbf{d}e)^{\frac{1}{\mathbf{s}}}}{1 + \mathbf{d}^{\frac{1}{\mathbf{s}}}} \right]^{\mathbf{s}} \frac{e^{-1} - R}{R - 1} \Leftrightarrow \\ \frac{f + R(W - f)}{f + eR(W - f)} = \left( \frac{\mathbf{a}}{1-\mathbf{a}} \left[ \frac{e + (\mathbf{d}e)^{\frac{1}{\mathbf{s}}}}{1 + \mathbf{d}^{\frac{1}{\mathbf{s}}}} \right]^{\mathbf{s}} \frac{e^{-1} - R}{R - 1} \right)^{-1/\mathbf{s}}. \quad (32)$$

Define the parameter  $r > 0$  by

$$r = \frac{1 + d^{\frac{1}{s}} e^{\frac{1-s}{s}}}{1 + d^{\frac{1}{s}}}. \quad (33)$$

The value of  $r$  depends on the preference parameter  $s$  and the exchange rate in the devaluation state  $e$ .  $r = 1$  if  $s = 1$ , while  $r > 1$  if  $s > 1$  and  $e < 1$ . From equations (32)-(33), the optimal portfolio is

$$f^* = WR \frac{r \left[ \frac{a(e^{-1} - R)}{(1-a)(R-1)} \right]^{\frac{1}{s}} - 1}{r \left[ \frac{a(e^{-1} - R)}{(1-a)(R-1)} \right]^{\frac{1}{s}} (R-1) + [e^{-1} - R]}. \quad (34)$$

As can be seen from (34),  $f$  depends on the difference (actually the ratio) of the expected returns on foreign and domestic currency assets  $a(e^{-1} - R) + (1-a)(1-R)$ .

In the log case  $s = r = 1$ , and (34) reduces to  $f^* \equiv WR \left[ \frac{a}{R-1} - \frac{1-a}{e^{-1} - R} \right]$ .

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