Consumption Smoothing and the

Time Profile of Self-Enforcing Agreements

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I. Introduction

This paper analyzes the time structure of a self-enforcing agreements in a repeated relationships between a principal who can commit to contract terms and an agent who cannot commit. A common feature of such agreements is the use of a promise of a higher payoff in the future to deter deviation from the agreement in the current period. This "back-loading" of payoffs is a feature of agreements where one party has an incentive to deviate from the agreement, and is used to relax the incentive (i.e. no deviation) constraint of that party. An efficient agreement will thus have the property that the payoff to the constrained rises over time, while that of the unconstrained party declines over time.

The literature contains a number of examples of agreements with this feature. Lazear (1981) considers the case of a labor contracts when workers may shirk on the job, and shows the efficient contract is one in which the payment to the worker is less than the marginal product in the early period of the contract and exceeds the marginal contract in the later stages. Thomas and Worrall (1994) model the taxation of earnings of a multinational firm by a host country in situations where the host country cannot commit to its tax rate. The efficient tax profile in this case rises over time, because the promise of higher tax revenues in the future will deter the host country from reneging and taxing away all of the current period profits. Bond and Park (2002) examine a trade agreement between asymmetric countries, and show that an efficient agreement will also have this back-loading feature when the no deviation constraint is binding for only one of the two countries.

Recently, Ray (2002) has analyzed a quite general principal-agent model. in which the principal can commit to contract terms but the agent cannot, in order to derive a general result on the time structure of self-enforcing agreements. He proves that an efficient agreement will reach the payoff that is most favorable to the agent in finite time, which establishes that a form of back-loading is a pervasive feature of self-enforcing agreements.[1] While the back-loading of payoffs to the agent is a useful tool for relaxing the no deviation constraint, it has an efficiency cost in situations where one of the parties to the agreement has a finite intertemporal elasticity of substitution. The optimal agreement will thus represent a trade-off between the desire to relax the incentive constraint and the desire to smooth the payoffs to the party with a finite intertemporal elasticity of substitution.

The fact that the payoff that is most favorable to the agent is reached in finite time suggests a sense in which the consumption smoothing motive is of a second order of importance relative to the desire to relax incentive constraints. For example, consider a model in which the payoff most favorable to the agent is reached at time T, and the agent's payoff remains at that point forever. The payoff to the agent is thus rising prior to time T but fully smoothed after T, indicating that the benefit to additional payoff smoothing in the neighborhood of T is dominated by the desire to relax the agent's incentive constraint.

Ray's result is obtained under the assumption that the discount factor is sufficiently low that none of the first best agreements are sustainable. This paper relaxes this assumption by examining the case in which the discount parameter is such that only one first best agreement, the one yielding the most favorable payoff to the agent, is supportable. It is shown that for a broad

[1]In the most general model considered by Ray, the payoff of the agent does not necessarily rise montonically over time because agreements can have a periodic structure. In a more restricted model wherepayoffs are linear in the transfers, he shows that the payoff to the agent msut rise over time and reach the best stationary program that is most favorable to the agent in finite time.

class of utility functions, the payoff to the agent will rise over time but will not reach the payoff that is most favorable to the agent in finite time. For these payoff functions, the payoff smoothing incentive is sufficiently strong that the payoff most favorable to the agent is only reached asymptotically.

In the case where the discount parameter is low (i.e. no first best agreements can be supported), attempts to postpone reaching the agreement most favorable to the agent has a first order efficiency cost, because the effort is below the efficient level for all contracts. However, when the discount paraemeter is sufficiently high that efficient effort levels can be supported, the cost of postponing the agreement most favorable to the agent approaches zero in the neighborhood of that agreement. These results thus provide insights about the trade-off between production efficiency and payoff smoothing in self-enforcing agreements.

II. A Principal-Agent Model

To illustrate the role of consumption smoothing in characterizing the time profile of an optimal self-enforcing agreement, we consider a simple principal agent model where the agent has a payoff smoothing motive. Specifically, the payoff to the agent in a period is $u(w - e)$, where w is the wage paid by the principal, e is the effort exerted by the agent, and u is a strictly concave function. The output produced by the agent is $h(e)$, with the payoff to the principal in a period being $h(e) - w$. The functions h and u are assumed to have derivatives that are strictly bounded above and below.

An agreement between the principal and agent will be an infinite sequence of wage and effort levels, $A = \{(w_0, e_0), (w_1, e_1),.....\}$. Letting $\beta \in (0,1)$ be the discount rate and $A_t = \{(w_t, e_t), (w_{t+1}, e_{t+1}), ...\}$ the sequence of contract terms from t onward under agreement A, the average discounted payoff to the respective parties under A from time t onward is

$$v_t(A_t) = (1-\beta)\left[\sum_{s=0}^{\infty} u(w_{t+s} - e_{t+s})\,\beta^s\right]$$

$$\pi_t(A_t) = (1-\beta)\left[\sum_{s=0}^{\infty} \left(h(e_{t+s}) - w_{t+s}\right)\beta^s\right] \tag{1}$$

As a benchmark, we begin with the case where effort is observable and contractible. The locus of efficient contracts can be derived by choosing *e* and *w* to maximize the average payoff to the principal, given an average payoff of *v* to the agent. Any such contract will specify the efficient level $e_t = e^*$ for all t, where $h'(e^*) = 1$. Since the agent's payoff is strictly concave, the wage rate in an efficient contract will also be a constant, $w^*(v) = u^{-1}(v) + e^*$. The first best frontier can be obtained by substituting the efficient wage and effort levels into the principal's payoff, $\pi^*(v) = h(e^*) - u^{-1}(v) - e^*$. This frontier will be strictly concave in *v* due to the strict concavity of *u*.

In addition to being efficient, a contract must also provide the respective parties a payoff at least as large as can be earned if no contract is reached. Assuming that the outside option of the principal is 0, the maximum payoff attainable by the agent, v^*_{max}, is defined by $\pi^*(v^*_{max}) = 0$. Letting $\bar{v}$ be the outside option of the agent, the efficient and individually rational payoffs will be described by the payoff frontier $\pi^*(v)$ for $v \in [\bar{v}, v^*_{max}]$.. The contract in this case would be the outcome of a bargaining game between the principal and the agent over the set of efficient and individually rational agreements.

A. Self-Enforcing Agreements

Now consider the case in which the effort of the agent is not contractible and can only be observed by the principal with a one period lag. In order for an agreement to be self-enforcing for the agent, it must provide a payoff level at each time t that is no worse than can be choosing the optimal deviation from the agreement and pursuing the outside option $\bar{v}$. Since effort is costly to the agent, the optimal deviation from the agreement is to accept the wage and choose $e_t = 0$. The incentive compatibility constraint at time t will then require that

$$Z(w_t, e_t, v_{t+1}) \equiv (1-\beta)\big(u(w_t - e_t) - u(w_t)\big) + \beta\big(v_{t+1} - \bar{v}\big) \geq 0 \tag{2}$$

Equation (2) can be used to illustrate how the contract terms can be used to relax the incentive constraint for an agreement. A lower effort level and a promise of greater payoff to the agent in the future will both make deviation less attractive by raising the payoff to the agent under the agreement. With strictly concave utility, an increase in w_t will also make deviation less attractive because it raises the payoff under the agreement by more than the payoff in the event of deviation. If the utility function is linear, then the wage will have no effect on the incentive to deviate.

In order for an efficient agreement yielding per period payoffs $\{v, \pi^*(v)\}$ to satisfy (2), the discount parameter must be sufficiently high that it satisfies

$$\beta \geq \beta^c(v) \equiv \frac{u(w^*(v)) - u(w^*(v) - e^*)}{u(w^*(v)) - \bar{v}} \tag{3}$$

It can be seen from (3) that the critical discount parameter required to sustain an agreement, $\beta^c(v)$, is decreasing in the payoff received by the agents. A higher wage paid to the agent in all periods relaxes the incentive constraint (2) in two ways: it raises the future value of the agreement, v_{t+1}, and it reduces the current period gain from deviation when u is strictly concave. Note also any agreement yielding the agent a payoff exceeding the reservation value will be self-enforcing for β sufficiently large, since $\beta^c(v) < 1$ for $v > \bar{v}$.

Our objective is to characterize the efficient frontier for agreements that must satisfy the self-enforcing constraint (2), which will be denoted $\Omega(v)$. The function $\Omega(v)$ is obtained by finding the agreement A that maximizes π_0, subject to the requirement that (2) hold for each t and that $v_0 \geq v$. In particular, we are interested in the dynamics of agreements in the case where a subset of the agreements on the first best frontier are incentive compatible. Therefore, we will focus on the case in which $\beta = \beta^c(v^*_{max})$, so that $\Omega(v^*_{max}) = \pi^*(v^*_{max}) = 0$. For $v < v^*_{max}$, the incentive constraint will be binding and $\Omega(v) < \pi^*(v)$.

This problem can equivalently be stated as the dynamic programming problem:

$$\begin{aligned} \Omega(v_t) = \max_{w_t, e_t, v_{t+1}} \; & (1-\beta)(h(e_t) - w_t) + \beta\,\Omega(v_{t+1}) \\ & + \lambda_t Z(w_t, e_t, v_{t+1}) + \mu_t\big((1-\beta)u(w_t - e_t) + \beta v_{t+2} - v_t\big) \end{aligned} \quad \textbf{(4)}$$

where λ_t is the Lagrange multiplier associated with the no deviation constraint (2) and μ_t is the multiplier associated with the constraint that the agent's payoff be at least v_t.

It is shown in the appendix that the necessary conditions for choice of e_t and w_t imply

$$h'(e_t) = 1 + \lambda_t u'(w_t) \quad \textbf{(5)}$$

The effort level in an efficient self-enforcing contract will be below the first best level in any period in which $\lambda_t > 0$. A reduction in the effort level is desirable because it lowers the incentive of the agent to deviate from the agreement. The necessary conditions for choice of the wage rate at times t and t + 1 yields the following condition

$$u'(w_t - e_t) \geq \frac{1 + \lambda_t u'(w_t)}{\lambda_t + \mu_t} \geq \frac{1 - \lambda_{t+1}\left(u'(w_t - e_t) - u'(w_t)\right)}{\lambda_t + \mu_t} \geq u'(w_{t+1} - e_{t+1}) \qquad \textbf{(6)}$$

Since u is assumed strictly concave, (5) ensures that $w_t - e_t \leq w_{t+1} - e_{t+1}$. The net payoff to the agent will be strictly increasing between any two periods in which the incentive constraint is binding in at least one of the periods. This also means that the continuation value of the agent is non-decreasing over time, since the fact that $w_t - e_t$ is a non-decreasing sequence implies $v_t \leq v_{t+1}$. An increase in the wage rate at time t + 1 will relax the incentive constraint at both time t and t + 1, whereas an increase in the wage at time t will only relax the incentive constraint at time t. Therefore, the payoff to the agent in the efficient contract will be backloaded in order to induce effort on the part of the agent.

Finally, the necessary conditions for choice of e_t and e_{t+1} yield

$$\frac{h'(e_t)}{u'(w_t - e_t)} = \lambda_t + \mu_t \leq \lambda_t + \mu_t + \lambda_{t+1} = \frac{h'(e_{t+1})}{u'(w_{t+1} - e_{t+1})} \qquad \textbf{(7)}$$

Condition (7) shows that the ratio of the marginal benefit of effort to the marginal cost of effort will be non-decreasing in the efficient contract, and will be strictly increasing if $\lambda_{t+1} > 0$. The inequality in (7) arises from the fact that an increase in effort at t + 1 will tighten the incentive constraint at both t and t + 1, which suggests a sense in which effort is more costly at later stages in the contract. Note that this does not necessarily mean that the level of effort will fall over time in an efficient contract, because (6) guarantees that marginal cost of effort to the agent is non - increasing over time. Thus, (7) could be consistent with either a rising or falling effort level over time.

The implications of conditions (5) - (7) for self-enforcing contracts are illustrated in Figure 1, which shows iso-payoff contours for the principal and agent in a period t with $\lambda_t > 0$. With a binding incentive constraint at time t, the iso-profit contour of the firm will be steeper than the indifference curve of the agent at e_t from (5). The AA contour is the locus of values of w and e at which $h'(e)/u'(w-e) = h'(e_t)/u'(w_t - e_t)$, and has slope $(u'h'' + h'u'')/(h'u'') > 1$. The inequalities in (7) indicate that $\{w_{t+1}, e_{t+1}\}$ must lie to the left of the AA locus if $\lambda_{t+1} > 0$. The inequalities in (6) establish that $w_{t+1} - e_{t+1} > w_t - e_t$.

The results in equations (5) - (7) will hold for all choices of the discount parameter. The role of the discount parameter arises through its impact on whether the deviation constraint (2) will bind. It is shown in the appendix that if $\beta = \beta^c(v^*_{max})$, then $Z(w_t, e_t, v_{t+1}) = 0$ for all t and $\lambda_t > 0$ if $v_t < v^*_{max}$. When the only first best contract that is sustainable is the one that is most favorable to the agent, the effort level will be below the first best level for all promised payoffs to the agents that are less than v^*_{max}. Combining this result with (5) - (7) yields the following result:

Proposition 1: Suppose that $\beta = \beta^c(v^*_{max})$ and an agreement between the principal and the agent specifies an average payoff $v < v^*_{max}$ to the agent at time t. The efficient contract will have the properties

a. $e_t < e^*$ b. $v_{t+1} > v_t$ c. $Z(w_t, e_t, v_{t+1}) = 0$

d. $\frac{h'(e_t)}{u'(w_t - e_t)} \leq \frac{h'(e_{t+1})}{u'(w_{t+1} - e_{t+1})}$

The implication of Proposition 1 for the behavior of the continuation values in the contract can be illustrated in Figure 2, which shows the first best frontier, $\pi^*(v)$, and the incentive-constrained frontier, $\Omega(v)$. For $v_t < v^*_{max}$, a first best agreement is not incentive compatible and $\Omega(v) < \pi^*(v)$. Part (b) shows that the contract becomes increasingly favorable to the agent over time, so the continuation values in the contract will be moving rightward over time along the frontier. Since the payoff to the agent is an increasing sequence on the compact set $[\bar{v}, v^*_{max}]$, its limit must be v^*_{max}.

One question of interest is how quickly the payoff to the agent will reach v^*_{max}. The speed of the approach to v^*_{max} involves a tension between the desire to back-load the payoffs to relax the incentive constraint and the desire to smooth the consumption path of the agent. This can be seen most clearly by noting that if the payoff to the agent is linear in w - e, so that the consumption smoothing motive is absent, the optimal policy will involve a jump to v^*_{max} in the second period of the agreement. In this case there is no cost to the agent of raising the degree of back-loading, so the problem of obtaining efficient effort can be achieved in one period.[2]

Ray (2002) has shown that if the discount parameter is sufficiently low that none of the first best paths are attainable, then the agreement will reach the payoff on the frontier that is most favorable to the agent in finite time. The case considered here differs in that we allow for one path on the first best frontier that is self-enforcing, and examine whether the payoff that is most

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[2]The approach to v^*_{max} can be slowed down by imposing a liquidity constraint, such as requiring $w_t - e_t \geq -A$ for all t.

favorable to the agent will be reached in finite time in this case. A contradiction of the assumption that v^*_{max} will be reached in finite time can be established if it can be shown that shown that there exists a feasible adjustment in $\{w_{T-1}, e_{T-1}, w_T, e_T\}$ that leaves agent welfare unaffected and increases the payoff to the principal.

To construct such an adjustment, we first note that for the initial agreement to be optimal, it must satisfy $Z(w_{T-1}, e_{T-1}, v_T) = 0$ and $Z(w_T, e_T, v^*_{max}) = 0$ when evaluated at $w_T = w^*_{max}$ and $e_T = e^*$ by Proposition 1c. This condition requires that the initial value of w_{T-1} and e_{T-1} satisfy

$$u(w_{T-1}) - u(w_{T-1} - e_{T-1}) = u(w^*_{max}) - u(w^*_{max} - e^*) \qquad \textbf{(8)}$$

In order to maintain incentive compatibility at time T (and holding $v_{T+1} = v^*_{max}$), the adjustment in time T agreement terms must satisfy $dw_T = \dfrac{u'(w^*_{max} - e^*)}{u'(w^*_{max} - e^*) - u'(w^*_{max})} de_T$. The impact of this adjustment on the time T payoffs is

$$\frac{d\pi_T}{de_T} = -\frac{dv_T/de_T}{u'(w^*_{max} - e^*)} = \frac{-(1-\beta)u'(w^*_{max})}{u'(w^*_{max} - e^*) - u'(w^*_{max})} < 0 \qquad \textbf{(9)}$$

Since $\pi_T = 0$ at $v_T = v^*_{max}$, the only feasible adjustments will result in $dw_T < de_T < 0$. This adjustment raises the payoff to the principal and reduces the payment to the agent at time T.

In order for v_{T-1} to remain constant given $de_T < 0$, the change in agreement terms at time T -1 must satisfy $\beta dv_T - u'(w_{T-1} - e_{T-1})(dw_{T-1} - de_{T-1})(1-\beta) = 0$. To maintain incentive compatibility at time T-1 with $dv_{T-1} = 0$ requires $dw_{T-1} = 0$ from (2). Combining these results, we have the requirement that

$$de_{T-1} = \frac{\beta dv_T}{(1-\beta)u'(w_{T-1} - e_{T-1})} \tag{10}$$

A lowering of the return to the agent at time T will require a lowering of effort by the agent at T-1 to maintain incentive compatibility. This adjustment will have the benefit of smoothing consumption to the agent, but it comes at the cost of a first order reduction in output at time T-1 because of the reduce effort by the agent (i.e. $h'(e_{T-1}) > 1$). The effect of this adjustment on the time T-1 payoff to the principal is $d\pi_{T-1} = (1-\beta)h'(e_{T-1})de_{T-1} + \beta d\pi_T$. Substituting into this result for de_{T-1} from (10) and for $d\pi_T$ from the first equality in (9) yields

$$d\pi_{T-1} = \beta\left[\frac{h'(e_{T-1})}{u'(w_{T-1} - e_{T-1})} - \frac{1}{u'(w^*_{max} - e^*)}\right]dv_T \tag{11}$$

If the bracketed expression in (11) is negative when evaluated at the $\{w_{T-1}, e_{T-1}\}$ consistent with (8), then a continuation value for the agent of $v_T = v^*_{max}$ is not optimal.

The following examples illustrate that (11) will in general not be satisfied

[Examples to be provided]

C. Discussion

It is useful to compare the results of the previous section with the case in which $\beta < \beta^c(v^*_{max})$ as considered by Ray. In this case no first best agreements will be incentive compatible, so the incentive constrained payoff frontier will lie strictly inside the first best frontier illustrated in Figure 2. Let v_{max} denote the point on the payoff frontier that is most favorable to the agent. It is straightforward to show that the results of Proposition 1 will extend to this case, so that the no deviation constraint will bind for all t and v . LettingThe no deviation constraint The dynamics of the agreement are essentially the same as that in Proposition 1 except that the efficient effort level is not incentive compatible for any w and thus will never be achieved as part of the agreement.

Letting v_{max} denote the point on the payoff frontier that is most favorable to the agent and e_{max}, assume that there exists a time T such that $v_t = v_{max}$ for t ≥ T. As in the previous case, we can construct an adjustment in $\{w_{T-1}, e_{T-1}, w_T, e_T\}$ that holds v_{T-1} constant to see if the payoff of the principal can be raised. In order to maintain incentive compatibility at time T, the adjustment in the agreement at T must satisfy $dw_T = \dfrac{u'(w_{max} - e_{max})}{u'(w_{max} - e_{max}) - u'(w_{max})} de_T$. The impact of this

adjustment on the time T payoffs is

$$\frac{d\pi_T}{de_T} = \left[(h'(e_{max}) - 1)\,(1 - \beta) - \frac{dv_T/de_T}{u'(w^*_{max} - e^*)} \right] \qquad \textbf{(12)}$$

Comparing with (7), it can be seen that a change in e_T has a first order effect on output because the effort level at T is below the efficient level.

The adjustment in the T-1 contract terms will correspond to those in the previous case, with $dw_{T-1} = 0$ and de_{T-1} given by (8). Substituting these results and (10) into $d\pi_{T-1} = (1 - \beta)h'(e_{T-1})de_{T-1} + \beta d\pi_T$ yields

$$d\pi_{T-1} = \beta\left[\left(\frac{h'(e_{T-1})}{u'(w_{T-1} - e_{T-1})} - \frac{1}{u'(w_{max} - e_{max})}\right)\frac{dv_T}{de_T} + (1 - \beta)(h'(e_{max}) - 1)\right]de_T \qquad \textbf{(13)}$$

The payoff to the principal can be improved by reducing the effort level at time T if the bracketed expression is negative. Note however that the bracketed expression must be postiive for e_T in the neighborhood of e_{max} because h/(e_{max}) > 1. Therefore, it will be optimal to jump to the payoff that is most favorable to the agent for v_T in the neighborhood of v_{max} In this case, the payoff most favorable to the agent will be reached in finite time as shown by Ray.

C. Separable Payoffs for the Agent

Further insight about the role of the form of the utility function and the dynamics of the efficient contract can be obtained by considering the case in which the agent's utility is separable in income and effort, $u = f(w) - g(e)$, where u is an increasing and strictly concave function and g is increasing and strictly convex.

The first best frontier with separable utility is derived by maximizing the payoff to the principal, subject to $f(w) - g(e) \geq v$. This optimization yields first best wage payments, $w^*(v)$, and first effort levels, $e^*(v)$, that equate the marginal rate of substitution between effort and wage of the principal and agent, $h'(e_t) = g'(e_t)/f'(w_t)$. It is straightforward to show that the first best wage is increasing in v and the first best effort level is decreasing in v.[3] As in the previous case, we can define v^*_{max} as the payoff to the agent in the first best agreement that yields zero profit to the principal, $h(e^*(v^*_{max})) - w^*(v^*_{max}) = 0$.

[3] Forming the Lagrangean problem for the first best frontier and solving yields $de^*/dv = g'\mu^* f'/\Delta < 0$ and $dw^*/dv = -(h'' - \mu^* g'')/\Delta > 0$, where $\mu^* > 0$ is the Lagrange multiplier associated with the agent's payoff and $\Delta \equiv -[(h''-\mu^* g'')(f')^2 + \mu^* f'(g')^2] > 0$.

With separable utility, the no deviation constraint of the agent simplifies to

$$Z(e_t, v_{t+1}) = -(1-\beta)\, g(e_t) + \beta\,(v_{t+1} - \bar{v}) \qquad \textbf{(14)}$$

Changes in the wage rate have no impact on the gains from deviation in this case, so that the only ways to relax the agent's no deviation constraint are to lower the current effort level or to raise the promised future payoff under the contract.

The necessary conditions for the dynamic programming problem for this case can used to generate the analogous conditions that govern the choice of effort levels and wages in an efficient contract.

$$h'(e_t) = \frac{g'(e_t)}{f'(w_t)} + \lambda_t g'(e_t)$$

$$f'(w_t) = \frac{1}{\mu_t} \geq \frac{1}{\mu_t + \lambda_t} = f'(w_{t+1}) \qquad \textbf{(15)}$$

$$\frac{h'(e_t)}{g(e_t)} = \lambda_t + \mu_t \leq \lambda_t + \mu_t + \lambda_{t+1} = \frac{h'(e_{t+1})}{g'(e_{t+1})}$$

The first condition shows that the effort level will be the first best level (given the current wage) in any period where the no deviation constraint is binding. The second condition establishes that the wage rate will be non-decreasing over time in the optimal contract, and will be strictly increasing if $\lambda_t > 0$. The final condition indicates that the ratio of the marginal benefit of effort to the marginal cost be non-increasing over time in the optimal contract, and will be strictly increasing if $\lambda_{t+1} > 0$. With separability, the marginal cost of effort is independent of the wage rate, so this condition will require that the effort level must be non-increasing. Since the wage rate must be non-decreasing and the effort level non-increasing, it must also be true that $v_{t+1} \geq v_t$ in this case. Furthermore, $v_{t+1} > v_t$ if $\lambda_t > 0$.

Suppose we consider the case in which $\beta = \beta^c(v^*_{max})$, so that the only sustainable first best payoff is the one that is most favorable to the agent.[4] It is shown in the Appendix that in this case the no deviation constraint (14) must bind with $\lambda_t > 0$ for any t at which $v_t < v^*_{max}$. Therefore, we must have $v_t < v_{t+1}$. Note however that if the no deviation constraint binds at each t, then the fact that the payoff to the agent is non-decreasing means that the effort level of the agent must be non-decreasing as well. The only way that this result can be consistent with (15), which requires that the effort level of the agent be non-increasing over time, is for $e_t = e^*(v^*_{max})$ for all t. The effort level will jump immediately to the first best level in the efficient contract that yields the highest payoff to the agent. The wage rate will be at the level yielding the highest payoff to the agent in all periods after the initial (t = 0) period. This yields the following characterization:

Proposition 2: If the agent's payoff function takes the separable form f(w) - g(e) and $\beta = \beta^c(v^*_{max})$, then the optimal self-enforcing contract has the properties

a) $e_t = e^*(v^*_{max})$ for all t

b) $v_t = v^*_{max}$ and $w_t = w^*(v^*_{max})$ for all $t > 0$

In this contract the effort level of the agent will be below the efficient level associated with the initial period payoff to the agent, because $v_0 < v^*_{max}$ and $e^*(v)$ is a decreasing function.

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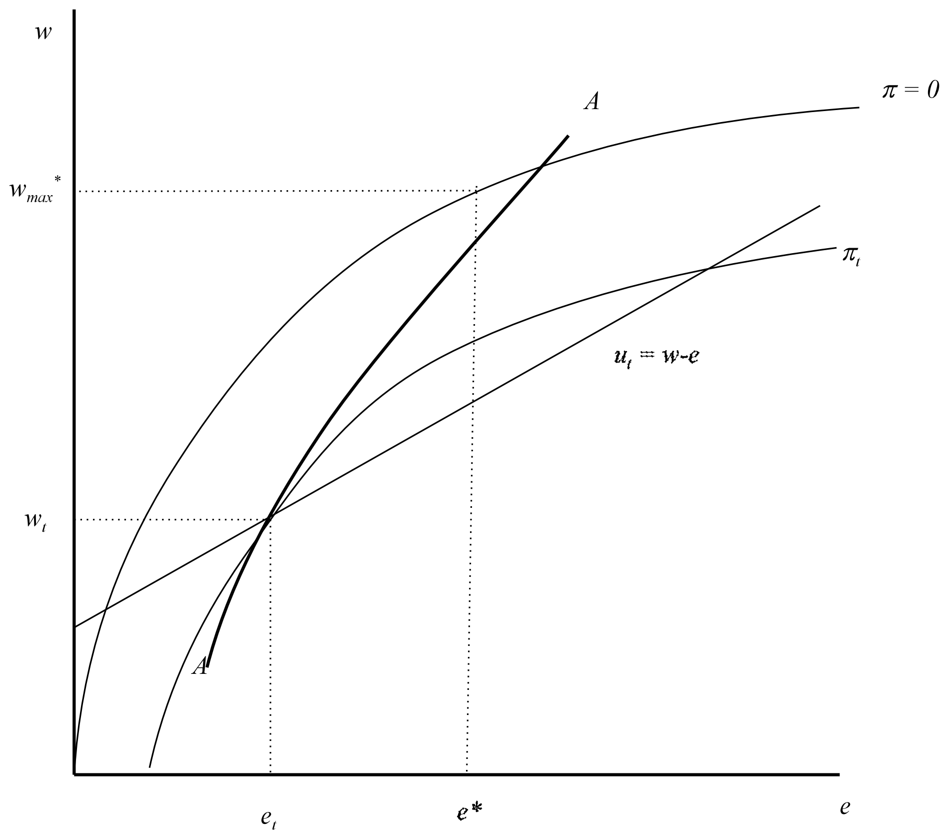
[4]Using (3) for this form of the agent's payoff function, $d\beta^c/dv = -[\beta f' dw^*/dv - g' de^*/dv]/(u(w^*)-\bar{v}) < 0$. Thus, the easiest first best agreement to sustain is the one that yields the highest payoff to the agent, as in the previous example.



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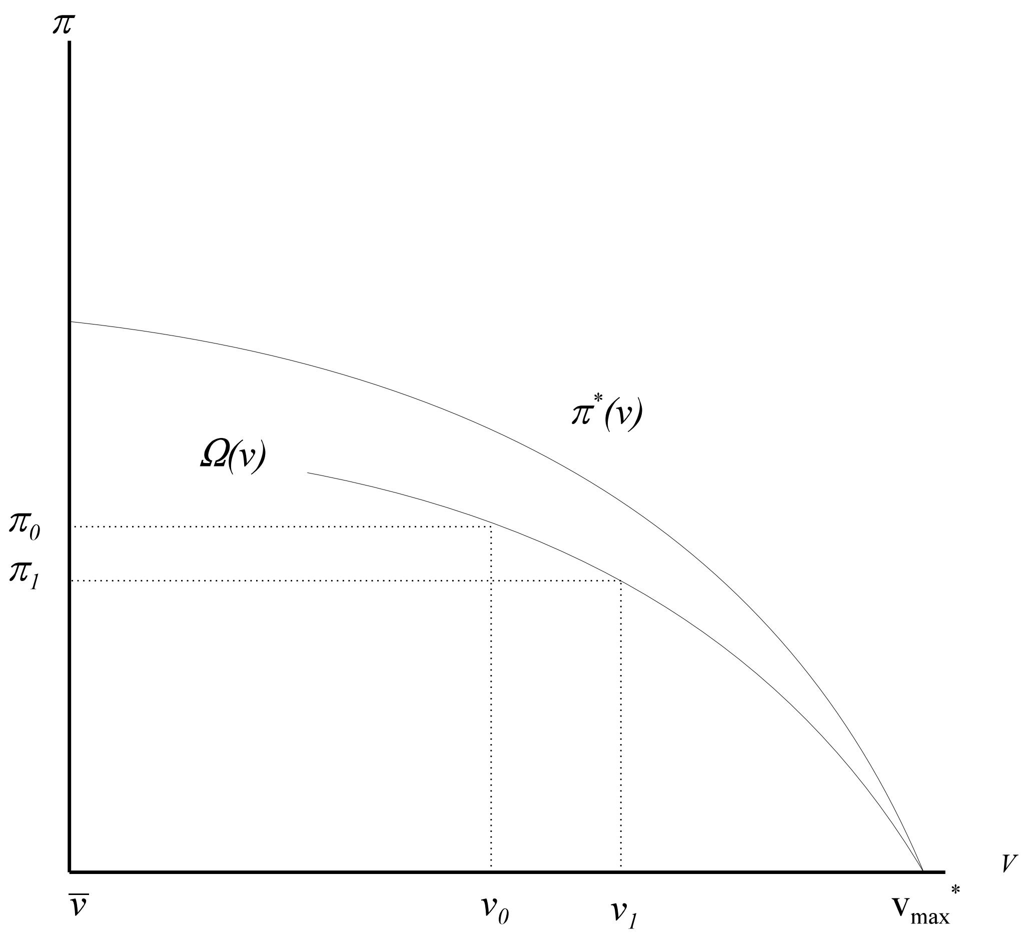

Figure 2

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Appendix

Proof of Proposition 1: Substitution of the value function into itself yields the following maximization problem:

$$\Omega(v_t) = \max_{w_t, w_{t+1}, e_t, e_{t+1}, v_{t+2}} (1-\beta)(h(e_t) - w_t) + \beta(1-\beta)(h(e_{t+1}) - w_{t+1}) + \beta^2 \Omega(v_{t+2})$$
$$+\lambda_t Z(w_t, e_t, (1-\beta)\, u(w_{t+1} - e_{t+1}) + \beta v_{t+2}) + \beta\lambda_{t+1} Z(w_{t+1}, e_{t+1}, v_{t+2}) \quad \text{(A.1)}$$
$$+ \mu_t\Big((1-\beta)u(w_t - e_t) + \beta(1-\beta)u(w_{t+1} - e_{t+1}) + \beta^2 v_{t+2} - v_t\Big)$$

where λ_t is the Lagrange multiplier associated with the incentive constraint at time t and μ_t is the multiplier associated with the payoff constraint at time t.

The necessary conditions for choice of the wage rates and effort levels yield:

$$\begin{aligned}
&(i) \quad w_t: && (\lambda_t + \mu_t)\, u'(w_t - e_t) = 1 + \lambda_t u'(w_t) \\
&(ii) \quad w_{t+1}: && (\lambda_t + \mu_t)\, u'(w_{t+1} - e_{t+1}) + \lambda_{t+1}\Big(u'(w_{t+1} - e_{t+1}) - u'(w_{t+1})\Big) = 1 \\
&(iii) \quad e_t: && h'(e_t) = (\lambda_t + \mu_t)\, u'(w_t - e_t) \\
&(iv) \quad e_{t+1}: && h'(e_{t+1}) = (\lambda_t + \mu_t + \lambda_{t+1})\, u'(w_{t+1} - e_{t+1})
\end{aligned} \quad \text{(A.2)}$$

Equation (5) follows from (i) and (iii), (6) follows from (i) and (ii), and (7) follows from (iii) and (iv).

Proof of Proposition 1: To prove the Proposition, we first establish two results in the case where $\beta = \beta^c(v^*_{max})$.

(i) $e_t = e^*$ iff $v_t = v^*_{max}$

$v_t = v^*_{max} \Rightarrow e_t = e^*$ follows from the fact that the first best solution is sustainable for $v = v^*_{max}$

with our choice of discount parameter. To establish the reverse implication, suppose that $e_t = e^*$ and $v_t < v^*_{max}$. The no deviation constraint (2) will then require that $Z(w_t, e^*, v_{t+1}) \geq 0$, with $Z(w^*_{max}, e^*, v^*_{max}) = 0$. Since $v_t > v^*_{max}$ is infeasible and Z is decreasing in w_t it follows that (2) can be satisfied only if $w_t \geq w_{max}$. However, $w_t - e^* \geq w_{max} - e^*$ combined with the result from (6) that w_t - e_t is non-decreasing yields $v_t \geq v^*_{max}$, which is a contradiction.

(ii) $Z(w_t, e_t, v_{t+1}) = 0$ for all t in an efficient agreement, with $\lambda_t > 0$ for $v_t < v^*_{max}$.

If $v_t = v^*_{max}$, then $Z(w^*_{max}, e^*, v^*_{max}) = 0$ and the first best contract is sustainable from the assumption that $\beta = \beta^c(v^*_{max})$. Next suppose that $v_t < v^*_{max}$ and $Z(w_t, e^*, v_{t+1}) > 0$. The necessary conditions for choice of e_t and w_t would require $e_t = e^*$ with $\lambda_t = 0$, but this contradicts result (i). Therefore, we must have $\lambda_t > 0$.

Parts a and c of the Proposition follows from result (ii) and the necessary condition (5). Substituting $\lambda_t > 0$ in (5) and (6) establishes parts a and b of Proposition 1. Part (c) follows directly from (ii). Part d is a result of (7) and the fact that $\lambda_t > (=)\ 0$ if $v_t < (=)\ v^*_{max}$.||

Separable Utility

The necessary conditions for the optimization problem (A.1) in this case simplify to

$$\begin{aligned} &(i) \quad w_t: && \mu_t f'(w_t) = 1 \\ &(ii) \quad w_{t+1}: && (\lambda_t + \mu_t) f'(w_{t+1}) = 1 \\ &(iii) \quad e_t: && h'(e_t) = (\lambda_t + \mu_t)\, g'(e_t) \\ &(iv) \quad e_{t+1}: && h'(e_{t+1}) = (\lambda_t + \mu_t + \lambda_{t+1})\, g'(e_{t+1}) \end{aligned} \tag{A.3}$$

These conditions can then be combined to yield ().

Proof of Proposition 2: Suppose that $\lambda_t = 0$ for $v_t < v^*_{max}$, which from (A.3i) and (A.3iii) requires that $w_t = w^*(v_t)$ and $e_t = e^*(v_t)$. The no deviation constraint will then require that

$$g(e^*(v_t)) \leq \beta(v_{t+1} - \bar{v})/(1 - \beta) \tag{A.4}$$

The discount parameter has been chosen so that

$$g(e^*(v^*_{max})) = \beta(v^*_{max} - \bar{v})/(1 - \beta). \tag{A.5}$$

Since $e^*(v)$ is decreasing in v and g is an increasing function, $g(e^*(v)) > g(e^*(v^*_{max}))$. The promised payoff to the agent must satisfy and $v_{t+1} \leq v^*_{max}$, so combining these two results, it is clear that (A.4) cannot hold for a discount parameter that satisfies (A.5). Therefore, we have a contradiction and we must have $\lambda_t > 0$ for $v_t < v^*_{max}$.