

## **Risk Rationing and Activity Choice in Moral Hazard Constrained Credit Markets**

### **Abstract**

This paper explores the interaction of asymmetric information and risk preferences in the performance of the credit market. A model of contract design in the presence of moral hazard is developed in which a risk neutral lender offers contracts to a risk averse entrepreneur who owns a potentially profitable investment project. The model gives rise to the potential for quantity rationing and an additional form of non-price rationing called *risk rationing*. Both quantity and risk rationed farmers would seek a credit contract in a first best - or symmetric information - world but end up without a contract when information is asymmetric. Whereas quantity rationed farmers are "involuntarily" excluded because they are denied a contract, risk rationed farmers "voluntarily" withdraw from the market because the limited set of contracts available under asymmetric information imply too much risk. The paper points to the need for incorporating this additional form of non-price rationing in empirical work on credit markets.

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# 1 Introduction

In a competitive world of symmetric information and costless enforcement, credit contracts could be written conditional on borrower type and behavior. Borrowers would then have access to loans under any interest rate-collateral combination that would yield lenders a zero expected profit. However, as a large literature has shown, information asymmetries and enforcement costs that make such conditional contracting infeasible will restrict the set of available contracts, for example eliminating as incentive incompatible high interest rate, low collateral contracts.<sup>1</sup>

As has been emphasized in the literature, this contraction of contract space can result in quantity rationing in which potential borrowers who lack the wealth to fully collateralize loans are involuntarily excluded from the credit market and thus prevented from undertaking high return investments.

The principal contribution of this paper is to show that this same contraction of contract space can result in another form of non-price rationing that we label “risk rationing.” Risk rationing occurs when lenders, constrained by asymmetric information, shift sufficient contractual risk to the borrower so that the borrower voluntarily withdraws from the credit market even when she or he has the necessary collateral wealth to qualify for a loan contract.<sup>2</sup> The private and social economic costs of risk rationing are similar to those of more conventional quantity rationing. Like quantity-rationed individuals, the risk rationed individuals will retreat to lower expected return activities. Moreover, under mild assumptions about the nature of risk aversion, risk rationing will, like quantity rationing, predominately affect lower wealth individuals and firms.

The distinction between quantity and risk rationing highlights the fact that a firm’s activity choice depends both on the feasibility of activities and the preference ranking over available activities. The theory literature has focussed primarily on the former. The latter is important,

<sup>1</sup> Summaries of this literature include: Hillier and Ibrahimo (1993), Jaffee and Stiglitz (1990), and Besley (1995).

<sup>2</sup> Like an interest rate increase, an increase in contractual risk will also help equilibrate the loan market by reducing demand and is thus another form of non-price rationing.

however, because the fact that a high return project is made feasible through credit access does not necessarily imply that it will be chosen. When insurance markets are missing, the ordering of preferences depends on the nature of risk preferences. If firms or individuals are risk neutral, the ordering is based solely on expected return, so that attention can be focussed solely on feasibility. If, instead, farmers are risk averse then the higher moments of the distribution of returns of each activity also enter into the preference ordering. The willingness of risk averse agents to trade expected return for a reduction in risk creates the potential that the credit market can fulfill the dual role of overcoming liquidity constraints and serving as a partial substitute for missing insurance markets. The degree to which the credit market fulfills these dual roles and for whom will be shown to depend critically upon the nature of the information environment and farmers' risk preferences.

The distinction between quantity and risk rationing is also important from the perspective of empirical work. The econometrics of credit rationing have struggled with the fundamental problem of distinguishing individuals with zero loan demand given the cost of capital from quantity rationed individuals. To solve this problem, some studies have resorted to the econometrics of unobserved regime switching (*e.g.*, Bell *et al.*, 1997). Others have employed ancillary sample information to distinguish individuals with positive demand from those without. For example, Kochar (1997) uses loan application as a signal of positive loan demand. While the first approach is subject to statistical limitations, use of loan application as a necessary signal of positive demand is highly problematic in the presence of quantity rationing, as Mushinski (1999) argues.<sup>3</sup>

In an effort to obtain more reliable indicators of positive loan demand, several recent enterprise surveys have added questions inquiring about reasons why firms do not apply for loans. Not only do such questions reveal significant numbers of discouraged firms that do not apply for loans because they know they will not get them (what Mushinski calls preemptively-rationed), they also

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<sup>3</sup> If loan application is costly and individuals know that quantity rationing is a possibility, they will only apply for loans for which they have expect to have a reasonable probability of success.

Table 1: Risk and Quantity Rationed Firms

	<i>Peru</i>			<i>Guatemala</i>		
	<i>Non-Price Rationed</i> Quantity	<i>Risk</i>	<i>Price Rationed</i>	<i>Non-Price Rationed</i> Quantity	<i>Risk</i>	<i>Price Rationed</i>
<i>%</i>	36.7	17.2	46.1	31.1	13.7	55.2
<i>Wealth (\$)</i>	13,336	9,396	23,771	21,510	6,024	38,972
<i>Input (\$/ha)</i>	451	454	868	NA	NA	NA
<i>Income (\$/ha)</i>	653	593	919	NA	NA	NA

reveal that significant numbers of non-applicant firms that reported loan needs were discouraged from applying for loans by fear of losing collateral in the event of default. The modeling reported in this paper in fact grew out of an effort to make theoretical sense of the empirical report of fear-driven non-borrowers.

Table 1 reports data on risk-rationing from two recent surveys, one of agricultural enterprises in Peru (Boucher 2000) and the other of rural farm and non-farm enterprises in Guatemala (Barham *et al.*, 1996). As can be seen, the percentage of risk rationed enterprises is between 14 and 17% and these constitute some 30% of all non-price rationed firms. Failure to account for risk rationed households as non-price rationed would clearly have a major effect on the analysis of the significance of the efficiency of credit markets under asymmetric information.

Table 1 also displays some additional information on risk-rationed versus other types of firms. Given the relative homogeneity of agricultural producers in the Peru survey, we can glean a meaningful idea of the activity choice of risk rationed producers by looking at their use of inputs as well as net-income produced per-unit land. As can be seen, the risk rationed firms appear similar to the quantity rationed, with both inputs and income some 30 to 50% below that of price rationed producers. In both the Peru and Guatemala datasets, we see that risk-rationed and quantity rationed producers are similarly located in the lower deciles of the wealth distribution.

We turn now to more formally develop a theory of risk rationing in moral hazard-constrained credit markets. The next section lays out a simple model of entrepreneurial behavior under uncertainty and describes the structure of credit contracts. Section 3 explores the implications of

asymmetric information - which takes the form of the lender's inability to observe the borrower's effort level - on the existence and terms of the optimal credit contract and demonstrates the potential for quantity and risk rationing. Section 4 shows the dependence of activity choice and investment levels on endowment distribution by examining the comparative statics of contract terms and entrepreneurial wealth. Section 5 concludes.

## 2 Key Assumptions and Model Structure

Every agent  $i$  has an initial wealth endowment,  $W_i$  and must choose between an entrepreneurial or a wage labor activity. Wage labor pays a certain income of  $\omega$ . Gross entrepreneurial income,  $X$ , is generated according to the following stochastic process:

$$X = x(k, j(e)) \begin{cases} x_s \text{ if } j = s \text{ and } k \geq \underline{K} \\ x_f \text{ if } j = f \text{ and } k \geq \underline{K} \\ 0 \text{ if } k < \underline{K} \end{cases} \quad (1)$$

where  $k$  is capital input sunk into the entrepreneurial project,  $j$  is the state of nature (realized after capital is committed), and  $e$  is the agent's level of effort. There are two states of nature: success ( $j = s$ ) and failure ( $j = f$ ), with income under success greater than under failure:  $x_s > x_f$ . The agent influences the probability of success through choice of effort—which can be either high ( $e = H$ ) or low ( $e = L$ ). Let  $\phi^e$  be the probability of success under effort level  $e$ . The probability of success is increasing in effort so that  $\phi^H > \phi^L$ . The entrepreneurial project has a fixed capital requirement per-hectare,  $\underline{K}$ . If the capital requirement is not met, output is zero independent of the realized state of nature. Additional capital beyond  $\underline{K}$  has zero marginal productivity. Under the fixed capital requirement, agents with insufficient wealth endowments ( $W < \underline{K}$ ) are unable to self-finance production.<sup>4</sup>

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<sup>4</sup> The assumption of a fixed project size is made for analytical simplicity. The general conclusions are not altered under the less restrictive assumption of a diminishing returns, variable input technology. This continuous input size case is discussed in chapter four of Boucher (2000).

## 2.1 Autarchic Self-Finance

Agents have access to a riskless savings activity, and wealth not invested in the productive activity yields a gross return of  $1 + r^a$ . Net entrepreneurial income under autarchic self-finance would thus be:

$$y_j^a = x(k, j) - (1 + r^a)k \quad (2)$$

Note of course that liability is unlimited under self-finance. Letting  $\bar{y}_H^a = \phi^H x_s + (1 - \phi^H)x_f - (1 + r^a)k$  denote expected entrepreneurial income under high effort and  $\bar{y}_L^a$  denote the same thing under low effort, we make the following assumptions:

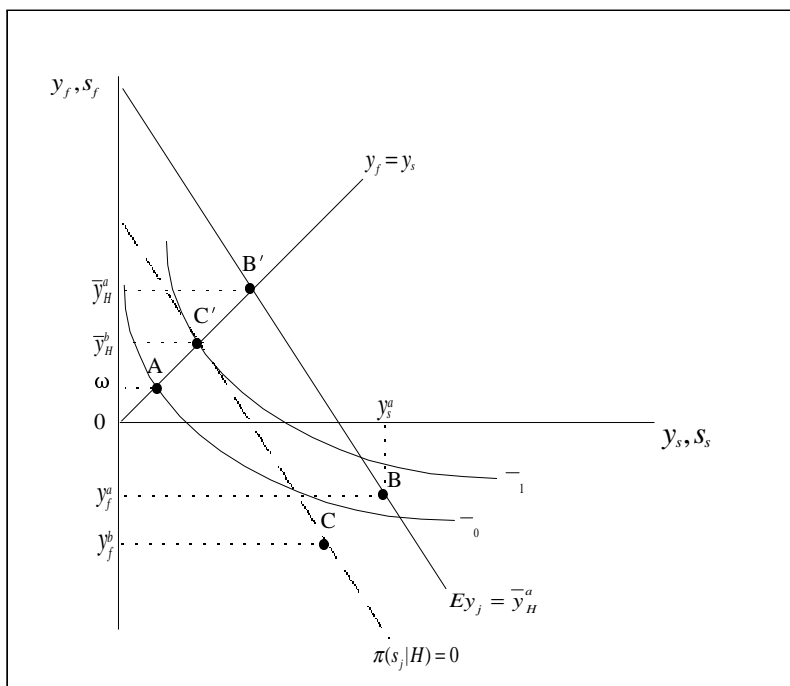
$$\bar{y}_H^a > \omega > 0 > \bar{y}_L^a \quad (A1)$$

$$y_s^a > 0 > y_f^a \quad (A2)$$

>From A1, expected net income exceeds the wage rate if high effort is applied but is negative under low effort. By making realized net income negative under failure, A2 makes liability a non-trivial issue.

Using these assumptions we can graphically portray these payoffs in state contingent income space as shown in Figure 1. Point *A* denotes the certain payoff received under wage labor. More generally, any payoff point along the 45-degree, or full insurance, line yields consumption which is independent of the state of nature. The net income pair  $(y_s^a, y_f^a)$  shown as point *B* represents the outcome under autarchic self-finance of the entrepreneurial project. Note that we can define a locus of state-contingent payoffs that would yield an expected value identical to the entrepreneurial project under high effort. Specifically, this locus is given by state-contingent incomes,  $(y_s, y_f)$ , such that  $\phi^H y_s + (1 - \phi^H)y_f = \bar{y}_H^a$ , or  $y_f = \frac{\bar{y}_H^a}{1 - \phi^H} - \left(\frac{\phi^H}{1 - \phi^H}\right) y_s$ . The downward sloping line that passes through point *B* in Figure 1 illustrates this locus. Note that if  $\bar{y}_H^a = \omega$ , this locus would pass through point *A*. Under assumption A1, it of course lies to the northeast of point *A* along the 45 degree line.

Figure 1: Indifference curves in income/payoff space



## 2.2 Credit Contracts

The capital costs of the entrepreneurial project can also be potentially funded by loans. We assume that competitive lenders must earn a rate of return  $r^b$  on loans in order to cover the cost of capital. We further assume that  $r^b > r^a$  with the difference between the two rates reflecting lenders' costs of intermediation.

One possible loan contract would be the unlimited liability (or fully collateralized) contract that offers the borrower the payoff of the form:

$$y_j^b = x(k, j) - (1 + r^b)k. \quad (3)$$

This unlimited liability contract guarantees the lender a gross return of  $k(1 + r^b)$  and an expected profit - denoted by  $\pi$  - of zero. With  $r^b > r^a$ , the payoffs to the borrower under this contract will appear in Figure 1 as point  $C$ , strictly to the southwest of the self-finance payoff pair denoted as point  $B$ . Following the notational convention established above, let  $\bar{y}_H^b = \phi^H x_s + (1 - \phi^H)x_f -$

$(1 + r^b)k$  denote expected entrepreneurial income conditional on high effort. To keep the problem meaningful, we will further assume that:

$$\bar{y}_H^b > \omega > 0 > \bar{y}_L^b \quad (\text{A3})$$

$$y_s^b > 0 > y_f^b \quad (\text{A4})$$

Assumption A3 implies that, conditional on high effort, the project yields a return greater than the wage rate even at the higher capital cost,  $r^b$ . Note that A2 implies that the unlimited liability loan contract requires a transfer of collateral wealth to the lender under failure, thereby demonstrating the infeasibility of high liability contracts for low wealth entrepreneurs.

In addition to the unlimited liability loan contract, there is a full suite of loan contracts that (conditional on high effort) will yield expected borrower and lender income identical to that given by the contract represented at point  $C$ . While it is conventional to express a loan contract in terms of the nominal interest rate and collateral requirement, alternative contracts can also be expressed in terms of the state-contingent payoffs they offer to the borrower (and lender). As with autarchic self-finance, we can define the locus of state contingent payoff pairs,  $(s_s, s_f)$ , that conditional on high effort yield an expected entrepreneurial income equal to  $\bar{y}_H^b$ . Illustrated in Figure 1 as the dotted, downward sloping line, this locus is given by  $s_f = \frac{\bar{y}_H^b}{1-\phi^H} - \left(\frac{\phi^H}{1-\phi^H}\right) s_s$  and is parallel to the entrepreneur's iso-income lines under self-finance.

Note that points along that locus that lie to the northwest of the unlimited liability loan contract  $C$  represent higher nominal interest rates (*i.e.*, lower payoff to the borrower under success) and lower liability or collateral (*i.e.*, a higher payoff to the borrower under failure). Point  $C'$  along that locus represents the full-insurance loan contract in which the borrower's income is independent of project success or failure.

Finally, note that, conditional on high effort, every payoff pair along the dotted locus yields the lender an expected gross return of  $k(1 + r^b)$  and an expected rate of return on money lent of exactly  $r^b$ . Therefore call this the lender's zero profit locus or participation constraint. Contracts

must be to the southwest of the participation constraint in order for the lender to at least break even.

### 2.3 Agent Preferences

Before turning to the analysis of the credit market, we need to specify preferences which guide agent choice and behavior. Assume that the agent's well being depends both on his state contingent consumption and effort level:

$$U(C_j) - B(e) \quad (4)$$

where  $U()$  is a standard utility function,  $C_j$  is state contingent consumption and the additively separable component,  $B(e)$ , represents the *disutility* of effort. To keep matters simple, we scale effort disutility such that under low effort  $B(L) = -B$ , and under high effort  $B(H) = 0$ .

Expected utility for the agent under the different activity and finance choices becomes:

$$V(C_j, e|W, c_0) = \begin{cases} EU(c_0 + W + y_j^k|e = H) \text{ under high effort} \\ EU(c_0 + W + y_j^k|e = L) + B \text{ under low effort} \\ U(c_0 + W + \omega|e = H) \text{ under wage labor} \end{cases} \quad (5)$$

where  $c_0$  is a consumption minimum that is assumed to be exogeneously guaranteed to the agent, and the superscript  $k = a, b$  is used to denote the finance regime.<sup>5</sup>

For later analysis it is useful to note that the slope of the agent's indifference curves in state-contingent income space conditional on high effort is:

$$y'_f(y_s) \equiv \frac{\partial y_f}{\partial y_s} \Big|_{\bar{V}} = - \left( \frac{\phi^H}{1 - \phi^H} \right) \left( \frac{U'(C_s)}{U'(C_f)} \right) \quad (6)$$

where  $\bar{V}$  is the expected utility level corresponding to the indifference contour. Indifference curves are downward sloping and convex to the origin. The marginal rate of substitution (MRS) between income under success and income under failure is decreasing, reflecting the desire of risk averse agents to smooth consumption across states.

<sup>5</sup> The agent's liability in a credit contract is limited to their wealth,  $W$ . The consumption minimum prevents the lender from offering contracts which drive the agent's consumption to zero, in which case there would always exist incentive compatible contracts and quantity rationing would never occur.

Figure 1 displays these indifference curves for two expected utility levels:  $\bar{V}_1$  that passes through the full insurance credit contract  $C'$ ; and,  $\bar{V}_0$  that passes through the certain wage income option. As drawn, this figure illustrates how risk rationing could occur. An agent would clearly prefer to undertake the entrepreneurial activity under the full insurance (and some other limited liability) credit contract to wage labor. However, under the unlimited liability credit contract, the agent would prefer the risk-free wage activity. Such an individual would be risk rationed if the only credit contract that is competitively available to him or her (under asymmetric information) is the unlimited liability contract.

Finally, because a very high initial wealth level would insulate the agent's utility from the success or failure of the entrepreneurial project, we assume that wealth levels are always low enough that the agent prefers high effort to low effort under the unlimited liability conditions of autarchic self-finance.<sup>6</sup>

### 3 Price and Non-Price Rationing in Credit Markets

As the discussion in the prior section makes clear, the nature of credit rationing, credit access and activity choice will depend on the set of contracts over state-contingent income space that are offered to agents. This section analyzes the set of contracts that lenders will make available under two alternative informational assumptions. The first is that information about the agent's choice of effort is costlessly observable to lenders and is contractable, meaning that loan contracts that specify high effort are enforceable. The second assumption is that the agent's choice of effort is a hidden action and that contracts cannot meaningfully specify agent effort.

Under the assumptions made in the prior section, expected project returns conditional on low effort are negative. The only way a lender could expect to recover costs if low effort were chosen by

<sup>6</sup> Formally we assume that  $W < \widehat{W}$  where  $\widehat{W}$  is implicitly defined as the level of wealth that just makes the agent indifferent between high and low effort under self-finance so that:

$$U(c_0 + \widehat{W} + y_s^a) - U(c_0 + \widehat{W} + y_f^a) = \left(\frac{B}{\Delta}\right)$$

where  $\Delta = \phi^H - \phi^L$ . Note that for  $W > \widehat{W}$  the expected benefit of high effort becomes less than its disutility cost.

the borrower would be if the contract offered the borrower negative expected income. Clearly such contracts would be of no interest to borrowers, so under symmetric information, lenders will only offer contracts that specify high effort. Under asymmetric information, the lender cannot directly specify the agent's effort level and therefore must consider how contractual payoffs indirectly affect the agent's choice of effort. The lender will only offer contracts that are incentive compatible in the sense that their payoff structure makes it optimal for the agent to choose high instead of low effort. For a contract to be incentive compatible, it must yield the borrower higher expected utility under high than low effort. Noting that the net expected utility gain to the agent for choosing high effort will be:

$$\eta(s_s, s_f, W) = [U(c_0 + W + s_s) - U(c_0 + W + s_f)]\Delta - B \quad (7)$$

(where  $\Delta = \phi^H - \phi^L$ ), we can define an incentive compatible contract as an  $s_s, s_f$  pair such that

$$\eta(s_s, s_f, W) \geq 0 \quad (8)$$

Expression (8) will be called the incentive compatibility constraint (ICC).

### 3.1 Credit Markets under Symmetric Information

Under symmetric information a loan contract is a triplet -  $(s_s, s_f, e)$  - that specifies the state contingent borrower payoffs and the effort level. We have already seen that lenders will only offer contracts specifying the high effort. In a competitive loan market the optimal contract,  $(s_s^*, s_f^*, H)$ , maximizes the agent's expected utility while guaranteeing the lender non-negative expected profits.

The payoffs of the optimal contract solve the following program:

$$\underset{S_s, S_f}{Max} \quad EU(c_0 + W + s_j | e = H) \quad (9a)$$

$$\text{subject to: } \pi(s_j | e = H) \geq 0 \quad (9b)$$

$$-s_j \geq W; \quad j = s, f \quad (9c)$$

The constraints 9b and 9c are the lender's participation constraint and the agent's limited liability constraint respectively. Note that the agent's payoff is not restricted to be non-negative. A negative payoff requires the farmer to hand over some of his assets and thus is equivalent to a collateral requirement.

Combining the first order necessary conditions with respect to the two payoffs yields:

$$\frac{\frac{\partial V}{\partial C_s}}{\frac{\partial V}{\partial C_f}} = -\frac{\phi^H}{1 - \phi^H} \quad (10)$$

The above expression states that, for a given expected income level, the optimal contract equates the agent's MRS of state contingent consumption with the ratio of success to failure probabilities. Recalling that the agent's MRS equals  $\frac{\phi^H}{1 - \phi^H} \frac{U'(C_s)}{U'(C_f)}$ , equation 10 implies that the optimal contract must equalize consumption across states. The lender's participation constraint pins down the unique optimal contract at the point  $(\bar{y}_H^b, \bar{y}_H^b)$  along the 45°line.<sup>7</sup> In Figure 1 the optimal contract is at point  $C'$ , which exhibits the familiar tangency condition - so that the rate at which the agent is willing to trade state contingent income (as defined by the MRS) is equal to the rate at which he is able to do so (as defined by the slope of the lender's participation constraint).

We now turn to the agent's activity choice among the three option: debt-finance with the optimal contract, self-finance, and wage labor. Since the optimal contract provides full insurance, the MRS at this contract is independent of agent wealth. As such, the optimal contract itself is independent of agent wealth. Since the optimal contract is identical for all agents and yields a certain income of  $\bar{y}_H^b$ , debt-finance will always be strictly preferred to the wage activity.

The comparison between self-finance and debt-finance is less clear. On one hand, by choosing self-finance, farmers avoid the finance cost of the credit contract - equal to  $(r^b - r^a)\underline{K}$ - and thus earn a higher expected income than under debt-finance. On the other hand, self-finance implies greater risk, as reflected by the movement away from the full insurance line in Figure 1. Risk averse farmers weigh this risk-return tradeoff in choosing between these two options. The relative

<sup>7</sup> It is easy to show that under symmetric information the limited liability constraint, (9c), will not bind.

weights assigned to risk and expected return are reflected in the curvature of indifference curves.

In order to explore the dependence of the activity ranking on the nature of risk preferences, consider Figure 2, which depicts indifference curves of a poor agent, with wealth  $W_0$ , and a rich agent with wealth  $W_1$ . In Figure 2,  $V^a(W_0)$  and  $V^a(W_1)$  are the poor and rich agent's indifference curves through the self-finance payoff at point  $B$  and  $V^b(W_1)$  is the rich agent's indifference curve through the optimal credit contract at point  $C'$ . If the indifference curve through the self-finance payoff crosses the full insurance line to the southwest of the optimal contract, the agent will prefer debt-finance to self-finance. The opposite holds if it crosses to the northeast of the optimal contract. At one extreme, a risk neutral agent's indifference curve through  $B$  would coincide with the iso-income line through  $B$  and cross the full insurance line at point  $B'$ , to the northeast of the optimal contract. A risk neutral farmer would not be willing to trade any expected income for a reduction in risk, as reflected by the fact that points  $B$  and  $B'$  are on the same iso-income line. A risk averse farmer, in contrast, is willing to exchange some expected return for a reduction in risk. This is reflected in a MRS less than  $\frac{\phi^H}{1-\phi^H}$  at point  $B$ . The smaller slope through  $B$ , combined with the fact that the slope of the indifference curves of all farmers along the full insurance line is equal to  $-\frac{\phi^H}{1-\phi^H}$ , implies that a risk averse indifference curve through  $B$  will cross the full insurance line to the southwest of  $B'$ . The more "curved" or bowed-in toward the origin an indifference curve is, the farther down the full insurance line toward the origin the intersection will occur. If the indifference curve is sufficiently "curved", the intersection will occur to the southwest of the optimal contract, implying that the farmer prefers debt-finance to self-finance.

As the Arrow-Pratt coefficient of absolute risk aversion measures the relative curvature of an indifference curve, we can use it to formalize these intuitions and examine the relationship between wealth and activity choice. Further differentiation of equation (6) with respect to wealth yields:

$$\text{sign} \left[ \frac{\partial y'_f(y_s)}{\partial W} \right] = -\text{sign} [U''(C_s)U'(C_f) - U''(C_f)U'(C_s)] = \text{sign} [R_s - R_f]; \quad (11)$$

where  $R_j$  is the coefficient of absolute risk aversion evaluated at consumption in state  $j$ . From



the relationship between wealth and optimal activity choice. Let  $p(y_j; W)$  be the full insurance risk premium associated with the risky prospect,  $y_j$ , for a borrower of collateral wealth,  $W$ . Following Pratt (1964), the risk premium is implicitly defined by:

$$EU(c_0 + W + y_j) = U[c_0 + W + Ey_j - p(y_j; W)]. \quad (12)$$

The risk premium tells us how much certain consumption the agent is willing to give up to completely eliminate the risk associated with a given income prospect. Since the poor agent in Figure 2 is indifferent between debt-finance and self-finance, it must be the case that  $p(y_j^a; W_0) = \bar{y}_H^a - \bar{y}_H^b$ , or the marginal agent's risk premium associated with self-finance equals the finance cost of the optimal contract ( $r^b - r^a$ ). The larger is the risk premium, the more certain income an agent is willing to give up relative to self-finance to achieve income smoothing. Equivalently, a larger risk premium implies that we must move further down the full insurance line toward the origin to find the riskless contract which yields the same expected utility as self-finance. In Figure 2, the poor agent's risk premium is given by the horizontal distance between points  $C'$  and  $B'$ , while the wealthy agent's is given by the horizontal distance between points  $D$  and  $B'$ . Since DARA implies that the risk premium associated with a given contract decreases with wealth, the intersection of the indifference curve through  $B$  becomes closer and closer to point  $B'$  as farmer wealth increases.

To summarize, under symmetric information, all agents have access to the unique optimal contract  $(\bar{y}_H^b, \bar{y}_H^b, H)$ . All agents undertake the entrepreneurial project. As depicted in Figure 2, let  $W_0$  denote the wealth level such that an agent is indifferent between the self-finance payoff and the optimal contract. Then, maintaining the assumption of DARA, agents with wealth greater than  $W_0$  carry out the project with their own funds while those with wealth less than  $W_0$  choose debt-finance. Two things are worth noting. First, if we assume that  $\underline{K} < W_0$ , then there will exist a class of agents with wealth  $\underline{K} < W < W_0$  who are able to self-finance the project but instead choose debt-finance. For this group -  $p(y_j; W) > \bar{y}_H^a - \bar{y}_H^b$  - so that their willingness to

pay to eliminate the risk of self-finance is greater than the finance cost of the credit contract. For these "insurance seekers", the credit market provides a substitute (although imperfect) for the missing insurance market. Farmers with wealth greater than  $W_0$  prefer to self-finance since the cost of full insurance is greater than their willingness to pay. Second, even with a perfect credit market, the absence of an insurance market implies a welfare loss. If actuarially fair insurance were available, then all farmers with wealth greater than  $\underline{K}$  would purchase insurance, self-finance the project and earn the state independent income  $\bar{y}_H^b$  - represented by point  $B'$  in Figure 2.

### 3.2 Credit Markets under Asymmetric Information

Asymmetric information requires altering the optimization program given in equations (9a) - (9c) by adding the incentive compatibility constraint which was defined by equations (7) and (8). A contract is incentive compatible if the agent's return to high effort,  $\eta(s_j, W)$  is non-negative. Equation (7) shows that under asymmetric information lenders must provide a contractual incentive to agents in order to offset the disutility cost of high effort. This implies that incentive compatible contracts offer payoffs that are strictly greater under success than failure.

Denote the locus defined by  $\eta(s_j, W) = 0$  as the *zero return curve*, which gives the set of contracts such that the incentive compatibility constraint binds or, equivalently, the agent's return to high effort is zero. As in the case of agent indifference curves, the shape of the zero return curve depends on the nature of risk preferences. Total differentiation of equation (7) yields:

$$s'_f(s_s)|_{\bar{\eta}} = \frac{\frac{\partial \eta}{\partial s_s}}{\frac{\partial \eta}{\partial s_f}} = \frac{U'(C_s)}{U'(C_f)} \quad (13)$$

Equation (13) shows that the zero return curve is upward sloping with a slope less than unity. Concavity of the utility function implies that a \$1 increase in the success payoff requires a less than proportionate increase in the failure payoff in order to maintain a constant return to the agent's high effort.

The curvature of the zero return curve less clear. Differentiation of equation (13) yields:

$$s_f''|_{\bar{\eta}} = \frac{U''(C_s)U'(C_f) - U'(C_s)U''(C_f)s_f'|_{\bar{\eta}}}{[U'(C_f)]^2} \implies \text{sign}[s_f''|_{\bar{\eta}}] = \text{sign}[R_f s_f'|_{\bar{\eta}} - R_s] \quad (14)$$

Since  $s_f'|_{\bar{\eta}} < 1$ , CARA and IARA preferences imply a concave zero return curve. The more appealing case of DARA yields an ambiguous result. If downside risk aversion is “sufficiently strong” then the level curves will be convex.<sup>8</sup>

The positive slope of the zero return curve, combined with convexity of indifference curves, implies that the optimal contract, if it exists, is unique and occurs at the intersection of the lender’s zero-profit contour and the zero return curve. The positive slope of the zero return curve also implies that the optimal contract will be the incentive compatible contract that requires the least collateral (has the highest failure payoff).

Figure 3 shows the situation for an agent with positive loan access. The upward sloping curve,  $\eta = 0$ , is the zero return curve. The constrained optimal contract occurs at point  $C''$ . While contracts between  $C'$  and  $C''$  yield higher expected utility, the lender will not make them available because they are not incentive compatible. The more severe is the incentive problem, the greater is the liability (risk) the agent must assume in a credit contract.<sup>9</sup> In Figure 3, as the incentive problem worsens, the zero return curve shifts down, moving the constrained optimal contract away from the full insurance contract,  $C'$  and towards the full liability contract at  $C$ . The inefficiency of the constrained optimum is seen from the divergence of the agent’s and lender’s MRS at the optimal contract. The potential gains from trade that would occur with movements along the lender’s zero-profit curve towards the full insurance contract are prevented by the inability of the agent to commit to high effort for any contract to the northwest of the zero return curve.

When information is asymmetric, the best contract available entails some risk and thus, from

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<sup>8</sup> The graphs presented in the rest of the paper will assume this holds. This assumption is made for ease of presentation. Since the optimal contract occurs at the intersection of the agent’s zero net return curve and the lender’s zero profit curve, the principal results only depend on the zero net return curve being upward sloping, and not on its curvature.

<sup>9</sup> The severity of the incentive problem is given by the size of  $B/\Delta$ . Thus the incentive problem is increasing in the agent’s private benefit of low effort and decreasing in the probability differential of success under high versus low effort.



of the contract, which serves as the credit rationing mechanism. Risk rationing is of particular concern, both in terms of equity and efficiency, since it may cause certain classes of farmers to voluntarily forego a higher return project in favor of the certain, but low return alternative.

The agent with wealth  $W$  in Figure 3 is clearly risk rationed. If it were available, the agent would accept the full insurance contract. The information asymmetry, however, pushes the constrained optimal contract to  $C''$ . Since the agent's expected utility associated with this contract,  $V_c^b(W)$ , is less than the utility of wage labor,  $V^\omega(W)$ , she will reject the loan contract. This agent is risk rationed because she has access to a credit contract which would raise her expected income, however the excessive risk of the constrained optimal contract leads her to prefer the low return, but certain option of wage labor.<sup>10</sup>

Conditional on positive credit access and sufficient wealth to self-finance, an agent's credit demand and activity choice can be summarized with reference to the certainty equivalent of each activity. Let  $p_c^b(W) \equiv p(s_j^*(W); W)$  be the risk premium associated with the constrained optimal credit contract and  $p^a(W) \equiv p(y_j^a; W)$  be the risk premium associated with self-finance. Then the agent's optimal activity choice is:

$$Activity\ Choice = \begin{cases} Wage\ labor\ if : \omega > \max [\bar{y}_H^b - p_c^b(W), \bar{y}_H^a - p^a(W)] \\ Debt - finance\ if : \bar{y}_H^b - p_c^b(W) > \max [\omega, \bar{y}_H^a - p^a(W)] \\ Self - finance\ if : \bar{y}_H^a - p^a(W) > \max [\omega, \bar{y}_H^b - p_c^b(W)] \end{cases} \quad (15)$$

Note that the certainty equivalent of any risky prospect can be represented in income space as the intersection of the indifference curve through that prospect with the full insurance line. This permits a clear interpretation of the preference ordering of the activities as we need only find the certainty equivalent furthest to the northeast on the full insurance line to determine the optimal activity.

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<sup>10</sup> As drawn in Figure 3, the agent also prefers wage labor to self-finance. It is possible that an agent that would choose the full insurance contract under symmetric information would choose self finance under asymmetric information, and thus perceive an increase in expected income as a result of the information asymmetry. This would be the case if the self finance point in Figure 3 were just to the northwest of the  $V_c^b(W)$  along the lender's zero profit curve.

## 4 The Economics of Risk Rationing: Wealth, Optimal Contracts and Activity Choice

Having explored credit demand and activity choice *conditional* on a positive supply, we now must explore the supply side of the credit market to ask: 1) What types of agents will have access to a contract? And 2) How do the terms of credit contracts vary across agents? The availability and terms of contracts will vary across agents because when agents are risk averse, the return to high effort is a function of wealth. For example, under DARA, poor agents are more sensitive to income differentials across states and have greater incentive to choose high effort to minimize the probability of failure. The dependence of the return to high effort on wealth has two implications for the optimal contract, which are summarized by two propositions. Proposition 1 discusses the existence of a contract.<sup>11</sup>

**Proposition 1** *Under asymmetric information, the minimum collateral requirement for access to a credit contract is  $W^*$ , where:  $W^* = \phi^H \{U^{-1} [U(c_0) + \frac{B}{\Delta}] - c_0\} - \bar{y}_H^b$ . Farmers with collateral wealth greater than or equal to  $W^*$  have access to at least one contract while those with collateral wealth less than  $W^*$  face quantity rationing.*

Figure 4 provides a graphical depiction of Proposition 1. The zero return curve is drawn for agents with three different wealth levels, with  $W^0 < W^* < W^1$ . A non-empty contract set requires that there exist at least one contract which simultaneously satisfies the incentive compatibility, limited liability, and (lender's) participation constraints.  $W^*$  is the unique wealth level such that there exists a contract - point  $B$  in Figure 4 - that simultaneously satisfies all three constraints with equality. The feasible set for the agent with wealth  $W^*$  contains a single contract. For this agent, any movement away from point  $B$  would violate at least one of the constraints. To see this, consider the following two implicit definitions:

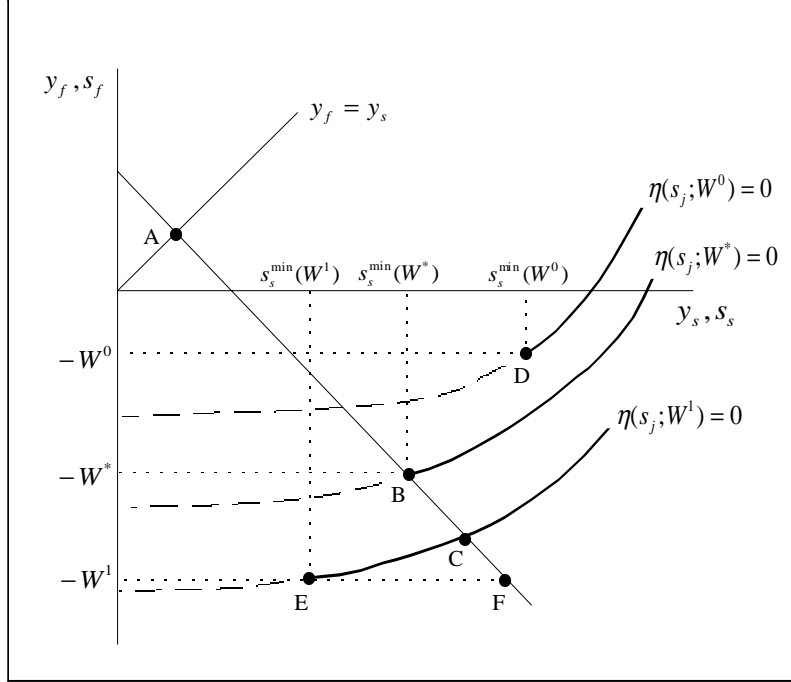
$$s_s^{\min}(W) : \eta(s_s^{\min}, -W; W) = 0 \quad (16)$$

$$s_s^{\max}(W) : \pi(s_s^{\max}, -W) = 0 \quad (17)$$

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<sup>11</sup> The proof of Propositions 1 and 2 are given in the appendix.

Figure 4: The effect of wealth on the existence and terms of the constrained optimal contract



>From equation (16),  $s_s^{\min}(W)$  is the minimum incentive compatible success payoff which is consistent with limited liability, and is found at the intersection of the zero return curve and the curve  $s_f = -W$ . From equation (17),  $s_s^{\max}(W)$  is the maximum success payoff consistent with limited liability and the lender's break even constraint, and is found at the intersection of the lender's zero profit line and the curve  $s_f = -W$ . A non-empty contract set requires  $s_s^{\min}(W) \leq s_s^{\max}(W)$ . Since  $W^*$  satisfies all three constraints with equality,  $s_s^{\min}(W^*) = s_s^{\max}(W^*)$ .

As collateral wealth increases, two things happen. First, the lender must increase the contractual risk of each contract to ensure that the agent chooses high effort. As a result, the zero return curve shifts down and, since the lender's zero profit curve is independent of agent wealth,  $s_s^{\max}(W)$  increases.<sup>12</sup> Second,  $s_s^{\min}(W)$  decreases. To see this consider points  $D$ ,  $B$ , and  $E$  in Figure 4. Each of these contracts requires the agents' full collateral under failure and yields a

<sup>12</sup> To see that the zero benefit curve shifts down with an increase in wealth, note from equation (7) that  $d\eta|_{s_s=0} = 0 \Rightarrow \frac{ds_f}{dW} = \frac{U'(C_s) - U'(C_f)}{U'(C_f)} < 0$ .

zero return to high effort. Consumption under success is the same for all three farmers and equal to the consumption minimum,  $c_0$ . Since  $C_s = c_0 + W + s_s$ , contract  $E$ , which is on the wealthiest agent's zero return curve, must have the smallest success payoff. The result of these two effects is that the feasible contract set expands with wealth. The feasible set for the wealthiest agent is composed of the triangle formed by points  $C$ ,  $E$ , and  $F$ . Meanwhile, the feasible set for the poorest agent is empty. There are no incentive compatible contracts with collateral less than or equal to  $W^0$  that yield the lender non-negative profits.

Examination of the expression for  $W^*$  in Proposition 1 yields the following comparative static results:  $\frac{\partial W^*}{\partial B} > 0$ ,  $\frac{\partial W^*}{\partial \phi^L} > 0$ , and  $\frac{\partial W^*}{\partial r^b}$ ; while the sign of  $\frac{\partial W^*}{\partial \phi^H}$  is ambiguous. An increase in either  $\phi^L$  or  $B$  augments the moral hazard problem by reducing the agent's return to high effort. The lender compensates by requiring additional collateral, thereby forcing some lower wealth agents out of the market through quantity rationing. A decrease in the lender's cost of funds,  $r^b$ , relaxes the lender's participation constraint and makes some lower collateral contracts available. An increase in  $\phi^H$ , the probability of success under high effort, has an ambiguous effect on  $W^*$ . On one hand, the agent has more incentive to choose high effort, which would reduce the collateral requirement. However, an increase in  $\phi^H$  also increases the lender's MRS between success and failure payoffs - which increases the additional collateral required for a unit increase in the success payoff.

For agents who meet the minimum collateral requirement, an increase in collateral wealth has two opposing effects on the expected utility of debt-finance. Recalling that  $V_c^b$  is the agent's expected utility under the constrained optimal debt contract, these two effects are seen in the following equation:

$$\frac{dV_c^b(s_j^*(W), W)}{dW} = \sum_{j=s,f} \frac{\partial V_c^b}{\partial s_j} \frac{\partial s_j^*}{\partial W} + \frac{\partial V_c^b}{\partial W} \quad (18)$$

The first term on the RHS of equation (18) is the indirect wealth effect which gives the change in an agent's expected utility if he were to receive the optimal contract of a wealthier agent. The second term is the direct wealth effect which gives the change in expected utility from additional

wealth, holding constant the optimal contract. The following proposition describes the relative strengths of these two effects.

**Proposition 2** *Conditional on a non-empty feasible set (ie.  $W > W^*$ ), the following results hold:*

$$(i). \frac{\partial V_c^b}{\partial W} > 0; \quad (ii) \sum_{j=s,f} \frac{\partial V_c^b}{\partial s_j} \frac{\partial s_j^*}{\partial W} < 0; \quad (iii) \frac{dV_c^b}{dW} > 0.$$

The first component of Proposition 2 says that the direct wealth effect is positive since, holding the contract terms constant, an increase in collateral wealth raises consumption in both states. In contrast, (ii) says that the indirect wealth effect is negative, or that an agent with greater wealth prefers the optimal contract of an agent with less wealth to his own optimal contract. As wealth increases, the zero return curve shifts down so that the optimal contract slides southeast along the lender's zero profit contour. Since an agent's MRS at any contract below the full insurance line is less than that of the lender, an agent's indifference curve will always cross the lender's zero profit contour from below. Thus any southeast movement along the zero profit contour corresponds to a decrease in a farmer's expected utility. In Figure 4, the wealthiest agent would prefer the optimal contract of the agent with wealth  $W^*$  at point  $B$  to his own contract at point  $C$ . Finally, (iii) implies that the direct wealth effect dominates the indirect wealth effect so that the net effect of an increase in collateral wealth on the expected utility of debt-finance is positive.

We now seek to put together the demand and supply side of the story and see if we can map the relationship between wealth and optimal activity choice. Recall that  $V^a$ ,  $V_c^b$ , and  $V^\omega$  are the agent's expected utility under self-finance, debt-finance with the constrained optimal contract, and wage labor respectively. We define the following three marginal wealth levels:

$$W^{m1} : V^a(W^{m1}) = V^\omega(W^{m1}) \leftrightarrow \bar{y}_H^a - p^a(W^{m1}) = \omega \quad (19a)$$

$$W^{m2} : V_c^b(W^{m2}) = V^\omega(W^{m2}) \leftrightarrow \bar{y}_H^b - p_c^b(W^{m2}) = \omega \quad (19b)$$

$$W^{m3} : V^a(W^{m3}) = V_c^b(W^{m3}) \leftrightarrow \bar{y}_H^a - p^a(W^{m3}) = \bar{y}_H^b - p_c^b(W^{m3}) \quad (19c)$$

Each of the wealth levels in equations (19a) - (19c) is such that an agent is indifferent between two activities. The strategy for mapping wealth into activity choice is to see if we can unambigu-

Table 2: A mapping of wealth into rationing mechanism and activity choice

Risk Preferences	Activity Comparison					
	(A) Self-finance vs. Wage labor		(B) Debt-Finance vs. Wage labor		(C) Self-finance vs. Debt-finance	
	<i>sign</i>	$\frac{d[V^a(W^{m1}) - V^\omega(W^{m1})]}{dW}$	<i>sign</i>	$\frac{d[V_c^b(W^{m2}) - V^\omega(W^{m2})]}{dW}$	<i>sign</i>	$\frac{d[V^a(W^{m1}) - V_c^b(W^{m2})]}{dW}$
DARA		+		?		+
CARA		0		-		+
IARA		-		-		?

ously determine which activity becomes preferred when the three marginal farmers are given an additional unit of collateral wealth. The three things which factor into an agent's comparison of two activities are: 1) The expected return of the activity; 2) The riskiness associated with the activity; and 3) The agent's willingness to bear risk. As should be clear by now, the expected return of each activity is independent of agent wealth. The risk associated with land rental and self-finance are also independent of wealth while, from Proposition 2, the riskiness of debt-finance increases with collateral wealth. Finally, how an agent's willingness to bear a particular risk varies with wealth will depend upon the nature of risk preferences. Table 2 summarizes the impact of collateral wealth on activity choice for three alternative specifications of risk preferences.

Column A shows the impact of an increase in collateral wealth on the agent who is indifferent between self-finance and wage labor. As wealth increases, the only thing that (potentially) changes is the agent's willingness to bear the risk associated with self-finance. If risk preferences are described by DARA, this willingness increases or, alternatively, the self-finance risk premium decreases. From equation (19a) this implies that  $\frac{d[V^a(W^{m1}) - V^\omega(W^{m1})]}{dW} > 0$  so that any agent with wealth greater than  $W^{m1}$  prefers self-finance to wage labor. The opposite holds if risk preferences are IARA while under CARA an extra unit of wealth has no impact on the agent's comparison between the two activities.

The effect of wealth on the remaining two comparisons is less clear because both involve a comparison with the constrained optimal credit contract, the terms of which are endogenous

to agent wealth. Consider the agent with wealth  $W^{m2}$ , who is indifferent between wage labor and debt-finance. From Proposition 2, we know that the expected utility from debt-finance is increasing in wealth. The expected utility from wage labor, however, is also increasing in wealth. The sign of  $\frac{d[V_c^b(W^{m2}) - V^w(W^{m2})]}{dW}$  will then depend upon the relative strengths of these two effects. Since the riskiness of debt-finance increases (for all risk averse agents) it makes sense that if an agent becomes more sensitive to risk, then he would prefer the relatively less risk activity, which in this case is the wage activity. We can make use of the risk premium to formalize this intuition and confirm the entries of column B of Table 2. From equation (19b), an increase in wealth leads to a preference for debt-finance if the risk premium associated with the optimal contract is decreasing in wealth. It is straightforward to show that a local approximation of this risk premium is:

$$p_c^b(s_j^*(W); W) \cong \frac{R(\bar{y}_H^b)\sigma_s^2}{2} \quad (20)$$

where  $\sigma_s^2 = E[(s_j^* - \bar{y}_H^b)]$  is the variance of the optimal, contract payoff and  $R(\bar{y}_H^b)$  is the coefficient of absolute risk aversion evaluated at the riskless consumption level  $c_0 + W + \bar{y}_H^b$ . Differentiation of equation (20) with respect to  $W$  yields

$$\frac{dp_c^b}{dW} \cong \frac{1}{2} \left[ \sigma_s^2 \frac{\partial R(\bar{y}_H^b)}{\partial W} + R(\bar{y}_H^b) \frac{\partial \sigma_s^2}{\partial W} \right] \quad (21)$$

Equation (21) decomposes the change in the risk premium into a direct and indirect wealth effect. The first term in square brackets is the direct effect and measures, holding the optimal contract constant, how much more or less risk averse the borrower becomes as wealth is increased. This term is positive, zero, and negative as risk preferences are described by IARA, CARA, and DARA respectively. The second term is the indirect effect and tells us how much additional insurance an agent would seek to compensate for the increased risk of the optimal contract. Proposition 2 showed that, for all risk averse agents, the optimal contract terms shift southeast along the zero-profit contour and thus the contract becomes more risky. As a result, the indirect effect is always positive. The more risk averse the agent is, the greater will be the indirect effect. Under

IARA, the direct and indirect effects reinforce each other - both tending to drive the risk premium up and credit demand down. The same net result obtains under CARA since the direct effect is zero and the indirect effect is positive - and independent of wealth. Under the more plausible case of DARA, however, the direct effect is negative while the indirect effect is positive. Under DARA, the net impact of increasing wealth thus depends on the relative strength of the two effects.

Finally, column C of Table 2 describes the impact of collateral wealth on the individual who is indifferent between debt-finance and self-finance. The previous two comparisons each involved the certain prospect of wage labor. This provided an anchor which allowed the optimal activity choice to depend solely on the change in the agent's willingness to bear the risk of a single prospect - either self-finance or debt-finance. In this final comparison, in contrast, we must compare across two risky prospects. The question becomes, does an increase in wealth make an agent more or less willing to bear the additional risk of self-finance relative to debt-finance?

Consider first the case of DARA. If an agent is indifferent between two risky prospects, one of which has a higher expected return but greater variance, then, under DARA, intuition might suggest that a wealthier agent would prefer the higher return, higher risk project. Using this logic, agents with wealth greater than  $W^{m3}$  would prefer self-finance to debt-finance. This intuition is reinforced by the fact that as wealth increases the risk of self-finance stays constant while that of debt-finance increases. Unfortunately, this intuition is not so easily supported analytically. From equation (19c), this intuitive result requires that  $\frac{d[V^a(W^{m1}) - V_c^b(W^{m2})]}{dW} > 0$  or, equivalently, that the self-finance risk premium decreases more than the risk premium of the optimal contract. While DARA implies that  $\frac{\partial p^a(W)}{\partial W} < 0$ , we have seen that  $\frac{\partial p_c^b(W)}{\partial W}$  could be either positive or negative. This leaves us with a problem since assumptions about risk preferences using the Arrow-Pratt measure of absolute risk aversion inform us about the *direction* of change of the full insurance risk premium, but, in general, not about the *magnitude* of change. If, in addition, we assume that the coefficient of absolute risk aversion is concave, we are ensured that  $\frac{\partial p^a(W)}{\partial W} < \frac{\partial p_c^b(W)}{\partial W}$  so that the

intuitive result indeed holds.<sup>13</sup>

The comparison of column C is more straightforward under CARA. CARA implies  $\frac{\partial p^a(W)}{\partial W} = 0$  and  $\frac{\partial p_c^b(W)}{\partial W} > 0$  so that self-finance becomes more attractive. Finally, under IARA, both  $\frac{\partial p^a(W)}{\partial W} > 0$  and  $\frac{\partial p_c^b(W)}{\partial W} > 0$  so that the effect of wealth on activity choice is ambiguous.

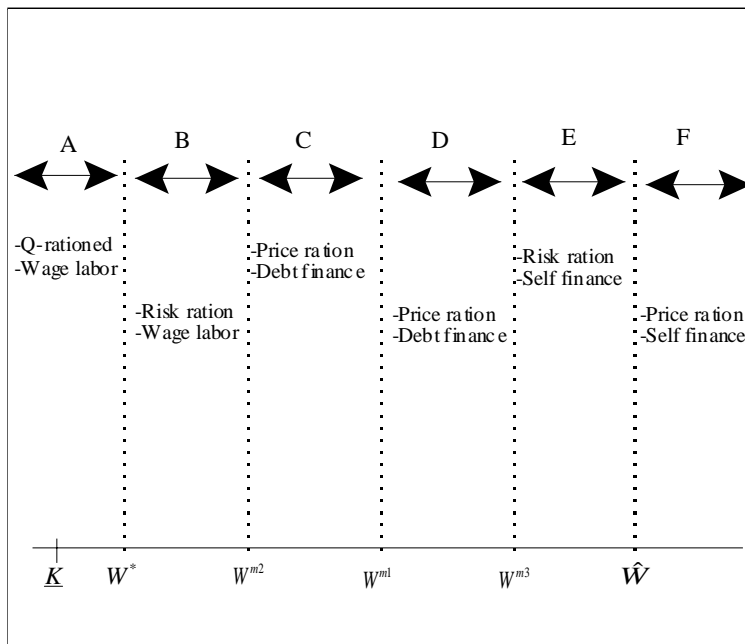
We can now review the main results of Table 2. Notice that in each column, the riskier of the two activities being compared appears first. Thus a (+) indicates that an increase in wealth leads the marginal agent to prefer the riskier of the two activities. Compare the first and third rows, which give the results for the opposing cases of DARA and IARA. In general, the results coincide with intuition since, under DARA, increasing collateral wealth makes the riskier prospect more attractive while under IARA the less risky prospect becomes more attractive. The two question marks both involve a comparison with debt-finance. This reflects the negative impact of wealth on contract terms in the presence of moral hazard. Under DARA, an increase in wealth would make an agent prefer the higher return from debt-finance relative to the wage activity if the contract terms stayed constant. Since the terms worsen, however, the agent may instead prefer wage labor. Similarly, under IARA an increase in wealth makes the less risky, debt-finance more attractive than self-finance if the contract terms are fixed. Again, however, the result is unclear when the worsening of contract terms is factored in.

As a means of concluding this section, consider Figure 5, which depicts one possible mapping of wealth into credit market rationing mechanism and optimal activity choice when risk preferences are described by DARA. The following ordering is assumed to hold:  $\underline{K} < W^* < W^{m2} < W^{m1} < W^{m3}$ . In addition, define  $\widehat{W}$  as the collateral wealth level such that a farmer is indifferent between self-finance and the optimal contract under symmetric information. It is easy to see that  $\widehat{W} > W^{m3}$ . By assuming that:  $\max\{\underline{K}, W^*\} < \min\{W^{m1}, W^{m2}, W^{m3}\}$ , we make all three activities feasible for the three marginal agents in order to fully examine the role of risk and moral hazard.<sup>14</sup>

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<sup>13</sup> One line proof...

Figure 5: A mapping of agent wealth into rationing mechanism and activity choice (DARA)



Consider the wealth ranges in ascending order:

A.  $W < W^*$ . In this range agents do not meet the minimum collateral requirement and are thus shut out of the credit market. All agents in this wealth range work in wage labor. Those that are able to self-finance -  $\underline{K} < W < W^*$  - still have relatively little wealth and thus prefer wage labor to the riskier prospect of self-finance. This is true for all agents with wealth less than  $W^{m1}$ .

B.  $W^* < W < W^{m2}$ . Agents in this wealth range have access to a credit contract. Whether or not they accept it depends upon the sign of  $\frac{d[V_c^b - V^\omega]}{dW}$  which, from Table 2, is ambiguous. The case depicted in Figure 8 is  $\frac{d[V_c^b - V^\omega]}{dW} > 0$ , which implies that the optimal contract contains too much risk relative to wage labor. These agents are risk rationed. Under symmetric information they would seek the first-best contract and debt-finance risky production. However, under asymmetric information they instead choose wage labor.

<sup>14</sup> Two of the six possible orderings of  $W^{m1}$ ,  $W^{m2}$ , and  $W^{m3}$  are inconsistent with DARA. These are  $W^{m1} > W^{m2} > W^{m3}$  and  $W^{m3} > W^{m2} > W^{m1}$ .

C.  $W^{m2} < W < W^{m1}$ . Under the assumption that  $\frac{d[V_c^b - V^\omega]}{dW} > 0$ , agents in this range are willing to accept the risk of the constrained optimal contract and choose debt-finance. They are price rationed since they would also choose debt-finance under symmetric information.

D.  $W^{m1} < W < W^{m3}$ . Agents in this range prefer self-finance to wage labor, however they still seek the partial insurance of the constrained optimal debt-contract.

E.  $W^{m3} < W < \widehat{W}$ . The partial insurance of the constrained optimal debt contract is too expensive for agents in this range so they opt for self-finance. They are risk rationed because, under symmetric information they would seek the full insurance credit contract.

F.  $\widehat{W} < W$ . This wealthiest group chooses to self-finance under both symmetric and asymmetric information and is thus classified as price rationed.

There are three things to be gleaned from Figure 5. First, the interaction of risk aversion and asymmetric information clearly breaks the independence between wealth distribution and credit market rationing mechanism/activity choice. Second, the nature of the mapping from wealth into activity choice depends upon the nature of risk preferences. For example, Figure 5 would look quite different if IARA instead of CARA had been assumed. Finally, even conditional on the specification of risk preferences, there remains some unresolved ambiguity in the mapping. In Figure 5, the location of groups (B) and (C) depends upon the sign of  $\frac{d[V_c^b - V^\omega]}{dW}$ , which is uncertain due to the opposing direct and indirect effect of wealth on credit demand.

## 5 Conclusion

The theoretical credit market literature has focussed on the potential for asymmetric information to give rise to quantity rationing and thereby prevent agents from undertaking investments due to insufficient liquidity. This paper has argued that information asymmetries impact entrepreneurs' investment decisions not only through the existence of credit contracts but also by affecting the terms of the optimal contract when it exists. This latter effect is particularly relevant in developing countries where risk averse agents have little or no access to insurance markets. In such an

environment, individuals may seek a credit contract both to overcome liquidity constraints and/or because it offers insurance against production or price shocks. When farmers are risk averse, the presence of moral hazard can alter investment patterns not only by eliminating an agent's access to a contract, but also by forcing the agent to bear additional risk in the constrained optimal contract. This creates the potential for a class of risk rationed agents who alter their activity choice relative to the first best scenario.

The mapping of credit market rationing mechanism and activity choice in endowment space has several ambiguities due to the opposing effects of wealth on the incentive problem and the willingness to bear risk. Even with these ambiguities, however, the identification of the category of risk rationed farmers is important since it suggests there is another class of farmers - in addition to those that are quantity rationed - for whom decentralized credit markets do not perform well. This suggests that empirical studies aimed at gauging the performance of credit markets should take risk rationed farmers into account or else risk over-estimating the "health" of the financial system

The paper also points to various directions for future work. First, the discrete nature of the model could be relaxed to more accurately reflect entrepreneurs' choices. In particular, the assumption of a fixed project size could be relaxed to allow for continuous capital inputs. The model could also be extended to incorporate the various means by which borrowers and lenders overcome information asymmetries in practice. For example, under monitored lending, the agent's effort level is monitored - either by the lender or by other agents in a group lending scheme - and a penalty is imposed if the farmer deviates from the agreed upon effort level. Conning (1996, 1999), for example, has taken initial strides along this line by developing a model which endogenizes the level of monitoring and institutional form under moral hazard. Extending the model in this direction is important since it can help explain the frequently observed coexistence of multiple institutional forms of credit delivery.

The model could also be extended to include the informal loan sector. Previous research has

tended to view the informal credit market in one of two ways. On one hand, the informal sector is portrayed as the recipient of “spillover” demand from the formal sector (Bell 1990, Bell et al 1996, 1997). In this view, farmers that are quantity rationed in the formal sector haven o alternative except to turn to informal lenders for their credit demand. Alternatively, borrowers may prefer the informal sector to the formal sector because the low transaction costs in the informal sector make it the lowest *effective* cost credit source (Kochar 1997, Chung 1995). The consideration of risk presented here raises another interpretation of the informal sector. With greater access to local information and lower cost monitoring technologies, informal lenders are better able to overcome information barriers. They may then be able to offer contracts with greater implicit insurance than formal sector contracts. This is consistent with empirical observation that informal lenders rarely require collateral. Even if informal contracts are more expensive in terms of the expected value of loan repayment, farmers may prefer them for their implicit insurance. This is quite a different picture from the traditional view of the village moneylender whose local power permits unlimited liability contract which imply extremely high contractual risk.

Appendix: Proofs of Proposition 1 and 2

**Proof.** Proposition 1. We first need to show that a farmer with collateral wealth  $W^*$  has a non-empty feasible contract set. From equations (16) and (17),  $s_s^{\min}(W)$  and  $s_s^{\max}(W)$  are implicitly defined by:

$$\eta(s_s^{\min}, -W; W) = [U(c_0 + W + s_s^{\min}) - U(c_0)] \Delta - B = 0 \quad (22)$$

$$\pi(s_s^{\max}, -W|e = H) = \bar{y}_H^b - \phi^H s_s^{\max} + (1 - \phi^H) W = 0 \quad (23)$$

$W^*$  is the wealth level such that  $s_s^{\min}(W^*) = s_s^{\max}(W^*)$ . Since  $s_s^{\min}$  satisfies both the incentive compatibility and limited liability constraints, while  $s_s^{\max}$  satisfies both the lender’s participation constraint and the limited liability constraint, the contract  $(s_s^{\min}(W^*), -W^*)$  is feasible for a farmer with wealth  $W^*$ .

Next we need to show that an increase in wealth maintains a non-empty feasible set while a decrease in wealth leads to an empty feasible set. The necessary and sufficient condition for a non-empty feasible set is that  $s_s^{\min} = s_s^{\max}$ . To see this, note that  $\frac{\partial \eta}{\partial s_s} > 0$  and  $\frac{\partial \pi}{\partial s_s} < 0$  so that any contract of the form  $(b, -W)$  where  $s_s^{\min} < b < s_s^{\max}$  is feasible. Now consider the effect of wealth on  $s_s^{\min}$  and  $s_s^{\max}$ . Inspection of equation (22) shows that  $\frac{\partial s_s^{\min}}{\partial W} = -1$ . As an additional dollar of wealth becomes available then, to hold consumption under success constant, the success payoff must be decreased by \$1. From equation (23),  $\frac{\partial s_s^{\max}}{\partial W} = \frac{1-\phi^H}{\phi^H} > 0$ . An additional dollar of wealth allows the lender to offer a larger success payoff and still earn zero profits. Thus for any agent with wealth  $W^1 > W^*$ , we have:  $s_s^{\min}(W^1) < s_s^{\min}(W^*) = s_s^{\max}(W^*) < s_s^{\max}(W^1)$ , and the necessary and sufficient condition for a non-empty feasible set is satisfied. In contrast, for any agent with collateral wealth  $W^0 < W^*$ , we have:  $s_s^{\max}(W^0) < s_s^{\min}(W^*) = s_s^{\max}(W^*) < s_s^{\min}(W^0)$ , which violates the necessary and sufficient condition for a non-empty feasible set.

Proposition 2. The expected utility from debt-finance for an agent with wealth  $W$  is:

$$V_c^b(s_j^*(W); W) = \phi^H U [c_0 + W + s_s^*(W)] = (1 - \phi^H) U [c_0 + W + s_f^*(W)] \quad (24)$$

>From equation (24), we have:

$$\frac{\partial V_c^b}{\partial W} = \phi^H U'(C_s) + (1 - \phi^H) U'(C_f); \quad (25a)$$

$$\frac{\partial V_c^b}{\partial s_s^*} = \phi^H U'(C_s); \quad (25b)$$

$$\frac{\partial V_c^b}{\partial s_f^*} = (1 - \phi^H) U'(C_f) \quad (25c)$$

(i): From equation (25a) it is clear that the direct wealth effect is positive. (ii): To find the indirect wealth effect we need equations (25b), (25c) plus the change in the optimal contract terms with wealth. Since the optimal contract solves the system  $\eta(s_j^*; W) = \pi(s_j^* | e = H)$ , we can use the implicit function theorem to find these two terms. Using Cramer's rule, the two terms of interest

are:

$$\frac{\partial s_s^*}{\partial W} = \frac{(1 - \phi^H) [U'(C_f) - U'(C_s)]}{\phi^H U'(C_f) + (1 - \phi^H) U'(C_s)} > 0 \quad (.26a)$$

$$\frac{\partial s_f^*}{\partial W} = \frac{-\phi^H [U'(C_f) - U'(C_s)]}{\phi^H U'(C_f) + (1 - \phi^H) U'(C_s)} = \left( \frac{-\phi^H}{1 - \phi^H} \right) \frac{\partial s_s^*}{\partial W} < 0 \quad (.26b)$$

Combining terms, the indirect effect becomes:

$$\sum_{j=s,f} \frac{\partial V_c^b}{\partial s_j} \frac{\partial s_j^*}{\partial W} = \phi^H [U'(C_s) - U'(C_f)] \frac{\partial s_s^*}{\partial W} < 0 \quad (27)$$

(iii): Finally, using equation (27), the total wealth effect is:

$$\frac{dV_c^b}{dW} = \sum_{j=s,f} \frac{\partial V_c^b}{\partial s_j} \frac{\partial s_j^*}{\partial W} + \frac{\partial V_c^b}{\partial W} = \frac{U'(C_f) U'(C_s)}{\phi^H U'(C_f) + (1 - \phi^H) U'(C_s)} > 0 \quad (28)$$

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