Rich Language and Refinements of Cheap-Talk Equilibria*

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Abstract

I formalize the notion that one cheap-talk equilibrium has a richer language than another. I show that the equilibria with the richest language coincide with the *S-Pareto efficient equilibria, i.e.* such that no other equilibrium makes the sender with some signal better off and does not make the sender with any signal worse off. I also study the stability of equilibria with respect to inflows of messages that enrich their language; more precisely, I study *recurrent equilibria*, i.e. the equilibria that are played repeatedly along the path that follows such an inflow of messages. I show that if an equilibrium is recurrent, then so are all equilibria having a richer language.

I provide examples of implausible cheap-talk equilibria that are neither S-Pareto efficient nor recurrent but are not refined away by the existing refinements, and I confront the predictions of my refinemnets and the existing ones with experimental evidence.

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1 Introduction

In addition to very important insights, cheap-talk models also provide implausible predictions. First, cheap-talk games always have equilibria in which messages provide no information. Even in games of pure common-interest, i.e. when the payoff functions of both agents coincides, there are equilibria where the sender communicates his true signal, as well as babbling equilibria where the sender sends each possible signal with the same probability. Second, truthful communication appears as an equilibrium in games when the sender would achieve the highest payoff if the receiver did not receive any information from him.

These implausible equilibria led to a search for refinements. The main line of research on refinements follows Farrell (1993). He, as well as Matthews, Okuno-Fujiwara and Postelwaite (1991), Rabin (1990), Rabin and Sobel (1996), and Zapater (1997), defines messages that are credible and should be believed. Intuitively, a message is credible if assuming that it will be believed, a sender with different information cannot benefit from sending that message. According to these papers, the receiver should believe in the sender's message whenever it is credible, because a rich common language should enable the sender to convince the receiver that he is truthful. All papers quoted above refer to rich, common language in an informal sense. Neither the papers quoted above nor other existing refinements[1] refine away all implausible equilibria. Some of them refine away too many equilibria, leaving sometimes the set of remaining equilibria empty. Others can be criticized for their inconsistent logic (the Stiglitz critique).

In the present paper, I formalize and study the consequences of the notion that one cheap-talk equilibrium has a richer language than another. Given an equilibrium, I identify each message - used in the equilibrium with a positive probability - with the information about the sender's signal that the message conveys and the receiver's action that the message implies, or more precisely, with a pair (τ, β) where τ stands

[1] I postpone a detailed discussion of the existing refinements and their relation to this paper to Section 6.

for a distribution of possible signals and β stands for a distribution of possible actions. Then I extend the equilibrium set of messages by all pairs (τ, β) that can be available when the equilibrium is played. I say that equilibrium E has a richer language than equilibrium F if the extended set of equilibrium messages for E contains the extended set of equilibrium messages for F. I show that equilibria with maximally rich language coincide with the S-Pareto efficient equilibria, i.e. such equilibria that no other equilibrium makes some type of the sender better off and does not make any type of the sender worse off.

I also study the stability of equilibria with respect to inflows of new messages. The intuition behind this concept can be explained briefly as follows: A population of agents plays initially some equilibrium. At some point, a fraction of senders randomly sends messages, which have not been used so far. Receivers form beliefs about the signals of those senders who use those new messages. Senders try to take advantage of those new messages. This, in turn, makes receivers update their beliefs. Which equilibria may appear along the path that follows such an inflow of new messages? Which equilibria have the property that players come back to the original equilibrium strategies? I show that equilibria with richer language have better stability properties. Namely, if some equilibrium satisfies the criterion from the first question, then so does all equilibria with a richer language, and only S-Pareto efficient equilibria can satisfy the criterion from the second question. I also characterize the equilibria that satisfy the two criteria for some specific classes of games.

The results are confronted with an experiment. In the experiment, students were playing first the role of sender and then the role of receiver. Each student was told a signal, and was supposed to submit an e-mail message. The e-mails, properly mixed, were forwarded back to students who, this time, picked up actions. Consistently with the spirit of rich language, there was no constraint on the text of e-mail messages, except the game from Example 2[2].

[2] This game was played twice, with no restriction on the text of messages and when the set of allowed messages consisted of "I have s_i", for all possible, not nessarily true, signals s_i.

2 Model

I study the following game G: A sender S gets a signal s from a set of possible signals T. For simplicity, assume that T is a finite set, $T = \{s_1, ..., s_n\}$, and the prior probability that S gets s equals $1/n$ for every $s \in T$. S sends a message $m \in M$ to a receiver R, and R takes an action a from a set of possible actions $A = \{a_1, ..., a_k\}$. Assume that A is a finite set. The payoffs of the players are represented by u_S and u_R, respectively, real-valued functions of R's action and S's signal. Players are allowed to use mixed strategies. A mixed strategy of the sender is denoted $\sigma : M \times T \to [0, 1]$, where $\sigma(m, s)$ stands for the probability that the sender with signal s sends message m. A mixed strategy of the receiver is denoted $\alpha : A \times M \to [0, 1]$, where $\alpha(a, m)$ stands for the probability of taking action a by the receiver who received message m. The receiver is assumed to form beliefs. The beliefs are represented by a function $\mu : M \times T \to [0, 1]$, where $\mu(m, s)$ stands for the probability assigned to the sender having signal s conditional on receiving message m.

Denote by $U_S(\sigma, \alpha \mid s)$ the expected payoff of the sender with signal s, and by $U_R(\sigma, \alpha)$ and $U_R(\sigma, \alpha \mid m)$ the expected payoff of the receiver and the expected payoff of the receiver who received message m, respectively, provided S plays according to strategy σ and R plays according to strategy α. I study Bayesian equilibria (M, σ, α) of G. I do consider various sets of message. Therefore I always specify the equilibrium set of messages M. For simplicity, I identify all Bayesian equilibria with the same strategies: σ, α, and different beliefs μ.

3 Examples

Now, I provide two examples to illustrate the problem of implausible equilibria.

Example 1 *Consider a pure common-interest game, where* $T = \{s_1, s_2\}$, $A = \{a_1, a_2\}$, *and the payoffs are represented by the entries of the following matrix:*

$$\begin{array}{ccc} & a_1 & a_2 \\ s_1 & 1,1 & 0,0 \\ s_2 & 0,0 & 1,1 \end{array} .$$

The first and the second entry represent the sender's and the receiver's payoff, respectively.

The game has two equilibria: a pooling equilibrium E, where S sends each message with probability $1/2$ *independently of his signal, and a separating equilibrium F, where S reveals his signal truthfully. In the pooling equilibrium, R picks up every possible action with probability* $1/2$*, and in the separating equilibrium, R responds to message* s_i *by taking action* a_i*.*

The equilibrium outcome E seems counter-intuitive, because the sender has a strong incentive to share his information with the receiver. Our intuition suggests that the separating equilibrium is more plausible.

Example 2 *Consider now a game with conflict of interests, where* $T = \{s_1, s_2\}$*,* $A = \{a_1, a_2, a_3\}$*, and the payoffs are represented by the following matrix:*

$$\begin{array}{cccc} & a_1 & a_2 & a_3 \\ s_1 & 1,3 & 0,0 & 2,2 \\ s_2 & 0,0 & 1,3 & 2,2 \end{array} .$$

The game has also a pooling equilibrium E and a separating equilibrium F. In the pooling equilibrium, R takes action a_3*, and in the separating equilibrium, R responds to message* s_i *by taking action* a_i*.*

The equilibrium outcome F seems counter-intuitive, because he has a strong incentive to keep the receiver uninformed. Again our intuition suggests that the pooling equilibrium is more plausible.

4 Equilibria with Richer Language

Messages used in different equilibria may be very different objects, e.g. they can be formulated in English, Polish, or cryptograms. Thus, the sets of messages used in different equilibria may be incomparable per se. However, the content of each message that is relevant for strategic reasons consists of the information about the sender's signal that the message contains and the receiver's action that the message implies. I shall identify messages with probability distributions over the sender's possible signals and probability distributions over the receiver's possible actions, i.e. elements of $\Delta T \times \Delta A$.

More precisely, consider an equilibrium $E = (M, \sigma_E, \alpha_E)$. Take a message $m \in M$ such that $\sigma_E(m, s) > 0$ for some $s \in T_S$. Let

$$\tau_m(s) = \frac{\sigma(m, s)}{\sum_{t \in S} \sigma(m, t)}, \tag{1}$$

$$\beta_m(a) = \alpha(a, m), \tag{2a}$$

and

$$i(m) = (\tau_m, \beta_m). \tag{3}$$

Then mapping $i : M \to \Delta T \times \Delta A$ identifies the equilibrium messages with elements of $\Delta T \times \Delta A$.

Proposition 1 *For every equilibrium $E = (M, \sigma_E, \alpha_E)$, there exist a set $M_E \subset \Delta T \times \Delta A$ and such that $i(M) \subset M_E$ and $(M_E, \sigma_E^{/}, \alpha_E^{/})$, where $\sigma_E^{/}(i(m), s) = \sigma_E(m, s)$ and $\alpha_E^{/} = \pi_2 \mid M_E$ is the projection onto ΔA, is an equilibrium. Moreover, there exist the smallest* $\inf M_E$ *and the largest* $\sup M_E$ *such sets of messages with respect to inclusion.*

Proof. Let

$$\inf M_E = \{(\tau_m, \beta_m) : m \in M \text{ and } \sigma(m, s) > 0 \text{ for some } s \in T\},$$

and

$$\sup M_E = \bigcup \left\{ M_E \subset \Delta T \times \Delta A : (M_E, \sigma_E^{/}, \alpha_E^{/}) \text{ is an equilibrium} \right\}.$$

■

I call each set $M_E \subset \Delta T \times \Delta A$ having the properties from Proposition 1 an equilibrium set of messages for E. I will identify equilibria (M, σ, α) and $(\inf M_E, \sigma^{/}, \alpha^{/})$. Now, I can compare the sets of messages used in different equilibria.

Definition 1 *Equilibrium E has a richer language that equilibrium F (quilibrium F has a poorer language that equilibrium E) if* $\sup M_F \subset \sup M_E$.

Intuitively, an equilibrium has a richer language if the set of messages that can be available when the equilibrium is played is larger. Proposition 2 provides two conditions, equivalent to that from Definition 1. Conditions (b)-(c) will be useful both in proofs and applications.

Proposition 2 *The following conditions are equivalent:*

(a) E has a richer language that F,

(b) $\inf M_F \subset \sup M_E$,

(c) $\inf M_F \cup \inf M_E$ *is an equilibrium set of messages for E.*

Proof. (b) $\Rightarrow$ (a) Since all types of the sender prefer playing $\sigma_E^{/}$ to sending any other message $m \in \sup M_E$, by (b), they prefer plaing $\sigma_E^{/}$ to sending any other message $m \in \inf M_F$. Since all types of the sender prefer playing $\sigma_F^{/}$ to any other message $m \in \sup M_F$, they prefer sending some message $m \in \inf M_F$ to sending any other message $m \in \sup M_F$. Thus, by transitivity, all types of the sender prefer playing $\sigma_E^{/}$ to sending any other message $m \in \sup M_F$. This yields that $\inf M_E \cup \sup M_F$ is an equilibrium set of messages for E, and so $\inf M_E \cup \sup M_F \subset \sup M_E$.

(a) $\Rightarrow$ (c) Indeed, every M_E such that $\inf M_E \subset M_E \subset \sup M_E$ is an equilibrium set of message for E.

(c) $\Rightarrow$ (b) By (c), $\inf M_F \cup \inf M_E \subset \sup M_E$. ■

In Examples 3 and 4, I apply the concept of having a richer language to the games from Examples 1 and 2.

Example 3 *Consider the pure common-interest game from Example 1. Then*

$$\inf M_E = \sup M_E = \{(\tau, \beta)\},$$

where $\tau(s_1) = \tau(s_2) = 1/2$ *and* $\beta(a_1) = \beta(a_2) = 1/2$*;*

$$\inf M_F = \{(s_1, a_1), (s_2, a_2)\},$$

and

$$\sup M_F = \{(\tau_p, a_1) : p > 1/2\} \cup \{(\tau_p, a_2) : p < 1/2\} \cup \{(\tau, \beta_p) : p \in [0,1]\},$$

where $\tau_p(s_1) = p$ *and* $\beta_p(a_1) = p$ *(see Figure 1). That is,* F *has a richer language than* E*.*

Example 4 *Consider now the game from Example 2. Then*

$$\inf M_F = \{(s_1, a_1), (s_2, a_2)\},$$

and

$$\sup M_F = \{(\tau_p, a_1) : p \geq 2/3\} \cup \{(\tau_p, a_2) : p \leq 1/3\};$$

$$\inf M_E = \{(\tau, a_3)\},$$

and

$$\begin{aligned} \sup M_E &= \{(\tau_p, a_1) : p \geq 2/3\} \cup \{(\tau_p, a_2) : p \leq 1/3\} \\ \cup\{(\tau_{2/3}, \beta_p^{13}) &: p \in [0,1]\} \cup \{(\tau_{1/3}, \beta_p^{23}) : p \in [0,1]\} \\ \cup\{(\tau_p, a_3) &: p \in [1/3, 2/3]\}, \end{aligned}$$

where $\beta_p^{ij}(a_i) = p$, $\beta_p^{ij}(a_j) = 1 - p$ *(see Figyre 2). That is,* E *has a richer language than* F*.*

There are two arguments for equilibria with richer language being more plausible than equilibria with poorer language. First, the relation of having a richer language can be viewed as a formalization of the notion that trusting is a focal policy, in the sense that the receiver believes in the largest set of messages consistent with rationality. Second, as it is shown in Section 5, equilibria with richer language are at least as stable with respect to inflows of new messages into the language as equilibria with poorer language.

Having a richer language is a binary relation. The relation suggests a candidate for refinement: the set of equilibria with maximally rich language. However, such equilibria may not exist if by maximally rich language we understand that no other equilibrium has richer or equally rich language.

Example 5 *Consider games where the payoff of S, $u_S(s, a)$, is independent of S's signal s and R's action a. Many of such games have multiple equilibria. It is easy to see that for every such a game and for every pair of its equilibria E and F, F has a richer language than E and E has a richer language than F .*

Further, I show the existence and characterize equilibria with maximally rich language if by maximally rich language we understand that no other equilibrium has strictly richer language. For each equilibrium E, define a class of equilibria

$$\mathcal{E} = \{F : \sup E = \sup F\}.$$

Proposition 3 *For every equilibrium E, equilibrium $F \in \mathcal{E}$ if and only if $U_S(\sigma_F, \alpha_F \mid s) = U_S(\sigma_E, \alpha_E \mid s)$ for every $s \in T$.*

Proof. If $U_S(\sigma_F, \alpha_F \mid s) \geq U_S(\sigma_E, \alpha_E \mid s)$ for every $s \in T$, then S weakly prefers playing σ_F to sending the messages that he sends in E. Thus, $\inf M_E \cup \inf M_F$ is an equilibrium set of messages for F. If $\inf M_E \cup \inf M_F$ is an equilibrium set of messages for F, then S always weakly prefers playing σ_F to sending the messages that he sends in E, and so $U_S(\sigma_F, \alpha_F \mid s) \geq U_S(\sigma_E, \alpha_E \mid s)$ for every $s \in T$.

That is, the following conditions are equivalent: (a) $\inf M_E \cup \inf M_F$ is an equilibrium set of messages for F, (b) $U_S(\sigma_F, \alpha_F \mid s) \geq U_S(\sigma_E, \alpha_E \mid s)$ for every $s \in T$. Thus, Proposition 3 follows from Proposition 2. ■

Definition 2 *Class $\mathcal{E}$ has a richer language than class $\mathcal{F}$ if E has a richer language than F.*

Class $\mathcal{E}$ has maximally rich language if no class $\mathcal{F} \neq \mathcal{E}$ has a richer language than $\mathcal{E}$.

Notice that the definition is independent of the choice of equilibria $E \in \mathcal{E}$ and $F \in \mathcal{F}$. The elements of the classes of equilibria with maximally rich language can be characterized as follows.

Definition 3 *Equilibrium E S-Pareto dominates equilibrium F if*

$$U_S(\sigma_E, \alpha_E \mid s) \geq U_S(\sigma_F, \alpha_F \mid s) \text{ for every } s \in T_S,$$

and

$$U_S(\sigma_E, \alpha_E \mid s) > U_S(\sigma_F, \alpha_F \mid s) \text{ for some } s \in T_S.$$

Equilibrium E is S-Pareto efficient if no equilibrium S-Pareto dominates E.

Theorem 4 *An equilibrium belongs to a class of equilibria with maximally rich language if and only if it is S-Pareto efficient.*

Proof. It follows from the equivalence of conditions (a) and (b) in the proof of Proposition 4. ■

Theorem 5 *For every game there exists a class of equilibria with maximally rich language.*

Proof. It follows from Theorem 4 and the fact that the set of equilibrium payoffs

$$\{(U_S(\sigma_E, \alpha_E \mid s))_{s \in T} : \ E \text{ is an equilibrium}\}$$

is a non-empty compact set. ■

Throughout the paper, I identify messages with the information about the sender's signal that they reveal and the receiver's response, i.e. with elements of $\Delta T \times \Delta A$.

One may find more appealing to identify messages only with the information about the sender's signal, i.e. with elements of ΔT. Moreover, identifying messages with elements of $\Delta T \times \Delta A$ implicitly assumes some generosity of the receiver. Indeed, suppose that both β and β' maximize the payoff of the receiver who believes that the sender's signal is distributed according to τ. If $m = (\tau, \beta), m' = (\tau, \beta') \in M_E$, then by choosing a messages, the sender chooses not only the information he wants to convey but, to some extent, also the receiver's response. In Appendix, I discuss an alternative notion of having a richer language, where messages are identified with the information about the sender's signal that they convey.

5 Equilibria Stable with respect to Inflows of New Messages

In this section, I look at the equilibria that are likely to prevail in dynamic processes with inflows of new messages. Several dynamic processes could be considered, but the choice of one of them would always be, to some extent, ad hoc. Thus, I shall adapt Rabin and Sobel's (1996) "sort of" dynamics. The idea is to characterize all strategies of both players that may appear along the path that follows an inflow of new messages, and then consider equilibria in those strategies.

Consider an equilibrium E, $\varepsilon > 0$, and a finite set of messages $N = \{m_1, ..., m_k\} \subset \Delta T \times \Delta A - \inf M_E$. Say, $m_i = (\tau_i, \alpha_i)$. Restrict attention to sets N such that there exists $\sigma^D : N \times T \rightarrow [0, 1]$, the sender's strategy, such that the distribution of the sender's types conditional on message m_i being sent and strategy σ^D being played is τ_i. Recall that $\sigma'_E : \inf M_E \times T \rightarrow [0, 1]$ represents the sender's strategy and $\alpha'_E : A \times \inf M_E \rightarrow [0, 1]$ represents the receiver's strategy in equilibrium E. Let $M = N \cup \inf M_E$, and let $\sigma_0 : M \times T \rightarrow [0, 1]$ denote the strategy, where the sender plays σ'_E with probability $1 - \varepsilon$ and σ^D with probability ε. Let $\alpha_0 : A \times M \rightarrow [0, 1]$ be defined by $\alpha_0(m) = \alpha'_E(m)$ for every $m \in \inf M_E$ and $\alpha_0(m_i) = \alpha_i$.

Define now sets $\Sigma_\infty(\varepsilon)$ and $\Theta_\infty(\varepsilon)$. Consider the following two cases: If for every $s \in T$ and $\sigma' : M \times T \rightarrow [0, 1]$, $U_S(\sigma'_E, \alpha_0 \mid s) \geq U_S(\sigma', \alpha_0 \mid s)$, then set $\Sigma_\infty(\varepsilon) =$

$\{\sigma_E^{/}\}$ and $\Theta_\infty(\varepsilon) = \{\alpha_E^{/}\}$. If $U_S(\sigma_E^{/}, \alpha_0 \mid s) < U_S(\sigma^{/}, \alpha_0 \mid s)$ for some $s \in T$ and $\sigma^{/} : M \times T \to [0,1]$, then define, by induction, sets $\Sigma_n(\varepsilon)$, $\Theta_n(\varepsilon)$, $n = 0, 1, ...$. Namely, let $\Sigma_0(\varepsilon) = \{\sigma_0\}$, $\Theta_0(\varepsilon) = \{\alpha_0\}$, let $\Sigma_{n+1}(\varepsilon)$ be defined by the following condition: A pure strategy $\sigma_{n+1} \in \Sigma_{n+1}(\varepsilon)$ if either $\sigma_{n+1} \in \Sigma_n(\varepsilon)$ or there is $\alpha^{/}$, which is a full-support convex combinations of all strategies from $\Theta_n(\varepsilon)$, such that σ_{n+1} is the sender's best response to $\alpha^{/}$. Similarly, let $\Theta_{n+1}(\varepsilon)$ be defined by the following condition: A pure strategy $\alpha_{n+1} \in \Theta_{n+1}(\varepsilon)$ if either $\alpha_{n+1} \in \Theta_n(\varepsilon)$ or there is $\sigma^{/}$, which is a full-support convex combinations of all strategies from $\Sigma_n(\varepsilon)$, such that α_{n+1} is the receiver's best response to $\sigma^{/}$ [Note that $\sigma^{/}$ assigns positive probabilities to all messages $m \in M$.] Sets $\Sigma_n(\varepsilon)$, $\Theta_n(\varepsilon)$ consists of pure strategies, and $\Sigma_n(\varepsilon) \subset \Sigma_{n+1}(\varepsilon)$, $\Theta_n(\varepsilon) \subset \Theta_{n+1}(\varepsilon)$. Therefore there exists an n such that $\Sigma_n(\varepsilon) = \Sigma_{n+1}(\varepsilon) = ...$, and $\Theta_n(\varepsilon) = \Theta_{n+1}(\varepsilon) = ...$ Let $\Sigma_\infty(\varepsilon)$ be the set of all convex combinations of strategies from $\Sigma_n(\varepsilon)$, and let $\Theta_\infty(\varepsilon)$ be the set of all convex combinations of strategies from $\Theta_n(\varepsilon)$.

In the dynamic interpretation, a population of agents initially plays equilibrium E. Their language consists of elements of $\inf M_E$. At some point, the fraction ε of senders sends new messages, elements of $\Delta T \times \Delta A - \inf M_E$, according to σ^D; receivers initially respond to these new messages with $\alpha_0(m_i) = \alpha_i$. In a more complete story, the fraction of senders can mistakenly think that they play a different game. If these new messages create no profitable deviation from the equilibrium strategy for any type of the sender, then senders ignore new messages and players continue to play E. Otherwise, each type of the sender may switch to his best response to α_0, and the receiver may switch to her best response to σ_0[3]. These best responses form sets $\Sigma_1(\varepsilon)$ and $\Theta_1(\varepsilon)$. Then players observe that their opponents play strategies from $\Sigma_1(\varepsilon)$ and $\Theta_1(\varepsilon)$, respectively; then senders may switch to strategies from $\Sigma_2(\varepsilon)$, the best responses to strategies from $\Theta_1(\varepsilon)$, and receivers may switch to strategies from $\Theta_2(\varepsilon)$, the best responses to strategies from $\Sigma_1(\varepsilon)$. Thus, $\Sigma_\infty(\varepsilon)$ and $\Theta_\infty(\varepsilon)$ can be

[3] I could alternatively assume that only those types of the sender who can profitably deviate from the equilibrium strategy may switch to one of their best responses to α_0. It would not change the results.

interpreted as strategies that may appear over time in the dynamic process initiated by the inflow of messages $m_1, ..., m_k$. The assumption that σ_{n+1} and α_{n+1} are the best responses to full-support combinations of strategies from $\Theta_n(\varepsilon)$ and $\Sigma_n(\varepsilon)$ means that the receiver assigns a positive probability to every strategy of the sender which has been used in the past.

Let $B(\varepsilon, E, m_1, ..., m_k)$ stand for the set of Bayesian equilibria $(\sigma, \alpha) \in \Sigma_\infty(\varepsilon) \times \Theta_\infty(\varepsilon)$. Further, let $B(E, m_1, ..., m_k)$ be the limit of sets $B(\varepsilon, E, m_1, ..., m_k)$ when $\varepsilon \to 0$, i.e. $B(E, m_1, ..., m_k) = \{(\sigma, \alpha) \in \Sigma \times \Theta$: there exist ε_n and $(\sigma_n, \alpha_n) \in B(\varepsilon_n, E, m_1, ..., m_k)$ such that $\varepsilon_n \to 0$ and $(\sigma_n, \alpha_n) \to (\sigma, \alpha)\}$. Of course, elements of $B(E, m_1, ..., m_k)$ are Bayesian equilibria[4].

Definition 4 *(a) An equilibrium E is quasi-stable if $E \in B(E, m_1, ..., m_k)$ for every finite set of messages $m_1, ..., m_k$.*

(b) If additionally $B(E, m_1, ..., m_k) = \{E\}$, then equilibrium E is stable.

(c) A set of equilibria $\mathcal{E} \neq$ is absorbing if $B(E, m_1, ..., m_k) \subset \mathcal{E}$ for any $E \in \mathcal{E}$ and $m_1, ..., m_k$.

(d) A set of equilibria $\mathcal{E}$ is recurent if $\mathcal{E}$ is absorbing and no proper subset $\mathcal{F} \subset \mathcal{E}$ is absorbing. Recurrent equilibria are the members of recurrent sets.

The intuition behind stable and quasi-stable equilibria is straightforward. To interpret recurrent equilibria consider a process where agents initially play some E. An inflow of new messages $m_1, ..., m_k$ initiates the transition from E to an equilibrium $F \in B(E, m_1, ..., m_k)$ [Theorem 6 shows that $B(E, m_1, ..., m_k)$ is a non-empty set]. When the transition comes to the end and agents play (at least approximately) F, another inflow of new messages $n_1, ..., n_k$ initiates the transition from F to an equilibrium from $B(F, n_1, ..., n_k)$. Then the recurrent equilibria can be interpreted as the equilibria that appear repeatedly over time in this process.

Theorem 6 *Let G be a sender-receiver game with a finite set of messages M.*

[4] In the interpretation, the limit when $\varepsilon \to 0$ captures the idea of the inflow of new messages, or the fraction of players who mistakenly think that they play a different game, being infinitesimally small.

(i) For every equilibrium E and messages $m_1, ..., m_k \in \Delta T \times \Delta A - \inf M_E$, $B(E, m_1, ..., m_k) \neq$.

(ii) There exists a recurrent set.

(iii) $\mathcal{F} \cap \mathcal{E} =$ if $\mathcal{F}$ and $\mathcal{E}$ are recurrent and $\mathcal{F} \neq \mathcal{E}$.

Proof. (**i**)By definition, $\Sigma_\infty(\varepsilon)$ and $\Theta_\infty(\varepsilon)$ are convex subsets of Σ and Θ. Again, by definition, for every full-support combination $\sigma \in \Sigma_\infty(\varepsilon)$ of the strategies from $\Sigma_n(\varepsilon)$, $\Theta_\infty(\varepsilon)$ contains the receiver's best response to σ from Θ. Since every $\sigma \in \Sigma_\infty(\varepsilon)$ is the limit of a sequence of full-support combinations of the strategies from $\Sigma_n(\varepsilon)$, $\Theta_\infty(\varepsilon)$ contains the receiver's best response to σ from Θ. Similarly, for every $\alpha \in \Theta_\infty(\varepsilon)$, $\Sigma_\infty(\varepsilon)$ contains the sender's best response to α from Σ. By Kakutani's Fixed Point Theorem, $B(\varepsilon_n, E, m_1, ..., m_k)$ is a non-empty set. Therefore, by compactness of Σ and Θ, so is $B(E, m_1, ..., m_k)$.

(**ii**) It follows from the following four observations: (1) every absorbing set is a compact subset of the set of all pairs (σ, α), where $\sigma : M \times T \to [0,1]$ and $\alpha : A \times M \to [0,1]$; (2) the intersection of a linearly ordered family of non-empty compact sets is a non-empty compact set; (3) the set of all equilibria of G is an absorbing set, and (4) the intersection of absorbing sets is an absorbing set.

(**iii**) It follows from (4). ■

In Example 6, I apply the concept of stability to the games from Examples 1 and 2.

Example 6 *Consider the pooling equilibrium E from Example 1, and messages $m_1 = (s_1, a_1)$, $m_2 = (s_2, a_2)$. Then for $n = 0, 1, ...$ and sufficiently small $\varepsilon > 0$, $\Sigma_n(\varepsilon) = \{\sigma_0, \sigma_{TRUTH}\}$, σ_{TRUTH} denotes the strategy where the sender with signal s_1 sends m_1 and the sender with signal s_2 sends m_2, $\Theta_n(\varepsilon) = \{\alpha_0, \alpha_1, \alpha_2\}$, α_i denotes the strategy where the receiver responds with a_1 to m_1, a_2 to m_2 and plays a_i in response to the unique message $m \in \inf M_E$ [$m = (\tau, \alpha)$, where $\alpha(a_1) = \alpha(a_2) = 1/2$, $\tau(s_1) = \tau(s_2) = 1/2$]. Indeed, σ_{TRUTH} is the best response to every convex combination of $\alpha_0, \alpha_1, \alpha_2$ with full support; for every full-support convex combination of σ_0 and σ_{TRUTH}, a_1 is the best response to m_1 and a_2 to m_2 and both a_1 and a_2 give the receiver the same payoff if the sender sends m.*

Therefore $B(E, m_1, m_2)$ consists of separating equilibria, where the sender plays σ_{TRUTH}, and the receiver responds with a_1 to m_1, a_2 to m_2. These equilibria are identified with F. Consider now the separating equilibrium F. Since each type of the sender obtains the highest possible payoff in F, $B(F, m_1, m_2) = \{F\}$ for every $\{m_1, ..., m_k\} \subset \Delta T \times \Delta A - \inf M_F$. This means that F is a unique stable equilibrium of the pure common-interest game from Example 1.

Consider the separating equilibrium F from Example 2. Let $m_1 = (s_1, a_1)$, $m_2 = (s_2, a_2)$ denote elements of $\inf M_F$, and $m = (\tau, a_3)$, where $\tau(s_1) = \tau(s_2) = 1/2$. Then for $n = 0, 1, ...$ and sufficiently small $\varepsilon > 0$, $\Sigma_n(\varepsilon) = \{\sigma_0, \sigma_{POOL}\}$, σ_{POOL} denotes the strategy where the sender sends m independently of his signal, $\Theta_n(\varepsilon) = \{\alpha_0, \alpha\}$, α denotes the strategy where the receiver responds with a_1 to m_1, a_2 to m_2 and plays a_3 in response to m. Therefore $B(E, m_1, m_2)$ consists of the pooling equilibrium, where the sender plays σ_{POOL} and the receiver plays α.

Consider now the pooling equilibrium E. Since each type of the sender obtains the highest possible payoff in E, $B(E, m_1, ..., m_k) = \{E\}$ for every $\{m_1, ..., m_k\} \subset \Delta T \times \Delta A - \inf M_E$. This means that E is a unique stable equilibrium of the game from Example 2.

The observation from Example 6 can be generalize as follows:

Definition 5 *Equilibrium $E = (\sigma_E^{/}, \alpha_E^{/})$ has highest S-payoffs if $U_S(\sigma_E^{/}, \alpha_E^{/} \mid s) \geq u_S(a, s)$ for all $s \in T$ and $a \in A$.*

That is, each type of the sender obtains in E the highest possible payoff.

Proposition 7 *(i) Every equilibrium E with highest S-payoffs is stable.*

(ii) If, additionally, no other equilibrium has the highest S-payoffs, then E is a unique stable equilibrium, and $\{E\}$ is a unique recurrent set.

Example 7 shows that both stable and quasi-stable equilibria may not exist.

Example 7 *Consider a slightly modified game from Rabin-Sobel (1996), Figure 5:*

$$\begin{array}{cccccccc} & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ s_1 & 0,0 & 5,4 & 4,4 & 1,5 & 0,0 & 0,0 & 3,3 \\ s_2 & 4,4 & 0,0 & 5,4 & 0,0 & 1,5 & 0,0 & 3,3 \\ s_3 & 5,4 & 4,4 & 0,0 & 0,0 & 0,0 & 1,5 & 3,3 \end{array}.$$

The game has a pooling equilibrium E_P, where R takes action a_7 independently of S' message, a separating equilibrium E_S, where R responds by action a_{i+3} to message s_i, and three equilibria (E_i, $i = 1,2,3$) that involve partial pooling. In E_i, type s_i reveals his signal truthfully inducing action a_{i+3}, and the other two types pool inducing action a_i.

Of course, E_S is S-Pareto dominated by E_P. I shall show that no equilibrium is quasi-stable and the set of all equilibria is recurrent[5]*. Let $m_i = (s_i, a_{i+3})$, $i = 1,2,3$, $m_i = (\tau_i, a_i)$, where $\tau_i(s_j) = 1/2$ for $j \neq i$, and $m_P = (\tau, a_7)$, where $\tau(s_i) = 1/3$ for $i = 1,2,3$. It can be easily shown that:*

(1) $B(E_S, m_P) = \{E_P\}$,

(2) for every i, $B(E_P, m_i) = \{E_i\}$,

(3) for any $i \neq j$, $B(E_i, m_j) = \{E_j\}$,

(4) $E_S \in B(E_1, m_2, m_3)$.

(1)-(3) yield that no equilibrium is quasi-stable and (1)-(4) yield that the set of all equilibria is recurrent. Intuitively, E_P is not quasi-stable, because any two types prefer to separate from the third type, and they do so whenever there is a message that enables them to do so. However, s_1 prefers to pool only with type s_3. Similarly, type s_3 prefers to pool only with type s_2, and type s_2 only with type s_1. Because of these conflicting interests, for some configuration of available messages, s_1 and s_3 pool together, separating themselves from s_1, for other configurations, s_3 and s_2 or s_2 and s_1 pool together. Moreover, the sender may end up with the full revelation of his

[5]It can be easily checked that the separating equilibrium is neither neologism-proof nor announcement-proof, and it can be shown, in a similar manner to Rabin-Sobel (1996), that the separating equilibrium is a recurrent mop.

signal. Finally, E_S is not quasi-stable, because any all types prefer to pool together, and they do so whenever there is a message that enables them to do so.

As it has been announced in Section 4, I shall show that equilibria with richer language have (weakly) better stability properties than equilibria with poorer language.

Proposition 8 *If equilibrium E S-Pareto dominates equilibrium F and F is in a recurrent set, then so is E.*

Proof. It has to be shown that if equilibrium $E = (\sigma_E, \alpha_E)$ S-Pareto dominates equilibrium F, then there exists messages $m_1, ..., m_k \in \Delta T \times \Delta A - \inf M_F$ such that $E \in B(F, m_1, ..., m_k)$. Let $\{m_1, ..., m_k\} = \inf M_E - \inf M_F$. Then $\Sigma_1(\varepsilon)$ contains all pure strategies σ satisfying the following condition:

$$\text{For every } s \in T, \text{ there exists } m \in \inf M_E \text{ with } \sigma_E^/(m, s) > 0 \text{ and } \sigma(m, s) = 1. \quad (4)$$

Similarly, $\Theta_1(\varepsilon)$ contains all pure strategies α satisfying the following condition:

$$\text{For every } m \in \inf M_E, \text{ there exists } a \in A \text{ with } \alpha_E^/(a, m) > 0 \text{ and } \alpha(a, m) = 1. \quad (5)$$

To show that $\sigma_E^/ \in \Sigma_\infty(\varepsilon)$ and $\alpha_E^/ \in \Theta_\infty(\varepsilon)$ it suffices to represent $\sigma_E^/$ as a convex combination of strategies satisfying (4), and $\alpha_E^/$ as a convex combination of strategies satisfying (5). $\sigma_E^/$ and $\alpha_E^/$ are behavior strategies. Since behavior and mixed strategies are equivalent in games of perfect recall, $\sigma_E^/$ and $\alpha_E^/$ can be represented as convex combinations of pure strategies. Those pure strategies have to satisfy (4) and (5), respectively, because for other pure strategies, either $\sigma(m, s) = 1$ when $\sigma_E^/(m, s) = 0$ or $\alpha(a, m) = 1$ when $\alpha_E^/(a, m) = 0$. ■

Proposition 7 implies that only S-Pareto efficient equilibria may be stable, and Example 7 shows that S-Pareto dominated equilibria may also belong to recurrent sets.

6 Comparison with Other Refinements

Standard refinements, e.g. Cho and Kreps' (1987) intuitive criterion, fail to refine the set of sequential equilibria in cheap-talk games. The intuition is that these refinements put restrictions on the beliefs at out of equilibrium information sets. However, every equilibrium outcome in cheap-talk games can be attained with no out of equilibrium information sets. Blume (1993) studies the effectiveness of perturbation refinements (e.g., Selten's perfection, Myerson's properness, Kohlberg and Merten's strategic stability, and Kalai and Samet's persistence). These criteria either do not refine away the pooling equilibrium in the pure common-interest game from Example 1 or refine away the pooling equilibrium, instead of separating equilibria, in the game from Example 2.

Farrell (1993) proposes a refinement designed for cheap-talk games. Farrell takes as an axiom that some messages are credible and believed. To see this idea, consider a cheap-talk equilibrium, and suppose that the receiver will believe in message "$s \in K$" for some $K \subset T$. Farrell defines such a message as credible when the sender's payoff to sending the message exceeds his equilibrium payoff if and only if indeed $s \in K$. He calls an equilibrium neologism-proof if there are no neologisms (credible messages "$s \in K$"), and he argues that neologism-proof equilibria are more plausible. To this end Farrell, similarly to the present paper, assigns an important role to a rich, common language, although in an informal sense. Namely, he argues that a rich, common language always enables the sender to send neologisms. Matthews et al (1991) apply similar logic to Farrell to define announcement-proof equilibria. The difference is that Matthews et al rather than a criterion for the credibility of single messages consider a criterion for the credibility of sets of messages. None of the two refinements guarantees existence. That is, it happens that no equilibrium is neologism- (announcement-) proof. There are S-Pareto dominated, neologism- and announcement-proof equilibria, which seem rather counter-intuitive (see Matthews et al (1991), games Γ_3 and Γ_7). Finally, the two criteria are not immune to the Stiglitz critique. The critique argues against the inconsistent logic behind those criteria. Namely, suppose that the receiver believes that some type of the sender sends a neologism. Then she rather does not respond to the equilibrium messages according to the equilibrium strategy, because she knows that those messages can come only from the other types.

Rabin and Sobel (1996) make a point similar to the Stiglitz critique. They argue that although neologism-proof and announcement-proof criteria identify equilibria susceptible to deviations, those criteria do not specify what the players are likely to do when they deviate. Rabin and Sobel propose to refine away only those equilibria that are susceptible to deviations (in the sense of Matthews et al), and that are not played repeatedly along the path that follow such a deviation. That is, the equilibria that do not persist in the long run. They call the equilibria that do persist in the long-run recurrent mops. Recurrent mops always exist and are immune to the Stiglitz critique.

Rabin's (1990) credible-message rationalizability and credible-message equilibria, and Zapater's (1997) proposal-proofness apply a similar logic to Farrell (1993), though in the spirit of rationalizability. Like I formalize the notion that trusting is a focal policy, those theories formalize the notion that honesty is a focal policy. Both refinements also guarantee existence and are immune to the Stiglitz critique.

The "Farrell's type" refinements seem to be successful in games with pure strategy equilibria. I do not know any game, which has only pure strategy equilibria, and those equilibria that seem implausible are not refined away by any "Farrell's type" refinement. Those refinements, however, seem much less successful when the some equilibria are in mixed strategies. Here is an example:

Example 8 *Consider now a slightly modified game from Zapater (1997), Example 5.1:*

$$\begin{array}{ccccc} & a_1 & a_2 & a_3 & a_4 \\ s_1 & 6,6 & 0,4 & 1,4 & 0,0 \\ s_2 & 0,4 & 6,6 & 1,4 & 0,0 \\ s_3 & 2,1 & 2,1 & 1,4 & 0,6 \end{array}.$$

The game has a pooling equilibrium E_P, where the receiver always takes action a_3, and semi-separating equilibria, where the sender with s_1 and s_2 reveals his signals truthfully, whereas the sender with s_3 randomizes between messages "I have s_1" and

"I have s_2", e.g., sends each of the two with probability $1/2$. Denote by E_{SS} the semi-separating equilibrium where the sender with s_3 randomizes with probability fifty-fifty.

Consider the pooling equilibrium. It can be easily checked that no announcement is credible according to Matthews et al. Therefore E_P is neologism-, announcement-proof, and it is a recurrent mop. It can also be showed, in a similar manner to Zapater, Example 5.1, that the pooling equilibrium is proposal-proof.

Of course, E_P is S-Pareto dominated by E_{SS}. I shall show that E_{SS} is stable whereas E_P belongs to no recurrent set. Let m_P, and m_i, $i = 1, 2$, denote, respectively, the pooling message responded by action a_3, and (s_i, a_i). Then for $n = 0, 1, ...$ and sufficiently small $\varepsilon > 0$, $\Sigma_n(\varepsilon) = \{\sigma_0, \sigma_{13}, \sigma_{23}\}$, where σ_{i3} denote the strategy where the sender with signals s_i and s_3 sends m_i, and the sender with signal s_j, $j \in \{1, 2\} - \{i\}$, sends m_j; $\Theta_n(\varepsilon) = \{\alpha_0, \alpha_1, \alpha_2\}$, α_i denotes the strategy where the receiver responds with a_i to m_i, and plays a_3 in response to m_j and m_P. Indeed, every strategy of the receiver, which is a full-support convex combination of $\alpha_0, \alpha_1, \alpha_2$, responds with a_3 to m_P and randomizes between a_i and a_3 in response to m_i, $i = 1, 2$. Therefore the sender's best response to the receiver's strategy is sending m_i whenever he has s_i and sending m_i, which gives a higher probability of response a_i whenever he has s_3, i.e. the sender's best response is σ_{13} or σ_{23}. Consider now a full-support convex combination of $\sigma_0, \sigma_{13}, \sigma_{23}$, $w_0\sigma_0 + w_{13}\sigma_{13} + w_{23}\sigma_{23}$. If $w_{13} \geq w_{23}$, then the probability of the sender having a_2 conditional on message m_2 being used is at least 2/3, and so the receiver's best response is a_2. The receiver's best response to m_P is a_3, because m_P is used with positive probability only under σ_0. Finally, the receiver's best response to m_1 is either a_1 or a_3, because m_1 is used by the sender with both s_1 and s_3 under σ_{13} and by the sender with s_1 under σ_0 and σ_{23}, and consequently, the probability of the sender having a_1 conditional on message m_1 being used is at least 1/2. Thus, the receiver's best response to $w_0\sigma_0 + w_{13}\sigma_{13} + w_{23}\sigma_{23}$ is α_0 or α_2.

Therefore $B(E_P, m_1, m_2) = \{E_{SS}\}$. Since each type of the sender obtains the highest possible payoff in the semi-separating equilibrium, $B(E_{SS}, m_1, m_2) = \{E_{SS}\}$ for every $\{m_1, ..., m_k\} \subset \Delta T \times \Delta A - \inf M_{E_{SS}}$.

Other papers (e.g. Blume et al (1993) and Wärneryd (1993)) argue that evolutionary pressures may force populations to interpret messages in systematic ways. They suggest narrowing the set of equilibria to evolutionary stable outcomes. Although better motivated, evolutionary approach is less powerful. It can guarantee meaningful communication in the pure common-interest game from Example 1, but it fails to guarantee no-communication outcome in the pure conflict game from Example 2 unless messages are costly.

Blume and Sobel (1995) provide yet another idea. They give the sender an additional opportunity to communicate: After the sender sends a message and the receiver forms her beliefs but before the receiver takes an action, the sender can send another message. In their model, this additional opportunity to communicate causes that some equilibria trump other equilibria. They call the equilibria that belong to the stable set with respect to their trumping relation communication-proof. The additional opportunity to communicate can trump pooling equilibrium in pure common-interest game from Example 1, but it cannot prevent information already communicated from being know, and so it cannot trump separating equilibria in the game from Example 2.

Finally, it should be mentioned that there are also implausible S-Pareto efficient equilibria, which are refined away by Farrell and others.

Example 9 *Consider the following game:*

$$\begin{array}{cccc} & a_1 & a_2 & a_3 \\ s_1 & 3,3 & 0,0 & 2,2 \\ s_2 & 0,0 & 1,3 & 2,2 \end{array}.$$

The game has a pooling and a separating equilibrium. The pooling equilibrium is neither neologism- nor announcement-proof, because message "I have s_1" is credible. One can also show that the pooling equilibrium is not a recurrent mop, and that credible-message criteria and proposal-proofness refine away the pooling equilibrium.

It follows from Proposition 9, however, that the separating equilibrium of the game from Example 9 is stable with respect to the inflows of new messages, and the pooling equilibrium does not belong to any recurrent set. Note also that there are natural, common generalizations of some "Farrell's type" refinements and the notion of having a richer language.

Consider, for example, Rabin's (1990) credible message equilibria. To define these equilibria Rabin places restrictions on the set of permitted strategies, and restricts attention to equilibria in permitted strategies. Let Σ_S and Σ_R stand for this set of permitted strategies. If we agree with Rabin, then we should restrict attention to the set of messages generated by strategies from Σ_S and Σ_R. More precisely, consider

$$\overline{M} = \{i(m) \in \Delta T \times \Delta A : m \in M,\ (\sigma, \alpha) \in \Sigma_S \times \Sigma_R\},$$

where $i(m)$ is defined by (1)-(3), and equilibria E such that $\inf M_E \subset \overline{M}$. Say that E has a richer credible language than F if $\overline{M} \cap \sup M_F \subset \overline{M} \cap \sup M_E$.

Example 10 *By Rabin (1990), Example 2, the only credible message equilibrium of the game from Example 9 is the separating equilibrium. Therefore the separating equilibrium has a richer credible language than the pooling equilibrium. Consider now a slightly modified game from Rabin (1990), Example 5:*

$$\begin{array}{ccccc} & a_1 & a_2 & a_3 & a_4 \\ s_1 & 4,2 & 3,3 & 1,0 & 0,2 \\ s_2 & 3,3 & 4,2 & 1,0 & 0,2 \\ s_3 & 1,0 & 1,0 & 3,3 & 0,2 \end{array}.$$

The game has a pooling equilibrium $E_P = (\sigma_P, \alpha_P)$, where the receiver takes action a_4 independently of the sender's message, and a semi-separating equilibrium $E_{SS} = (\sigma_{SS}, \alpha_{SS})$, where s_3 reveals his signal truthfully, whereas s_1 and s_2 send message "I have either s_1 or s_2". The receiver responds with a_3 to type s_3's message and she randomizes between a_1 and a_2 (with equal probabilities) in response to "I have either s_1 or s_2". It can be shown, in a similar manner to Rabin (1990), Example 5, that $(\sigma_P, \alpha_P), (\sigma_{SS}, \alpha_{SS}) \in \Sigma_S \times \Sigma_R$, and so both E_P and E_{SS} are credible message equilibria. However, it seems implausible that types s_1 and s_2 are unable to separate *themselves from s_3. Since $(\sigma_P, \alpha_P), (\sigma_{SS}, \alpha_{SS}) \in \Sigma_S \times \Sigma_R$, $\inf M_{E_P}, \inf M_{E_{SS}} \subset \overline{M}$; since E_{SS} S-Pareto dominates E_P, $\sup M_{E_P} \subset \sup M_{E_{SS}}$. Thus, E_{SS} has a richer credible language than E_P.*

7 Appendix

I present an alternative notion of having a richer language, where messages are identified with the information about the sender's signal that they convey. One possibility would be to say that equilibrium E has a richer language than equilibrium F if $\pi_1(\sup M_F) \subset \pi_1(\sup M_E)$, where π_1 stands for the projection of $\Delta T \times \Delta A$ onto ΔT. However, if the receiver responds differently to some message from $\pi_1(\sup M_F)$ when E is played and when F is played, then there is no reason to think that equilibrium E is more plausible than equilibrium F. Therefore I propose a slightly different, alternative notion of having a richer language[6].

Definition 6 *Equilibrium E has a strongly richer language than equilibrium F if $M_E = \sup M_F \cup \{m = (\tau, \beta) \in \inf M_E$: there is no $m' = (\tau', \beta') \in \sup M_F$ with $\tau' = \tau\}$ is an equilibrium set of messages for E.*

To interpret Definition 5 note that $\pi_1(\sup M_F) \subset \Delta T$ is the largest set of available messages (identified with the information about the sender's signal that they convey) that that can be available when equilibrium F is played. Definition 5 says that equilibrium E has a strongly richer language if there is a larger set of messages, $\pi_1(M_E)$ with $\pi_1(\sup M_F) \subset \pi_1(M_E) \subset \Delta T$, that can be available when equilibrium E is played, and such that the receiver responses to messages from $\pi_1(\sup M_F)$ when E is played coincide with those when F is played.

Proposition 3 shows that having a strongly richer language implies having a richer language. Example 3 shows that some equilibria may have a richer language, but not a strongly richer language, than others.

[6] If messages are identified with the information about the sender's signal (elements of ΔT), then there are several other alternative definitions of having a richer language. The differences between these definitions are rather subtle, and because of that, I restrict the discussion to Definition 5.

Proposition 9 *If E has a strongly richer language than F, then E has a richer language than F.*

Proof. It follows from Definition 5 and Proposition 2 as $\inf M_E \cup \inf M_F$ is contained in M_E from Definition 5, and so it is an endogenous set of messages for E. ■

Example 11 *Consider the following game considered by Rabin (1990):*

$$\begin{array}{ccccc} & a_1 & a_2 & a_3 & a_4 \\ s_1 & -1,3 & -2,0 & 0,2 & -3,2 \\ s_2 & -2,0 & -1,3 & 0,2 & -3,2 \end{array}.$$

The game has a separating equilibrium and a pooling equilibrium. The pooling equilibrium where R takes action a_3 has a richer language than the separating equilibrium. One can argue that the separating equilibrium is at least as plausible as the pooling equilibrium, because R may wish to penalize S who does not reveal his signal, and she can do so in a manner costless for herself, by action a_4.

One readily checks the pooling equilibrium where R takes action a_3 has a richer language than the separating equilibrium, but not a strongly richer language.

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