Polynomials and Factoring

Unit Lesson Plan

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Abstract

This paper will discuss, and give, lesson plans for all the topics included with polynomials and factoring for an Algebra 1 class. The notes, homework, and all class activities will be written out and explained.

This lesson plan will be discussed in order that it should be presented and will take into account all major issues with the chapter the lesson is on, which will be given before the lesson plan itself.

The first section of the paper will deal with the overlying generalities between all the lessons. This section will discuss the common misconceptions with polynomials and factoring, along with the standards that this paper is aimed at completing.
Overview

Discussion about topic:

In Algebra 1 the topic of Polynomials and Factoring is a very important in many aspects. Understanding polynomials along with how to factor them lays the ground work for a multitude of different topics. To fully understand any higher level Math one must have a firm background in Algebra. A weak understanding in Algebra is usually what causes most people to fear and hate math.

Throughout the entire unit all three main types of learning (Kinesthetics, Audio, and Visual) will be implemented into the teaching strategy.

The homework will be short in length but difficult to answer in terms of content. The student will have to fully understand the content of the day’s lesson to answer the homework fully. This is how it will be for all sections.

Motivation:

The motivation behind learning any math is the real world applications of it. Every math has some type of real world application; some are just harder to see than others.

The other major motivation is the way of thinking that math instills. The logic behind all mathematics is what makes math useful in everyday life. The purpose of a math class, in my opinion, should stress the logical way of thinking instead of “learn this for the EOG exams”.
The reason why students have trouble with math is their way of thinking. They just learned the material on the exam then forgot. If the students learn how to think mathematical then it is highly unlikely that they will forget the material.

It is also the teachers fault when the student fails to learn the mathematical way of thinking. Either he/she did not put enough emphasis on how to think about the problems or didn’t make them think at all. There are several different teaching strategies that facilitates the students to the adoption of the new mind set.

Misconceptions:

The common misconceptions about polynomials are, or rather is, which equations really are polynomials. Many people believe that any equation without a radical, exponential, or logarithmic operation or an equation with an imaginary number, in it is a polynomial. This is very wrong. The simple function one divided by x is not a polynomial. No rational function is a polynomial. The idea of a negative exponent is a topic of discussion that should be addressed during the explanation of the definition of a polynomial. Many students go on thinking that rational functions are a subset of the polynomials. This train of thought will lead to the misconceptions of many things, such as taking one divided by x to be a continuous function.

The common misconception with factoring is the fact that factoring itself is a form of simplification. It is a method of taking high degree polynomials and turning them into lesser degree polynomials. Most of the students in high school, from what I seen observing, had trouble understanding why they got the problem wrong with the right answer. The reason they
got the problem marked incorrect is because they did not simplify the answer they gave. Most students, again from my experience, think simplifying is having either a numerical answer or an equation with all like terms added. The students forget that having a factorable equation means the simplest form is the factored state. This misconception is not an overbearing problem. It just makes the answer rougher to look at.

Assessment:

The assessment of the unit will be broken down as follows:

~5 quizzes (given at the beginning of class: 5mins)

~1 exam (given on the eighth day)

~7 days of graded homework

While the quizzes and homework will be graded with a holistic rubric, the exam will be on an analytic rubric. For any of the three (quiz, homework, or exam) each answer should have a sentence or two written about why this particular student “knows” this is the correct answer. If no explanation is present with a given answer only half of the total credit for that problem can be obtained by said student.

Standards:
This unit is aimed at completing the first objective of the NC Standards Course of Study for Algebra.

**Number and Operations Competency**

**Goal 1:** The learner will perform operations with numbers and expressions to solve problems.

**Objectives 1.01:** ~ Write equivalent forms of algebraic expressions to solve problems.

~ Apply the laws of exponents.

~ Operate with polynomials.

~ Factor polynomials.

1.02: ~ Use formulas and algebraic expressions, including iterative and recursive forms, to model and solve problems.

1.03: ~ Model and solve problems using direct variation
Lesson 1: Classification of Polynomials

This lesson plan will utilize two main types of teaching: Discovery and Inquiry. This lesson is designed so the students figure out what the classification system is themselves. Since this chapter of Algebra 1 is nearly always taught in a traditional style, this lesson plan will try to diverge from the norm and do something different to try and simulate the student’s thoughts.

This will also incorporate cooperative learning in the lesson plan, as the majority of the class will be spent in groups.

Since this lesson is highly different there will be no time in the class for traditional notes. This is why there should be a handout ready for the students after the lesson that will remind them of what they, hopefully, came up with that day.

Before the class beings the manipulatives that will be used in the lesson should already be cut out and ready for the students when they get to class. The manipulatives for this lesson are only little flash cards with different polynomials on them.

Also, note that there are times beside each part of the lesson plan. These times assuming a 50min class block so that all these activities together will fill the block.

Lesson Plan 1:

-Break class up into groups of 2 or 3 and hand out the manipulatives, as well as material to make the posters, while explaining what the exercise is and what is expected of them, also telling them they will present their findings to the class afterwards. (5mins)

~The Exercise~ (10mins)
At this point in the class we would not have talked about polynomials and the common ways we group them together. The students should, however, have a firm grasp of what constants and variables are. The idea behind the exercise will be for the student to come up with their own method of grouping the polynomials together. There will be a couple of questions instructing them on what to do.

-The students will present their different classification systems. Not every group might get to present but everyone’s work should be put on a poster to go on the wall. (15mins)

~After the students present their own methods of classifications the class as a whole will talk about and discusses the highlights of each group. This should lead into the common system of classification of polynomials, if not the teacher should interject leading questions to push them towards the correct answer. (15mins)

~The remaining five minutes of class should be used to give the nights homework, hand out the notes typed up for the students, and to clear up any questions that might still be left. (5mins)

Students Handout Sheet: (Notes for student)

~Terms: are the pieces that the polynomial is actually made of. Terms may consist of constants, variables, or a combination of the two. The only restrictions on what a term can be are:

1. The Power of each Variable must be a non-negative, whole number.

2. Each constant used must be either an integer or a rational number.
Now some polynomials have the same number of terms, while others have different amounts of terms in them. We have groups for the polynomials with the same number of terms in them. Below are the common names for them.

One Term Polynomial  ---------  Monomial
Two Term Polynomial  ---------  Binomial
Three Term Polynomial  ---------  Trinomial
Four Term Polynomial  ---------  Fourth term Polynomial
X Term Polynomial  ---------  X Term Polynomial

(Where X can be any natural number, except 0)

If there is a common term for a four term polynomial, then I have not heard of it. Usually higher termed polynomials are just referred to as "Blank" Term Polynomial, where blank is the amount of terms in the polynomial.

~Coefficients: are the constants multiplied by a variable.

~Variables: are the unknowns in the expression. They can equal any number, which means they vary from time to time (hints the name: variable).

~Degree: of a polynomial is the highest exponent of all the terms in the expression. If a term has more than one variable you must add the powers of the two variables in order to get the degree of that term.

~Names of degrees:

Sometimes polynomials are named by their degrees along with the number of terms; so that a second degree trinomial would be called a quadratic trinomial.
The **standard form** of a polynomial is:

1. *All term with coefficients and variables must have the coefficient first*
2. *You must arrange all terms according to their degree from highest to lowest*
3. *The leading coefficient must be positive!*
4. *All like terms must be simplified*

Homework for Lesson 1:

Answer the following questions with an explanation of why your answer is correct.

1. Write out a standard form cubic trinomial.

**Answer:** $x^3+x^2+1$, it is cubic because of the power on the leading term and it has three terms making it a trinomial.

2. How many constants can you have in any given polynomial if it is in standard form?

**Answer:** 1, you must add all like terms in order for a polynomial to be in standard form which means only 1 constant can be present in standard form.

3. How many coefficients can you have in any given polynomial if it is in standard form?

(Hint for 3: Think of this in terms of the highest exponent)

**Answer:** The maximum number of coefficient in polynomials is the power of the highest variable – 1. The minimum is 1.

4. Explain why $1/x$ is not a polynomial.
**Answer:** From the definition of a polynomial, all variables must have a non-negative, whole number exponent. $1/x$ has a negative exponent.
Lesson 2: Adding and Subtracting Polynomials

This lesson plan will use two main types of teaching; Scaffolding and Inquiry. This lesson plan will utilize certain examples in order to get the students to think of the subject in depth.

This lesson plan will not use cooperative learning, but will encourage group discussion. This type of lesson plan can easily flip from being scaffolding to discovery, that is if the students are a talkative bunch and like to discuss things among their peers.

The notes for this lesson come from the discussion and the students are expected to take notes as they considered necessary. This should be made clear to the students at the start of class.

There will be a few examples put on the board or written in a power point. These will be very easy examples aimed at getting the students to start thinking about the material in depth.

The majority of the class time should be spent discussing subtraction of polynomials. This topic will lead into the next day’s lesson plan of distribution and polynomials. The students should have a firm grasp of this concept before they leave the class.

This plan has a short quiz at the beginning of the class period on the material of lesson one.

Lesson Plan 2:

Before the class being, be sure to have examples already thought out and ready to present to class. The examples need not be difficult and should actually start with very simple
addition, such as 2+2. The reason behind such simple examples is for the student to realize that you are adding like terms. The quiz should not take more than five minutes of the class time and should only be three questions long.

~Start of class; give the quiz on previous day’s lesson. After the quiz the class should begin discussing the topic of the day, thus Adding and Subtracting polynomials. You can start by asking how they think this is done, and let them explain their reasoning behind it. If a student knows how to add polynomials ask him to come demonstrate at the board. This could either take up to 5 mins, if the students don’t know, or could take up to 15 mins, if they do know. (10-20 mins including quiz)

Quiz

1. Is $3x^2 + 1 + x$ in standard form? Why or why not?

**Answer:** No, the x is behind the constant. All variables go from highest power to lowest power.

2. Classify the polynomial: $4x^2 + 1$

**Answer:** This is a quadratic binomial; quadratic because of the squared power and a binomial because it has two terms.

3. In the above two questions are the leading coefficients the same? Why or why not?

**Answer:** No, the leading coefficient for the first expression is 3, the second one is 4. $4 \neq 3$.

~If the students already know how to add skip this part, if they don’t then give them one of the simple examples of addition thought of before class. This should lead them to think about
adding like terms. Try to get a student to say, “Like Terms”, as it is something they learned the day before. (10mins)

~If the students knew how to add polynomials then being here next. The next question to ask the students is, “If this is how you add, then how do you subtract?” This is the topic that will lead into the next day’s discussion about distribution. Ask the students of the class if they have any idea on how to subtract polynomials, and if anyone does then ask that student to demonstrate this on the board. If the student is correct, ask said student to explain their reasoning. If the student is incorrect then give him one of the examples that were made before class. This example should be a simple one like \((2x-1)-(3x)\) or \((2x+2)-(1)\). Try and get the students to ask about a binomial case, and when they do be sure to have a example on hand to try and let them figure out. (20mins)

~The last 10mins of class should be devoted to clearing up questions and showing lots of examples. The homework should be handed out or written on the board for the students to copy. (10mins)

Homework for Lesson 2:

The question below should be answered both with mathematical notations and written explanation.
1. Add $1+3$. Explain why, or why not, $1$ and $3$ are polynomials. Do you get a polynomial after adding them or is $1+3$ a standard form polynomial itself?

**Answer:** $1+3$ is $4$. All of which are polynomials. They are actually all monomials. The reason $1+3$ is not a polynomial is because $1$ and $3$ are like terms and all like terms must be added in order to have a standard form polynomial.

2. Add $2x+1$. Explain why, or why not, these can be added together. Do you get a polynomial after adding them or is $2x+1$ a standard form polynomial itself?

**Answer:** $2x+1$ cannot be added together. The two are not like terms, therefore they cannot be added together. $2x+1$ is a polynomial itself.

3. Add $(2x+1)+(3x+5+2x^2)$. Explain why, or why not, these can be added together. Do you get a polynomial after adding them or is this a polynomial itself?

**Answer:** You always get a polynomial when added two polynomials together. You can add these because both are proper polynomials. The answer is $2x^2 + 5x +6$.

4. Subtract $2-4$. Do you get a polynomial after subtraction?

**Answer:** $-2$. Yes all constants are a type of monomial.

5. Subtract $2x-3$. Explain why, or why not, these can be subtracted. Do you get a polynomial after subtracting them or is $2x-3$ a polynomial itself?

**Answer:** $2x-3$ cannot be subtracted and it is a polynomial itself. They are not like terms, therefore they cannot be subtracted.
6. Subtract $2x+4-(x-6)$. Explain why, or why not, these can be subtracted. Do you get another polynomial after subtracting them?

**Answer:** $x+10$. These two can be subtracted because they have like terms that can be simplified together. The answer, $x+10$, is a polynomial.
Lesson 3: The Distributive Property and Polynomials

This lesson will focus on the Scaffolding and Inquiry based styles of teaching. It will lean more so towards the inquiry based learning. The students have already seen some distribution of polynomials at this point with subtraction of binomials. This lesson plan will take that into account and build off of this.

The notes, again like the previous lesson, will be up to the students to create. The students are free to take down anything they deem important. Examples will be given for them to copy. Please note that the students must be informed of this fact. At the beginning of class this should be mentioned or briefly discussed.

This lesson will not focus on cooperative learning; however it could be the case that a class discussion begins. Hopefully the class will want to discuss the previous day’s lesson and how it relates to this topic.

Be sure to have examples of subtraction of binomials and distribution of monomials. Multiplication of binomials is the next lesson that will be covered. This might require a good amount of examples, so a power point presentation of examples might be helpful. These examples should be of monomials being distributed into any type of polynomial.

Another quiz on the previous lesson should be held at the beginning of class. This quiz should take no longer than 5 minutes and should only be around 3 questions.

Lesson Plan 3:
~Start with the quiz of the previous lesson, after that use an example of subtraction of binomials. Let a student come to the board to work the problem. Then ask the students if they know exactly what happens with the minus sign and the second binomial. You can use 2x+1-(x-4) as an example. This could take some time for the students to understand that the minus sign is being distributed into the second binomial. (15mins including quiz)

Quiz

1. When adding polynomials, do you always end up with another polynomial?

Answer: Yes, when adding polynomials you always end up with another polynomial. The group of polynomials is closed under addition.

2. When subtracting polynomials, do you always end up with another polynomial?

Answer: Yes, when subtracting polynomials you always end up with another polynomial. The group of polynomials is closed under subtraction.

3. Simplify the expression: 3x^2-(2x^2-3x+1)

Answer: x^2+3x-1

~Next give an example like, 3x (2x), and ask the students if they know what to do with this. If none of the students can answer why ask them questions to lead them to the answer. Note that the students have not seen multiplication of variables at this point with this lesson. (10mins)

~After the students understand that you add powers on variables when multiplying give them an example of a binomial being multiplied by a monomial. 3x (2x+1) is a decent example.
Ask the students for the answer or any ideas that might lead to the answers and why they think this would be correct. (10mins)

~This is where the power point presentation comes into the lesson. There should be an ample number of examples on the presentation. You can ask students to come up and do them on the board or projector. There should be at least one example of monomial time’s monomial, monomial time’s binomial, and monomial time’s trinomial. (10mins)

~The last five minutes of class should be devoted again to answering any last minute questions and handing out homework. (5mins)

Homework for Lesson 3:

The question below should be answered in full with a mathematical notation answer along with an explanation of the answer.

1. What is x times x and why does it equal that?

Answer: The answer is $x^2$, and it is equal to that because when multiplying variable you add their exponents.

2. Is $2x+(x-2)$ an example of distribution with polynomials? If so what does this simplify too?

Answer: No this is not a example of distribution of polynomials, however it does simplify to $3x-2$. 
3. Is $2x-(x-2)$ an example of distribution with polynomials? If so what does this simplify too?

**Answer:** Yes, this is an example of distribution with polynomials because of the -1 in front of the $(x-2)$. This simplifies to $x+2$.

4. Is $2x(4x)$ an example of distribution with polynomials? If so what does this simplify too?

**Answer:** Yes, because this is clear multiplication. This simplifies too $8x^2$.

5. Is $3x(x-6)$ an example of distribution with polynomials? If so what does this simplify too?

**Answer:** Yes, once again this has multiplication in it. It simplifies too $3x^2-18x$. 
Lesson 4: Multiplying Polynomials

This lesson plan will be another discovery style of teaching. The student will work in groups and hopefully figure out how to multiply binomials with other polynomials (bigger than monomials).

These notes will either be presented to them in work sheet form or will be given as a power point online so they can access it at home and print it. It will have a couple of examples on it along with some explanation of key aspects of multiplying polynomials.

The groups the student work in for this lesson plan can either be new groups or the same groups from lesson one. Either way they have to present their findings to the class for the class discussion. Again, the time limit for the class might restrict the amount of groups that can present but the work should be made into a poster and put on the wall. It would be best not to call on the same groups to present their findings as in the first lesson.

There will be no quiz for the previous lesson. This lesson and the last lesson will have a combined quiz in lesson five.

Lesson Plan 4:

~At the start of class split them into groups and briefly explain the topic of this lesson. The students should also be informed of the notes, whether they will be handed to them at the end of class or whether they need to go online to get the notes themselves. Also hand out all the materials need to make the posters for the presentations. (10mins)

The Exercise
The students are to start by showing multiplication between integers, then variables, then integer and a variable, then between a monomial and binomial, and then lastly they are to show the multiplication between two binomials. The polynomials are too made up by the students as well. The last part of the exercise the students have not seen before. Hopefully working their way from the beginning of multiplication up to the multiplication of binomials they will be able to figure it out. If not then try and direct the students in the right direction. 
(15mins)

~The class should then present their findings. Ask the class if any of the groups got the last one before calling on a group. If any of the groups say they think they got it call on them. Do this for three to four groups, and only ask for them to show the last two examples. If none of the groups have it right being a class discussion on the topic and try to get the students to figure it out. If the students cannot figure out how to multiply two binomials together by ten minutes till the end of class, being to lecture on the topic. Just don’t forget to hand out homework
(15mins-25mins)

~If the students do figure out how to multiply them together give a round of examples and start to hand out the homework. Don’t forget to mention the notes online if they are not being handed out in class.

Notes for Lesson 4:
Geometry of Algebra

1. Is multiplying two binomials together, say \((2x+1)\) and \((x+2)\), a type of polynomial distribution? If so, explain why?

**Answer:** Yes it is. Any type of polynomial multiplication is distribution.

**Diagram:**

\[
(3z+5)(2z+7) = 6z^2 + 21z + 10z + 35 = 6z^2 + 31z + 35
\]
2. When multiplying \((2x+1)\) and \((x+2)\) do you get another polynomial? If so, classify the polynomial. Show what you get when you multiply the two together.

**Answer:** You get \(2x^2 + x + 2\), and yes this is another polynomial. This polynomial is a quadratic trinomial.

3. How do you simplify \((2x+1)^2\)? Is this a type of multiplication? If so, find the answer. (Hint: think of simplifying \(x^2\))

**Answer:** Any square means to multiply the squared item by itself, so this is actually \((2x+1)\times(2x+1)\). This is multiplication and simplifies to \((4x^2 + 4x + 1)\).

4. Is it possible to multiply \((2x+1)\) and \((2x^2+x+1)\) together? Please explain why you think so and provide a mathematical answer with it.

**Answer:** Yes it is possible and the answer is \((4x^3+2x^2+2x^2+2x+x+1)=4x^3 +4x^2+3x+1\)
Lesson 5: Prime Factorization and the Greatest Common Factor (GCF)

This lesson will be based off of the Inquiry style of teaching and will utilize the cooperative learning of the students. The students will be split into groups so that they may discuss in groups questions they have, and then they will present their questions to the class.

Be sure to have some examples of greatest common factors. Not many examples are needed for this lesson plan, as most of the class time will be spent asking the students questions.

The notes for this lesson should either be online for the students to print at home or handed to them in a worksheet at the end of class.

There will be a quiz on the previous two lessons at the beginning of this lesson. It should cover the multiplication of polynomials and the distributive property of polynomials.

Lesson 5:

~At the start of class there should be a five minute quiz on the previous two lessons. After the quiz split the class up into groups and tell them the topic of the day. Ask them if anyone has ever heard of primes before, if so ask them to explain. (10mins including quiz)

Quiz

1. Is distribution, with integers, a special form of multiplication? If yes explain if no give an example of why not.

Answer: Distribution can been seen as a “subset” of multiplication. Distribution is a special form of multiplication used with polynomials.
2. When dealing with polynomials is distribution the same thing as multiplication? Why or why not?

**Answer:** Yes, Distribution is multiplication for polynomials.

3. Is multiplication and distribution the same thing? Please explain your answer. (in other words, is multiplication and distribution the same thing?)

**Answer:** No, multiplication is not the same thing as distribution. Distribution is a special type of multiplication used with polynomials and other things.

~After the quiz write the following numbers on the board: 2, 3, 5, 7, 11, 13, 17, 23, 29, 37. Then ask the class to discuss among their groups the similarities between the numbers on the board. (5mins)

~After they have had ten minutes to discuss the numbers ask each group to tell the class their thoughts. The key thought wanted from the students is that the only numbers that can multiply to give you all those numbers are 1 and itself. The idea is for the students to come up with the definition of a prime number on their own. (10mins)

~After the definition of a prime number has been discussed give the students the numbers: 4, 6, 8, 10, 12, 14, 16. Tell them to discuss among their groups the similarities between the numbers. (5mins)

~Ask certain groups to tell the class what they have found. This should lead to the idea of the numbers all having 2 as a factor. Then ask the students to break down each of the
numbers so that it is all multiplication between prime numbers (which should have been just defined). (10mins)

~The last ten minutes should be used in discussing the idea of the greatest common factor between any numbers given. Such as the example in class with the set of numbers: 4,6,8,10,12,14,16; the greatest common factor was 2. This is also the time in which the homework should be handed out as well as the notes if they are printed out.

Notes for Lesson 5:

~**Prime Numbers**: are integers in which there are no factors other than itself and 1. In other words, the only way to multiply and get the number is by one and itself.

~Examples of **Prime Numbers**:
  
  2 <-- only way to get two by multiplication is with 1 and 2.
  
  3 <-- only way to get three by multiplication is with 1 and 3.

  5
  
  7

  etc.

~Examples of **Non-Prime Numbers**:

  4 <-- the factors of four are 1 and 4 (1*4=4) and 2 (2*2=4)
  
  6 <-- factors of six are 1 and 6 (1*6=6), 2 and 3 (2*3=6)
  
  8 <-- factors of eight are 1 and 8 (1*8=8), 2 and 4 (2*4=8)

  etc.

~**Prime Factorization**: is the method of breaking down non-prime numbers into a group of prime numbers. The idea of this is to take really large numbers and rewrite them as multiplication between smaller numbers. There are two ways of showing this. The first way is dividing the number you want to factorize by prime numbers (usually by 2 if the number is even). The second way is by splitting the number into two smaller numbers and continuing this pattern until you get to all prime numbers.
~Example of **Division by Prime**: 

96 ÷ 2 = 48  
48 ÷ 2 = 24  
24 ÷ 2 = 12  
12 ÷ 2 = 6  
6 ÷ 2 = 3  
3 ÷ 3 = 1  

96 = 2 * 2 * 2 * 2 * 3

~Example of **Factor Tree** (number splitting):

As you can see, it does not matter which method you prefer to use the answer should be the same no matter what method is used. Next we will discuss factoring Monomials.

~**Prime Factorization** of a **Monomial**: is the method of breaking down a monomial into a group of prime numbers along with variables. In other words, you break the monomial down into multiplication between prime numbers and variables.

~Example of **Prime Factorization** of a **Monomial**:

Factor: 48 * x^3 * y^2  
48 ÷ 2 = 24
$24 \div 2 = 12$

$12 \div 2 = 6$

$6 \div 2 = 3$

$3 \div 3 = 1$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

$X^{(3)} \div X = X^{(2)}$

$X^{(2)} \div X = X$

$X \div X = 1$

$X^{(3)} = X \times X \times X$

$Y^{(2)} \div Y = Y$

$Y \div Y = 1$

$Y^{(2)} = Y \times Y$

So that  $48 \times X^{(3)} \times Y^{(2)} = 2 \times 2 \times 2 \times 2 \times 3 \times X \times X \times Y \times Y$.

Now you can also rewrite this as  $48 \times X^{(3)} \times Y^{(2)} = 2^{(4)} \times 3 \times X^{(3)} \times Y^{(2)}$

Now that we can factor a term of a polynomial (monomials are a one term polynomial if you don't remember) we can start finding the Greatest Common Factor (GCF) of a polynomial.

~Greatest Common Factor (GCF): is the largest factor that all the terms in a given polynomial have.

~Examples of GCF:

Find the GCF of  $14 \times X^{(4)} + 7 \times X \times Y$ then show the factored form of the polynomial
Factors of $14X^4$  

$14 \div 2 = 7$  

$7 \div 7 = 1$  

$14 = 7 \times 2$  

$X \div X = 1$  

$X^4 \div X = X^3$  

$X^3 \div X = X^2$  

$Y \div Y = 1$  

$X^2 \div X = X$  

$Y \times 1 = Y$  

$X \div X = 1$  

$X \times X \times X \times X = X^4$  

$14X^4 = 2 \times 7 \times X \times X \times X \times X$  

$7X*Y = 7X*Y$  

(it was already in its prime state)

Final Answer: \[ \text{GCF} := 7 \times X \]

Homework for Lesson 5:

All answers should be explained thoroughly.

1. What is a prime number? Can an even number ever be prime? Explain.

Answer: Prime numbers are integers in which there are no factors other than itself and 1. In other words, the only way to multiply and get the number is by one and itself.

2. What is a factor? Can any number be a factor? If not, what numbers cannot be factors and why?
**Answer:** With the exception of 0, any number can be a factor. Factors are simply numbers that multiply together to give a bigger number.

3. Can two integers, say 5 and 15, have a greatest common factor? If yes what is the GCF and why, if no why not?

**Answer:** Yes, the GCF is 5. This is because both 5 and 15 have a factor of 5, and it is the biggest factor that both of these numbers have in common.

4. Can a GCF ever be a polynomial (such as $(2x+1)$)? Please explain.

**Answer:** Yes, when two polynomials have something in common you can “factor” out the common expression.
Lesson 6: Factor by Grouping

This lesson will use the Inquiry style of teaching. The students will have to share their thoughts with the entire class either by examples at the board or by explaining themselves at their seats. This lesson plan will not incorporate group work.

Notes should either be handed out in class or given to the students online to print out at home.

There will be a quiz on the previous lesson included in this lesson plan, which will be at the start of the class and should only take around five minutes.

Be sure to have a couple of examples on hand to help the students understand the topic. This lesson is not meant to give the answers to the student through examples, unless it comes down to that.

Lesson 6:

~At the start of class the quiz on lesson 5 should being. After the quiz tell the students briefly about what will be covered today. (10mins)

Quiz

1. What is a prime number? Can an even number be prime? If so give one? If not explain why.

Answer: Prime numbers are integers in which there are no factors other than itself and 1. In other words, the only way to multiply and get the number is by one and itself.
2. Can a polynomial be a factor? Explain.

**Answer:** Yes, when a polynomial has terms that have common factors you can “factor” out the common numbers or variables.

3. Give the prime factorization of $64x^2$.

**Answer:** $2*2*2*2*2*x*x$

~Next ask the students to look at the given problem, $(2x+1)*(3x+7)$. Ask them if this is a factored form. Tell them to explain their answer. If no one can figure it out within five minutes, show them this $2x(3x+7) +1(3x+7)$ and ask them if this can be factored. (10mins)

~After the previous example give them this expression: $x^3 +3x^2+2x+6$. Ask them if there is a way to simplify this expression. If no one can think of anything within five minutes give them the hint: Prime factorization. (15mins)

~Once a student understands ask them to the board to demonstrate how to factor by grouping, and tell them this is the actual term for what has been done. (5mins)

~The last ten minutes should be filled by putting random examples and calling students to the board to factor by grouping. The homework and notes, if printed out, should be handed out at this time. (10mins)

Notes for Lesson 6:

~**Factor by Grouping:** is a tool used on polynomials with more than three terms. Note that this can be used on polynomials with only three terms as well, but it is mainly used on four terms and above. The idea is to get the terms that are most alike together and then to "pull out" the
common factors between the two terms (in other words find their GCF).

Factor: \(x^3 + 2x^2 + 3x + 6\)

Step One: Collect the terms with the most things in common with each other. If you look at \(x^3\) and \(2x^2\) you can see that these two have much more in common than say \(x^3\) and \(3x\). You can see this by finding their respective GCF's. Which leaves you to group \(3x\) with 6.

Step Two: Find and factor out the GCF of each "group".

\[
\text{GCF of } x^3 \text{ and } 2x^2 \text{ is } x^2
\]

\[
\text{GCF of } 3x \text{ and } 6 \text{ is } 3
\]

We now rewrite the given polynomial as...

\[
[x^3 + 2x^2] + [3x + 6] = x^2(x + 2) + 3(x + 2)
\]

Step three: Simplify if possible.

If you look closely you can see that we can factor something out of both groups. Both groups have \(x + 2\) in them so if we factor them out we get....

\[
x^2(x + 2) + 3(x + 2) = (x + 2)(x^2 + 3)
\]

which is our final answer for this.

Homework for Lesson 6:

All questions should have an explanation of why you think this is the correct answer.

1. Can you use factor by grouping on a trinomial? Why or why not?
Answer: No because there are not enough terms to group together.

2. Factor: $3x^4+x^2+6x^3+3x$. How did you group the terms? Why did you group them this way?

Answer: $(3x^4+x^2)+(6x^3+2x)= x^2(3x^2+1)+2x(3x^2+1)=(3x^2+1)(x^2+2x)$. If you were to group them any other way you would not get something that is equal in both groups.

3. When factoring by grouping, does it matter in what order you group the terms? Why?

Answer: Yes it matter. Factoring by grouping is not associative.
Lesson 7: Factoring Trinomial

The final lesson in this unit will be an Inquiry based lesson using cooperative learning. This final lesson will utilize all the previous lessons knowledge. The students will be discussing the topic in their groups, which will lead to the last class discussion in this unit.

The notes for the lesson should either be handed out at the end of class or posted online for the students to print themselves.

Examples should be prepared in case the Inquiry based learning style does not facilitate the students learning with this topic; in which case traditional lecture will take course.

The quiz on lesson six should be given at the beginning of class.

Lesson 7:

~At the start of class give the quiz on lesson six. After the quiz break the class into groups to prepare for the discussions to be had on the topic of factoring trinomials. Give a brief introduction to the topic while the class is splitting into groups. (10mins)

Quiz

1. Can factor by grouping be used on any trinomial?

Answer: No, there are not enough terms.

2. Factor: $5x^4+10x^2+7x^2+14$

Answer: $(5x^2+7)*(x^2+2)$
~After the groups have been formed give them the trinomial, x^2+2x+1, and ask them if they think this is factorable. If a student answers yes ask them to explain why. If the answer is a snarky remark, such as “Of course it is factorable, otherwise you would not have mentioned it”, send the child to the office. Just kidding, but anyway, if no one answer with the correct response ask them what two linear binomials multiplied together give you. Write on the board, 
(x+1)*(x+1). (10mins)

~After the previous example is understood, ask the class how they would go about doing this multiplication backwards, or in other words, how would you start with the trinomial and end up with two binomials multiplied together. This should be for the group to discuss for a moment. If no one answers start by asking what the factors are of the last term. If starts the discussion then let the students figure the rest out the best they can by calling some students to the board to write down their groups thoughts. (15mins)

~The last fifteen minutes of class should be used to go over examples. Given a different trinomial to each group and ask them to factor it. Then ask each group to come to the board and explain why this is how you would factor their trinomial. Each group should have a trinomial that is different, in coefficients and signs. Also hand out the homework and notes before the end of class. (15mins)

Notes for Lesson 7:

~Factoring Quadratic Trinomials: The reason we want to factor these quadratic trinomials is we want to simplify the quadratic that is given to us. Writing polynomials in their factored forms can prove to be helpful in many situations. There are a lot of different ways to do
this. The method shown here is the standard way to factor. When factoring quadratic trinomials you should end up with two binomials being multiplied together.

Factor: \( x^2 + 2x + 1 \)

First we need to remember how we can build one of these before we can factor them. Let’s look at multiplying two binomials together. Take \((x+1)(x+1)\), using the foil method, \((x+1)(x+1)=x^2 + x + x + 1 = x^2 + 2x + 1\) which happens to be the polynomial we are trying to factor now. Now, notice how the middle term and last term are formed. These two are very important because they are tied together in how their signs form (the leading term is always positive in standard form which means the factors of the leading term will always be positive). Notice how the last term, usually a constant, is only formed by multiplication while the middle term is formed with addition. Here we go....

Step One: Factor leading term: \( x^2 \)

\( x \times x = x^2 \) this tells us that each binomial will have a leading term of \( x \).

Step Two: Factor out Last Term:

\( 1 \times 1 = 1 \)

This tells us that each binomial will have a last term of 1, but we do not know the sign of either of them nor do we know which sign the binomials go to.

Step Three: Figure out the signs of your factors: + or -?

This is where logic comes into play. Look at the sign of the last term in the trinomial. It happens to be positive in this case. There are two ways in which you can get a positive sign by multiplication those are either positive times a positive or a negative times a negative. Now look
at the second term of the trinomial. The sign is positive as well; which means we need to use two positive signs in our factored out binomials.

Step Four: Figure out which set of factors add to the middle coefficient: 2

\[1 + 1 = 2\]

So now we know that 1 and 1 are the constants we need.

So if we put all the clues from steps one to four together we get our answer of: \((x + 1) \cdot (x + 1)\)

Step Five: Check answer:

\[(x + 1) \cdot (x + 1) = x^2 + x + x + 1 = x^2 + 2x + 1\]

Factor: \(x^2 - 4x + 3\)

Step One: Factor Leading Term: \(x^2\)

\[x \cdot x = x^2\]

so that each binomial will start with x

Step Two: Factor Last Term: 3

\[3 \cdot 1 = 3\]

Step Three: Figure out the signs

First looking at the last term we see that it is positive. This tells us the signs in the binomials must either be positive or negative. Next look at the term in the middle, since it is negative the signs used in our answer must be negative as well.

Step Four: Figure out which set of factors add to the middle coefficient: -4

\[-3 - 1 = -4\]

Now add the clues to get the answer: \((x - 3) \cdot (x - 1)\)

Step Five: Check Answer:

\[(x - 3) \cdot (x - 1) = x^2 - 3x - x + 3 = x^2 - 4x + 3\]
Homework lesson 7:

Each answer should be accompanied by an explanation of why.

1. When factoring trinomials how do you determine the signs that will be in the binomials? What two terms do you look at for this, given that the trinomial and binomials are in standard form?

   **Answer:** You look at the sign of the third term and second term to determine the sign of the two binomials.

2. Are all trinomials factorable? Explain why and give examples.

   **Answer:** No, not all trinomials are factorable. Example: \(x^2+2+2\)

3. Is it possible for higher powered trinomials to be factored? Why?

   **Answer:** It is possible if the second and third term can apply to conditions set above.

4. Factor: \(x^2+5+12\). Please explain your reasoning behind your answer.

   **Answer:** Not factorable. There is no way for the factors of 12 (1,12 or 2,6 or 3,4) to add to 5 when all the signs are positive.
Unit Test

(rated for 30mins, with 15mins left for checking answers, and five minutes of relaxing time)

Please answer all questions with an explanation of why you think this is the correct answer, or give justification for the given answer. It is important that you give some type of justification for your answer, because if you don’t you will automatically lose half of the points for the problem. Also, if you actually read all the instructions then write my name backwards on the bottom right hand corner of the page for three bonus points on this test.

1. Define what a term is of a polynomial in your own words. (be sure to use mathematical vocabulary!).

**Answer:** They are the pieces that the polynomial is actually made of. Terms may consist of constants, variables, or a combination of the two. The only restrictions on what a term can be are: the power of each Variable must be a non-negative, whole number, and each constant used must be either an integer or a rational number.

2. Is $x^{-2} + x + 1$ a polynomial?

**Answer:** No, given that the power on the variable is negative this is not a polynomial.

3. What is the degree of a polynomial?

**Answer:** the highest exponent of all the terms in the expression. If a term has more than one variable you must add the powers of the two variables in order to get the degree of that term.
4. Do you always obtain another polynomial while adding and subtracting polynomials?

**Answer:** Yes, the group of polynomials is closed under the operations of addition, subtraction, and multiplication.

5. Is distribution a type of multiplication for polynomials?

**Answer:** Yes, distribution is an extension of multiplication. Distribution comes into play when you are multiplying polynomials together. However, multiplication is not a type of distribution.

6. Multiply the two binomials: \((2x^3+5x)*(3x^2+4x)\) (your explanation of this problem is given by showing your work!)

**Answer:**

\[
(2x^3+5x)*(3x^2+4x) = (2x^3)*(3x^2+4x)+(5x)*(3x^2+4x)
\]

\[
=6x^5+8x^4+15x^3+20x^2
\]

7. Is it possible to have a prime even number? Are all prime numbers odd?

**Answer:** There is only one even prime number and it is 2. No, not all odd numbers are prime. 9 is an example of an odd non-prime.

8. What is a GCF? Do you use GCF’s while factoring?

**Answer:** The GCF is the largest factor that all the terms in a given polynomial have. Yes, GCF’s are used in every type of factoring. You must know the GCF to get a polynomial into factored form.

9. Factor: \(4x^3+2x^2+2x+1\) (remember to show all work!)

**Answer:**

\[
4x^3+2x^2+2x+1 = (4x^3 +2x^2)+(2x+1)=2x^2(2x+1)+(2x+1)=(2x+1)(2x^2+1)
\]
10. Factor: $4x^2+3x-2$

**Answer:** $4x^2+2x-2$

Well factors of 4 are 1,4, and 2,2. Factors of -2 are -1,2 and -2,1. The middle term must be 2 which means the positive number must be the bigger one. There is also no way in which to use 1,4 along with 1,2 to get three, so we must use 2,2 and 1,2. We already stated the bigger number must be positive so we know the factors must be 2,2 and -1,2. So the answer is:

$$(2x-1)*(2x+2)$$