Writing Linear Equations Unit
Algebra I
McDougal Little- Algebra 1, Explorations and Applications

Standards/Objectives

NCTM Numbers and Operations Standard:
• Understand numbers, ways of representing numbers, relationships among numbers, and number systems
• Understand meanings of operations and how they relate to one another
• Compute fluently and make reasonable estimates

NCTM Algebra Standard
• Understand patterns, relations, and functions
• Represent and analyze mathematical situations and structures using algebraic symbols
• Use mathematical models to represent and understand quantitative relationships
• Analyze change in various contexts

NCTM Geometry Standard
• Analyze characteristics and properties of two- and three- dimensional geometric shapes and develop mathematical arguments about geometric relationships
• Specify locations and describe spatial relationships using coordinate geometry and other representational systems
• Apply transformations and use symmetry to analyze mathematical situations
• Use visualization, spatial reasoning, and geometric modeling to solve problems

NCTM Measurement Standard
• Apply appropriate techniques, tools, and formulas to determine measurements
• Understand measurable attributes of objects and the units, systems, and processes of measurement

NCTM Data & Analysis Standard
• Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
• Select and use appropriate statistical methods to analyze data
• Understand and apply basic concepts of probability

NCTM Problem Solving Standard
• Build on new mathematical knowledge through problem solving
• Solve problems that arise in mathematics and in other contexts
• Apply and adapt a variety of appropriate strategies to solve problems
• Monitor and reflect on the process of mathematical problem solving
NCTM Reasoning & Proof Standard
- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

NCTM Communication Standard
- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

NCTM Connection Standard
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

NCTM Representation Standard
- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

DPI Standards
Numbers and Operations: Competency Goal 1:
The learner will perform operations with numbers and expressions to solve problems.
1.01 Write equivalent forms of algebraic expressions to solve problems.
   a.) Apply the laws of exponents.
   b.) Operate with polynomials.
   c.) Factor polynomials.
1.02 Use formulas and algebraic expressions, including iterative and recursive forms, to model and solve problems.
1.03 Model and solve problems using direct variation.

Geometry and Measurement: Competency Goal 2:
The learner will describe geometric figures in the coordinate plane algebraically.
2.01 Find the lengths and midpoints of segments to solve problems.
2.02 Use the parallelism or perpendicularity of lines and segments to solve problems.

Data Analysis and Probability: Competency Goal 3:
The learner will collect, organize, and interpret data with matrices and linear models to solve problems.
3.01 Use matrices to display and interpret data.
**3.02** Operate (addition, subtraction, scalar multiplication) with matrices to solve problems.

**3.03** Create linear models for sets of data to solve problems.
   a.) Interpret constants and coefficients in the context of the data.
   b.) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

**Algebra: Competency Goal 4**

The learner will use relations and functions to solve problems.

**4.01** Use linear functions or inequalities to model and solve problems; justify results.
   a.) Solve using tables, graphs, and algebraic properties.
   b.) Interpret constants and coefficients in the context of the problem.

**4.02** Graph, factor, and evaluate quadratic functions to solve problems.

**4.03** Use systems of linear equations or inequalities in two variables to model and solve problems. Solve using tables, graphs, and algebraic properties; justify results.

**4.04** Graph and evaluate exponential functions to solve problems.

**Teaching Style:** Inquiry Based Questioning

**Technology:**
I used technology in several of my lessons. I feel that using technology keeps the students more engaged in the learning environment and they actually want to learn more. We will go to the computer lab to play interactive learning games, investigate real world examples online, and using graphing calculators to work with investigating graphs. Playing the interactive games online will keep students interested in learning and give them a goal to work towards. Going online and investigating real world examples will allow students to see how the math they are learning applies in the real world and in the rear future of college. Lastly, using the graphing calculators will let students see how graphs can be manipulated and how the different components make up the line.

**Discourse:**
Students will be keeping a math journal throughout our class. Many of the questions will be based on higher learning questions or about how math applies to the real world. By involving the students in the higher level thinking, it will keep them involved in the learning process. Also, students will be able to see how math applies to the real world.

**Problem Solving:**
I used a lot of real world examples throughout my unit lesson plan. Within these lessons, I tried to pick examples of realistic occurrences throughout the real world. I tried not to pick just one subject, but a subject that most (if not all) students could relate to. By picking subjects that students can relate to, they will see how math applies to their everyday lives.

**Misconceptions:**
The material that we cover during this unit is very basic so hopefully there will not be too many misconceptions. However, in order to try and keep any from happening, I will to do a lot of group work together, so students can see each others work. This also gives me an opportunity to
circulate around the room and help clear up any confusion that may arise. Making sure students can read order pairs correctly (x coordinate first, y coordinate second) from the very beginning will also help keep any problem from arising when using graphs and when finding slope. Teaching the students to check their work on their calculator will also help them from making careless errors.

**Special Needs/Diversity:**
Keeping students involved in the learning process will be much easier if I appeal to many different types of students. By doing lessons that are based upon both conversing and listening I can appeal to different types of learners (visual, audio). By picking real world examples that apply to students of every type of background, they can see how the material will apply to their lives. Also, doing group work will allow students to interact with each other and talk through problems together. Writing down problems and using visual aids along with technology will allow students to understand problems that otherwise might not translate well to them.

**Assessment:**
Students will be given homework assignments nightly and, they will be checked for completion but not for correctness. Math journals will be checked for completeness as well.

Students will complete an overall assignment that has both math problems that must be solved and problems that must be answered with words. This makes sure that the students actually understand what they are learning.
Lesson 1-Applying Rates and Ratios
Computer Lab Explorations

Outline: We are starting a new unit about Writing Linear Equations. In this first lesson students will learn to apply their previous knowledge of ratios to real world examples of rates. We will be in the computer lab today. The students will be doing some independent work and then we will come together and work through examples as a class. This lesson will be more of the students attempting to learn on their own first by playing online computer games to try and find the pattern that ratios/rates follow. The computer games will guide their initial learning and then I will supplement by going over any concepts during our class discussion.

Objectives: North Carolina Standard Course of Study: Algebra I-Numbers and Operations
   Competency Goal 1 - The learner will perform operations with numbers and expressions to solve problems.
   1.01 Write equivalent forms of algebraic expressions to solve problems.
   1.02 Use formulas and algebraic expressions, including iterative and recursive forms, to model and solve problems.

Essential Question: How do you find unit rates from words and graphs?

Learning Outcome: Compare real world rates.

Prior Knowledge: fractions
   Limited knowledge of ratios/proportions
   Multiplying and dividing
   Limited knowledge of working with units (days, miles, etc)

Procedure

Materials: Computer lab

Time Required for Lesson: 40-45 minutes

Activity
   1. Go to the website http://www.bbc.co.uk/skillswise/numbers/wholenumbers/

   Click on Ratio and Proportion in the bottom right hand column. Read the fact sheets on ratio and proportion. You may do as many of the worksheets, games and quizzes as you wish to familiarize yourself with ratios and proportions. Be sure to read
“Understanding Direct Proportion.” Be aware that this is a British site so some of the language and money symbols will be different from ours. (10-15 minutes)

2. We will go to http://www.purplemath.com/modules/ratio2.htm together. We will work all the way through cross multiplying, working through examples together on the board.

   It may be helpful for you to rewrite the fractions in their traditional form. That is change $\frac{1}{2}$ to $\frac{1}{2}$. This will make the fraction easier to read and will help you see your proportions better next to each other. (10-15 minutes)

3. Determine whether the following proportions are true.

   a. $\frac{7}{8} = \frac{73}{83}$ [no]   b. $\frac{2}{8} = \frac{3}{12}$ [yes]   c. $\frac{16}{3} = \frac{4.8}{0.9}$ [yes]   d. $\frac{1}{6} = \frac{3}{8}$ [yes]

   Students will work independently to get their answers and then we will go over them as a class.

   (5-10 minutes)

4. To see some practical applications of proportions go to:
   http://www.shodor.org/UNChem/math/r_p/ (10-15 minutes/remainder of class working on this and beginning the homework)

5. HOMEWORK: Explore other sites on the internet and find a problem that shows a practical application of proportions/rates. Find an article in a current newsmagazine (either on-line or print) that makes use of proportions. Write a paragraph about what you have learned concerning the value of proportions in the workplace. Record this in your math journal.

Lesson 2: Exploring Direct Variation
Patterns and Direct Variation

**Introduction:** We will be working with discovering what direct variation is and how to find it something directly correlates. We will work through the activity below in groups and then as a class to draw conclusions about direct variation.

**Objective:** North Carolina Standard Course of Study-Algebra I: Numbers and Operations Competency Goal 1: The learner will perform operations with numbers and expressions to solve problems.

1.02 Use formulas and algebraic expressions, including iterative and recursive forms, to model and solve problems.
1.03 Model and solve problems using direct variation.

**Essential Question:** How do you recognize and describe direct variation

**Learning Outcome:** Students will be able to explore relationships between real-world variables, such as standing height and kneeling height.

**Prior Knowledge:** working with ratios and rates
Fractions
How to read a graph
Plotting points on a graph

**Procedure**

**Materials:** meter stick
Graph paper/graphing calculator

**Time required for lesson:** 40-45 minutes

**Activity:** (25-30 minutes) The artist Leonardo da Vinci studied human proportions in order to make more accurate drawings. He observed that the kneeling height of a person is \( \frac{3}{4} \) of the person’s standing height.

**Main question of the activity:** Was Leonardo da Vinci’s hypothesis correct?

Students will work together in groups of 6 (groups will be chosen by the teacher ahead of time). The students will work together to measure and record the kneeling and standing heights of each person in the group.

1.) For each person, find the ratio of kneeling heights to standing height. Then fill in this table (students will copy this down from the board).

<table>
<thead>
<tr>
<th>Name</th>
<th>Standing Height</th>
<th>Kneeling Height</th>
<th>Kneeling height</th>
</tr>
</thead>
</table>

2.) After completing the table, students will go up to the board where a coordinate plane will be displaying. [The y-axis will represent the kneeling height and the x-axis will represent the standing height (cm).] Each student will plot the point that they found based on their values in the kneeling/standing height column of their table. Once all of the points have been placed on the graph, this will show a scatter plot of the relationship between the two sets of data. [DO NOT CONNECT THE POINTS]

<table>
<thead>
<tr>
<th></th>
<th>(cm)</th>
<th>(cm)</th>
<th>Standing height</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.) Based on Leonardo da Vinci’s observation we talked about earlier; write an equation for kneeling height as a function of standing height based on your measurements. Draw this graph on your paper.

4.) Inquiry Questions posed to pose to the class:
   a. Can you see a pattern from the graph?
   b. Can you explain what the pattern is doing?
   c. Why doesn’t everyone have the same equation? Who is the tallest in the class? Who is the shortest? What the similarities and differences between these two values?
   d. Do you agree with da Vinci’s observation? Why or why not?

Notes: (10-15 minutes)
- Whenever two variables have a constant ratio, they are said to show **direct variation**. The constant ratio is called the **constant of variation**.
- If we let x=standing height and y=kneeling height. We form the equation \(\frac{y}{x} \approx 0.75\). This means that 0.75 is the constant of variation.
• We can rewrite this equation to show that kneeling height (y) is a function of standing height (x).
• \( x \cdot \frac{y}{x} = 0.75x \) [multiply both sides of the equation by x]
• \( y = 0.75x \) [kneeling height is a function of standing height]
• **DIRECT VARIATION** can be described by an equation in the form: \( y = kx \), or \( \frac{y}{x} = k \), where \( k \neq 0 \). (**k represents the constant of variation**)
• In plain English, this translates to \( y \) varies directly with \( x \).
• Example:
  o Do earnings vary directly with the number of hours worked? If so, give the constant of variation and write an equation that describes the situation.
    
    | Hours worked | 2  | 3  | 4  | 5  |
    |--------------|----|----|----|----|
    | Earnings (dollars) | 11.50 | 17.25 | 23.00 | 28.75 |

  o Step one: Check to see if the data pairs have a constant ratio.
    • If they have a constant ratio then they will have a direct variation. If they do not have a constant ratio, then they will not be a direct variation.

    \[
    \begin{array}{c|c}
    \hline
    \text{Earnings} & \text{Hours Worked} \\
    \hline
    \frac{11.50}{2} & = 5.75 \\
    \frac{17.25}{3} & = 5.75 \\
    \frac{23.00}{4} & = 5.75 \\
    \frac{28.75}{5} & = 5.75 \\
    \hline
    \end{array}
    \]

    • Does this data set have a constant ratio? YES=5.75=DIRECT VARIATION
    • If we let \( E \)=earnings in dollars, and \( H \)= hours worked then our equation will be \( E=5.75H \)

• Direct variation equations are always lines through the origin. You can use this fact to see whether the data shows direct variation.
• Relate back to da Vinci scatter plot from beginning of class. Draw the line \( y=0.75 \) on the graph.
• All of the points on the scatter plot lie on or very close to a line through the origin of the coordinate plane.
If there is time left over at the end of class, students can start on their homework]

Homework:
p.109 #3,4,5,13,14,16  **#16 Should be completed in your math journal**

#3 Tell whether the data show direct variation. If it does, give the constant of variation and write an equation that describes the situation.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cable Subscribers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>5,100,000</td>
</tr>
<tr>
<td>1980</td>
<td>17,500,000</td>
</tr>
<tr>
<td>1985</td>
<td>35,430,000</td>
</tr>
<tr>
<td>1990</td>
<td>50,500,000</td>
</tr>
</tbody>
</table>

#4 and #5: Decide whether each scatter plot suggests that the data show direct variation. Explain.

4.) This graph does not begin at 0, therefore it does not show a direct variation.

5.) This graph does start at 0, therefore it does show direct variation.

13.) In her new sales job, Mina Joshi will not make any commissions until she has sold $1000 of merchandise. Then she will receive a 5% commission on every sale. Which graph shows the amount Mina will receive in commission? Explain your choice.
Answer: B-Mina doesn’t start receiving commission until she has sold over $1000 in merchandise so graph A does not work.

14.) Does the amount Mina receives in commissions vary directly with her sales? Give a reason for your answer.
   No, there is no direct variation because the graph does not go through 0. Also, the points do not have a constant ratio.

16.) Explain why the graph of an equation in the form $y=kx$ always goes through the origin. Give an example of a graph that shows direct variation and one that does not show direct variation.
   **Complete in your math journal.**

Resource:
Lesson 3- Pre-Slope
Personal trainers: Working with slope

Outline: Students will work together in pairs to begin to understand slope without a formal explanation beforehand of what slope actually is. The worksheet has a number of questions to guide the students thinking to a higher level. Some of the questions are more difficult, so students are allowed to work in pairs. The students will learn what slope is about before being formally introduced to it.

Objective: North Carolina Standard Course of Study-Algebra I: Algebra
Competency Goal 4: The learner will use relations and functions to solve problems.
4.01 Use linear functions or inequalities to model and solve problems; justify results.
   a.) Solve using tables, graphs, and algebraic properties.
   b.) Interpret constants and coefficients in the context of the problem.

Essential question: How might a personal trainer use slope to analyze a workout and plan for future training sessions?

Learning outcomes: Students will gain experience finding the slope of a line and will use the rate of change to solve problems.

Prerequisites: coordinate plane
Working with ordered pairs

Procedure

Materials
Worksheet: “Finding Slope”
Calculator

Time required for lesson: Approximately 45 minutes

Activity

1. As a class, read the problem on the worksheet “Finding Slope.” [Remind students to find points that are corner points of the grid (lattice points) for accuracy] (5 minutes)
2. Have students work with a partner of their choosing to complete the worksheet. (15-20 minutes)
3. When students have completed the worksheet, review the questions as a class. (15 minutes)
4. Conclude the lesson by brainstorming other possible careers that may use a linear model/line graph in order to find the constant rate of change (slope). For example: advertisers/journalists showing consumers graphs in the newspaper or magazine, ski resorts reporting the grades of their ski slopes, builders and architects determining the type of roof incline based on heating and air conditioning needs. For more details about these careers, see “Career Information” below. If students have questions about other
careers, you may want to look them up using the Bureau of Labor Statistics’ *Occupational Outlook Handbook*. (5 minutes)

**Homework:** Students may either look up one of the careers discussed at the end of class and how that career uses slope or they may research another career not discussed during class. The student must tell how slope is used for that particular career and give information about the education required for that profession, salary, and other basic information about the career. The students should write this up in a paragraph and record it in their math journal.

(Worksheet and Answer Key are attached on the subsequent pages)
Shaynia and McKenzie hired a personal trainer to condition for the local Lighthouse Run/Walk. The personal trainer, Matt, recorded their times and the distance walked.

The red and blue lines on the coordinate plane below show the distances that Shaynia and McKenzie walked over time. You and your partner each choose one of the lines.

a. Work on your own. Choose at least three different travel times. For each time find the distance traveled. Record your results in the table below.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Distance (m)</th>
<th>Rate (m/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Compare your tables. Which person is walking at a faster rate?

- Similarities:
- Differences:
c. How do the graphs show who is walking at a faster rate?

d. Suppose another friend joins the conditioning group and has a faster walking rate than the rates you found. How would a line graph of this friend’s distance walked over time compare with the graphs shown?

e. What other factors would affect the distance you can cover in a given amount of time?

f. How would the graph appear if the women stopped to get a drink of water at a nearby park?

g. How would the graph appear if their personal trainer had the women walk up hills?
Finding Slope
Answer key

Shaynia and McKenzie hired a personal trainer to condition for the local Lighthouse Run/Walk. The personal trainer, Matt, recorded their times and the distance walked.

The red and blue lines on the coordinate plane below show the distances that Shaynia and McKenzie walked over time. You and your partner each choose one of the lines.

a. Work on your own. Choose at least three different travel times. For each time find the distance traveled. Record your results in the table below.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Distance (m)</th>
<th>Rate (m/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0 meters/minute</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0 meters/minute</td>
</tr>
<tr>
<td>15</td>
<td>1500</td>
<td>100 meters/minute</td>
</tr>
<tr>
<td>15</td>
<td>1000</td>
<td>66.7 meters/minute</td>
</tr>
<tr>
<td>30</td>
<td>3000</td>
<td>100 meters/minute</td>
</tr>
<tr>
<td>30</td>
<td>2000</td>
<td>66.7 meters/minute</td>
</tr>
</tbody>
</table>

Walking Distances

![Graph showing distances walked over time for Shaynia and McKenzie.](image-url)
b. Compare your tables. Which person is walking at a faster rate?
- **Similarities**: Both Shaynia and McKenzie are walking at a constant speed
- **Differences**: Shaynia is walking at a faster speed than McKenzie. She is walking 100 meters per minute whereas McKenzie is only walking 66.7 meters per minute

**Shaynia is walking at a faster speed**

c. How do the graphs show who is walking at a faster rate?
- Shaynia’s line

d. Suppose another friend joins the conditioning group and has a faster walking rate than the rates you found. How would a line graph of this friend’s distance walked over time compare with the graphs shown?
- Their line would be steeper than Shaynia’s (and McKenzie’s) line

e. What other factors would affect the distance you can cover in a given amount of time?
- the terrain: if it was hilly vs. if it was flat
- rain vs a sunny day
- how fit the women are

f. How would the graph appear if the women stopped to get a drink of water at a nearby park?
- The line for each person would be horizontal because they are not increasing their distance over time

g. How would the graph appear if their personal trainer had the women walk up hills?
- The line for each person would not be as steep as it appears at this time

Resources:
Lesson 4- Slope
Learning to work with Slope

Outline: We will go over some basic notes about slope and do a few examples about different ways to find it as a class. Then the students will have some time to work with geoboards to work with how slope will look on a graph. The geoboard will allow students to work through a hands-on activity.

Objective: North Carolina Standard Course of Study: Algebra 1- Algebra
Competency Goal 4-The learner will use relations and functions to solve problems
4.02 Use linear functions or inequalities to model and solve problems; justify results.
   c.) Solve using tables, graphs, and algebraic properties.
   d.) Interpret constants and coefficients in the context of the problem.

Essential Question: How can you find the slope of a line?

Learning Outcome: Students will use geoboard to analyze the different movements/shapes of slopes.

Prerequisites: Ordered pairs
   Working with a coordinate plane
   Graph vocabulary (x axis, y axis, origin, etc)
   Graphing ordered pairs

Procedure

Materials: Geoboard

Time Required for Lesson: 50 minutes

Activity:
1.) Students will be provided definitions (topics listed below) on the dry erase board. These definitions will be briefly discussed and modeled for the students.
   - Slope defined as steepness
   - Slope defined as rise over run
   - Slope defined as a formula
   - Positive, Negative, Zero, and Undefined Slope
   Students will be asked to explain the relationship between the graph of a line and its slope. Students will be called to the board and asked to draw a random line for the class to analyze together.
   The steepness of a line is called its slope. The slope is a rate of change. The slope = \( \frac{y_2-y_1}{x_2-x_1} \) (The difference in the y values of two coordinates over the differences of the x values of two coordinates. (10-15 minutes)
2.) Examples
   - Graph the equation \( y=2x + 3 \). Find the slope of the line.
a. To graph the equation on a coordinate plane, make a table of values and plot the points.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

b. To find the slope of the line, choose two points and count the vertical change and the horizontal change between them.

Example: You given the points (1,3) and (-2,1). Find the slope of the line through each pair of points.

c. \[
\frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2-y_1}{x_2-x_1} = \frac{1-3}{(-2)-1} = \frac{-2}{-3} = \frac{2}{3}
\]

(5-10 minutes)

3.) Geoboard Activity

a. The students are split into groups of two (they will chosen by the teacher to try and avoid any incidents from taking place since rubber bands are being used). Each group is a Geoboard and rubber bands. (Close monitoring will ensure that no rubber bands get flicked towards the teacher!)

b. The following slopes will be displayed on the dry erase board for groups to graph on their Geoboard.

1. \( m = 2 \)
2. \( m = -1/2 \)
3. \( m = 3/4 \)
4. \( m = 0 \)
5. \( m = \text{undefined} \)

(15-20 minutes)

Homework: p.116 #12, 16, 18
Find the slope of the line through each pair of points

#12 (5,-1) and (1,6)
$\frac{6 - (-1)}{1 - 5} = \frac{7}{-4} = -\frac{7}{4}$

$\frac{5 - 5}{23 - (-12)} = \frac{0}{35} = 0$

$\frac{22 - (-3)}{(-11) - 7} = \frac{25}{-18} = -\frac{25}{18}$

Resource:

Lesson 5-Finding Equations of Lines
Investigating Linear Equations

Outline: As a class, we will work through various manipulations of equations on the calculator. Students will learn what the slope intercept form is, and how each part of the equation contributes to how the line is formed.

Objective: North Carolina Standard Course of Study: Algebra 1- Algebra
        Competency Goal 4-The learner will use relations and functions to solve problems
        4.03 Use linear functions or inequalities to model and solve problems; justify results.
        e.) Solve using tables, graphs, and algebraic properties.
        f.) Interpret constants and coefficients in the context of the problem.
        4.03 Use systems of linear equations or inequalities in two variables to model and solve problems. Solve using tables, graphs, and algebraic properties; justify results.

Essential Question: How do you write linear equations in slope-intercept form?

Learning Outcome: Students will use their calculators investigate the relationships between linear functions.

Prior Knowledge: Ordered pairs
        Working with a coordinate plane
        Graph vocabulary (x axis, y axis, origin, etc)
        Using graphing calculators

Procedure
Go over homework from the night before: (5-10 minutes)
Find the slope of the line through each pair of points
#12 (5,-1) and (1,6)
\[
\frac{6 - (-1)}{1 - 5} = \frac{7}{-4} = -\frac{7}{4}
\]
#16 (-12,5) and (23,5)
\[
\frac{5 - 5}{23 - (-12)} = \frac{0}{35} = 0
\]
#18 (7, -3) and (-11,22)
\[
\frac{22 - (-3)}{(-11) - 7} = \frac{25}{-18} = -\frac{25}{18}
\]

Activity

Materials: graphing calculator
**Time Required for Activity**: 40-45 minutes

**Part 1: Slope of the line**

1. Have students graph the line $y=x$ on their graphing calculator (This is the line which all others are variations of. Line cuts through the middle of quadrant I and III)
   - Have students identify the following information:
     - What is the slope of the line? (1)
     - Where does the line cross the y-axis? (origin (0))
     - Which direction does the top of the line aim? (right)
2. Have students keep the equation $y=x$ as $Y1$ and enter the equation $y=4x$ as $Y2$
   - Have students answer the same questions as above
     - slope = 4
     - crosses y-axis at origin (0)
     - line aims to the right
3. Have students enter $y=(1/4)x$ as $Y3$ (**make sure they use parentheses**)
   - Answer questions in part 1
     - slope = 1/4
     - crosses y-axis at origin (0)
     - line aims right
4. Ask these questions and draw conclusions:
   - What does the coefficient of $x$(slope) in the equation do to the line? (changes the angle of the line- if $m>1$ line will be very steep (above $y=x$), if $m<1$ line will be less steep (below $y=x$))
5. Have students graph the following on their calculator:
   - $Y1$ as $y=-x$
   - $Y2$ as $y=-4x$
   - $Y3$ as $y=(-1/4)x$
6. Ask students these questions:
   - To which direction do all these lines aim? (left)
   - What part of the equation makes them aim left? (-)
   - Is the steepness of $y=4x$ and $y=-4x$ the same? (yes, they just aim in different directions. $y=4x$ aims to the right and $y=-4x$ aims to the left)

***Students should now understand that the coefficient of the $x$ term in slope-intercept form ($y=mx+b$) is the slope of the line and it tells the direction the top of the line will aim as well as giving an idea of the steepness of the line.

**Part 2: Y-Intercept of the line**

1. Have students clear all equations in their calculator to begin part 2
2. Have students Graph the line $y=x$ for $Y1$ on the calculator, $y=x+5$ as $Y2$, and $y=x-3$ as $Y3$.
3. Have students identify the following:
   - Where does the line $y=x$ cross the y-axis? (origin (0))
   - Where does the line $y=x+5$ cross the y-axis? (5)
Where does the line $y=x-3$ cross the y-axis? (-3)

4. Have students identify from the equation where 0 and 5 appear. (1) $y=x+0$, (2) $y=x+5$, and (3) $y=x-3$. *This constant is the Y-intercept (point where the line crosses the y-axis)*

**Part 3: Testing their linear ability**

1. Have students look at these equations and tell you what the graph will look like before they verify it on the calculator.
   - $y=2x-6$ (slope: line will be steeper than $y=x$, aim right and y-intercept: line will cross y-axis at -6)
   - $y=\frac{-1}{2}x+2$ (line will be less steep compared to $y=x$ and will aim to the left and it will cross the y-axis at 2)

**Entire activity (40-45 minutes)**

**Homework:** Record in math journal the following question: Is $y=2x$ in slope intercept form? Explain why or why not.

Students will need to pick a topic that they would like to compare using two variables that they believe will have a linear relationship. The students can choose anything they wish that is classroom appropriate, but need to begin finding the data. They will have two nights to complete this assignment. They need to find at least ten data points to compare (ie. Weight vs age, kneeling height vs. standing height)

**Resource:**
Lesson 6-Writing an Equation of a Line
Finding the y-intercept

Outline: We will first work through a real world example of a hot air balloon so students can see how objects can follow a linear pattern. Then we will work through an example of finding equations using two points (building on our knowledge of finding slope). Then I will guide students through some remaining questions to help them to find why the graph of a horizontal line will look like and what the equation will be. Then they will work independently on a math journal question about vertical lines. Homework is assigned from the book to supplement the class work.

Objective: North Carolina Standard Course of Study: Algebra 1- Algebra
Competency Goal 4-The learner will use relations and functions to solve problems
4.04 Use linear functions or inequalities to model and solve problems; justify results.
   g.) Solve using tables, graphs, and algebraic properties.
   h.) Interpret constants and coefficients in the context of the problem.
4.03 Use systems of linear equations or inequalities in two variables to model and solve problems. Solve using tables, graphs, and algebraic properties; justify results.

Essential Question: How do you find the y-intercept of a graph?

Learning Outcome: Students will model real-world situations with linear equations.

Prior Knowledge: Working with a coordinate plane
   Slope/finding slope
   Reading ordered pairs off of a graph

Procedure
Go over homework from the night before: (3-5 minutes)
Record in math journal the following question: Is y=2x in slope intercept form? Explain why or why not.
   Yes it is in slope intercept form. The y-value is just equal to zero in this equation. This is the same as y=2x + 0.

Activity

Materials: graphing calculator

Time Required for Lesson: 40-45 minutes

7. Hot air balloon example: (5-10 minutes)
The hot air balloon rises at a rate of 110 ft/min. After one minute, the balloon has reached a height of 6135 feet. Write an equation for the balloon’s altitude $a$ as a function of time $t$.

Step one: Write the equation in the form $y=mx + b$

- This means altitude = slope · time + vertical intercept
  1. $a=mt + b$

Step two: Substitute the slope of the line for $m$ in the equation.
  - $a=110t + b$

Step three: Find the vertical intercept by substituting the coordinates of any point on the graph for $t$ in $a$ in the equation. Then solve for $b$. [choose the point $(1,6135)$]
  1. $6135 = 110(1) + b$
  2. $6135 = 110 + b$
  3. $6025 = b$

Step four: Substitute 6025 for $b$ in the equation $a=110t + b$.

So the equation is $a=110t + 6025$.

8. Find an equation through the points (2,6) and (3,1) (5-10 minutes)

- we need to find the equation in the form $y=mx + b$
- First we need to find “m” (slope)
  - Since we already have two points, we can just use those to find the slope.
    \[
    \frac{6-1}{3-2} = \frac{-5}{1} = -5
    \]
  - We can now substitute this in for $m$. $y=-5x + b$
- Now we need to find the $y$-intercept (vertical intercept)
  - To do this we must pick one of the ordered pairs (doesn’t matter which one, both will work). We will then substitute the coordinates in for $x$ and $y$ respectively in the equation.
    - $y=-5x + b$ (let’s pick the point (2,6))
      1. $6=-5(2) + b$
      2. $6=-10 + b$
      3. $16=b$
- Now we can substitute 16 in for $b$.
  - $y=-5x + 16$
- To check to see if this equation actually works, we can plug the equation into our calculator and check to make sure these two points appear on the line. If they do not, then you have the wrong equation.

Inquiry Based Questions: (10-15 minutes)

- From the second example that we did, substitute the coordinates of the second point for $x$ and $y$ in the equation $y=-5x + b$. Why do you get the same value for $b$?
- Graph the horizontal line through (1,2) and (5,2). Write an equation for this line.
  - What is the slope of any horizontal line?
  - What do all of the horizontal lines have in common?
Math Journal Question: Find an equation of the vertical line through the point (-2,3). Explain your reasoning. (5-10 minutes)

**Homework:** p.127 #2, 6, 7, 9, 18, 20

Students will also need to bring in the data they should have been collecting for tomorrow’s lesson.

**Directions for 2, 6, 7, 9:** Find an equation of the line with the given slope and through the given point

**#2:** Slope=1; (5,8)
\[ y = x + b \]
8 = 5 + b
b = 3
\[ y = x + 3 \]

**#6:** Slope = (-1/2); (1,6)
\[ y = mx + b \]
\[ y = (-1/2)x + b \]
6 = (-1/2)(1) + b
6 = -1/2 + b
b = 12
\[ y = (-1/2)x - 12 \]

**#7:** Slope= 0; (-2,5)
\[ y = mx + b \]
\[ y = 0(x) + b \]
b = 5
\[ y = 5 \]

**#9:** undefined slope; (1,1)
\[ x = 1 \] (We know it’s a vertical line since we know that the slope doesn’t exist, i.e.-x=?)

**Directions for 18,20:** Find an equation of the line through the given points

**#18:** (15,-3), (15,2)
\[ y_2 - y_1 \]
\[ x_2 - x_1 \]
\[ = \frac{2 - (-3)}{15 - 15} = \text{undefined slope (bottom equals 0)} \]
Since the slope is undefined, we know that the line must be vertical, x=15
Since both coordinates go through the x-value 15, we know the equation must be
\[ x = 15 \]

**#20** (-5,-7), (7,6)
\[ y_2 - y_1 \]
\[ x_2 - x_1 \]
\[ = \frac{6 - (-7)}{7 - (-5)} = \frac{13}{12} = \text{slope} \]
\[ y = mx + b \]
\[ y = \frac{13}{12}x + b \]
\[ 6 = \frac{13}{12}7 + b \]
\[ 6 = \left( \frac{91}{12} \right) + b \]

\[ b = \left( \frac{-19}{12} \right) \]

\[ y = \left( \frac{13}{12} \right) x - \left( \frac{19}{12} \right) \]

Resource:
Lesson 7-Modeling Linear Data
To be or not to be linear

Outline: With the information students had gathered on their own, they will bring the data to class and plot it on a coordinate plane. After plotting the data, students will find the line of best fit. By allowing the students to pick their own data, they will be more interested in their results and will learn how to find a line of best fit.

Objective: North Carolina Standard Course of Study: Algebra 1- Algebra
Competency Goal 4-The learner will use relations and functions to solve problems
4.03 Use systems of linear equations or inequalities in two variables to model and solve problems. Solve using tables, graphs, and algebraic properties; justify results.

Essential Question: How do you fit a line to data?

Learning Outcome: Students will make predictions from data they collected about the equation that models the data.

Prior Knowledge: Ordered pairs
Working with a coordinate plane
Slope
Writing Linear Equations

Procedure

Materials: graph paper
Rulers
Homework from the night before (data collected)

Time Required for Lesson: 50 minutes

Activity:

1. Students will use the two variables that they think may have a relationship (completed for their homework). For Example: Height Vs. Weight, Time at mall Vs. Money spent, Foot size Vs. How high you can jump, Fingernail length Vs. Finger length, etc.
2. Students then need to create a coordinate plane with the X and Y axis labeled with the two chosen variables, an appropriate numbering scale, and a title.
3. Students must plot the given data collection points.
4. After students have plotted their points, students will draw a best fit line for the given data. If the data has no correlation students may draw a line to continue with the forthcoming parts of the lesson.

*A best fit line is a line that comes as close as possible to all of the points on the plot. The line does not have to go through all of the points.
*Even if a few points are far away from the line, you can say that the data is linear. If the points are sporadic and don’t really follow a pattern for a line, then the data is not linear. (Examples will be drawn on the board)

5. Once the best fit line is drawn students will gather the slope and intercepts from their line. They will then find the equation of the line.
6. Finally, you may have a discussion concerning positive, negative, and no correlation or have students answer the given questions (see Question Sheet attachment)

Homework: Students will complete any of the Question Sheet that they did not get a chance to finish in class.


**Discussion Questions**

1. What other variables would have positive correlation? Explain and justify your choices.

2. What other variables would have negative correlation? Explain and justify your answer.

3. Given a graph of an equation and list the steps to determine the equation of the line in slope-intercept form.

4. In your math journal, write three real world examples of data that would model linear data.

Resource:
Lesson 8- Assessment

Outline: We will simply go over homework from two nights ago, and then the students will take their test.

Homework: (10-15 minutes)
Directions for 2, 6, 7, 9: Find an equation of the line with the given slope and through the given point

#2: Slope=1; (5,8)
y=x + b

8=5 + b
3=b
y=x + 3

#6: Slope = (-1/2); (1,6)
y=mx + b
y=(- 1/2)x + b
6= (-1/2) (1) + b
-12= b
y= (-1/2)x – 12

#7: Slope= 0; (-2,5)
y=mx + b
y=0(x) + b
y=b
5=b
y=5

#9: undefined slope; (1,1)
y=mx + b
x=1 (We know it’s a vertical line since we know that the slope doesn’t exist, i.e.- x=?)

Directions for 18,20: Find an equation of the line through the given points

#18: (15,-3), (15,2)
\[
\frac{y_2-y_1}{x_2-x_1} = \frac{-3-2}{15-15} = undefined \text{ slope (bottom equals 0)}
\]
Since the slope is undefined, we know that the line must be vertical, x=?
Since both coordinate go through the x-value 15, we know the equation must be x=15

#20 (-5,-7), (7,6)
\[
\frac{y_2-y_1}{x_2-x_1} = \frac{6-(-7)}{7-(-5)} = \frac{13}{12} = \text{ slope}
\]
y=mx + b
y= \left(\frac{13}{12}\right)x + b
6=\left(\frac{13}{12}\right) 7 + b
6=\left(\frac{91}{12}\right) + b
\[
\begin{align*}
\frac{-19}{12} &= b \\
y &= \frac{13}{12}x - \left(\frac{19}{12}\right)
\end{align*}
\]

Test will take the remainder of the class time (30 minutes)
Test: Linear Equations

Name:

Answer the following questions given that you will be given a pair of points that pass through a line.

**Slope:**
1. Define what the slope of a line is:

2. What is the formula for finding slope?

**Y-Intercept:**
1. Define what the y-intercept of a line is:

2. How do you find the y-intercept?

Determine slope, y-intercept, slope-intercept form of a line, and graph the line for the following pairs of points.

1. (2,5) and (7,-10)
   - Slope:
   - Y-intercept:
   - Slope-intercept form:
   - Graph above
2. (-1,3) and (-3,-1)
   Slope:
   Y-intercept:
   Slope-intercept form:
   Graph above

Resource:
http://www.netc.org/classrooms@work/classrooms/middleteam/assessing/equations.pdf
Test: Linear Equations

Name:

Answer the following questions given that you will be given a pair of points that pass through a line.

**Slope:**
1. Define what the slope of a line is: the steepness of the line, rate of change

2. What is the formula for finding slope?
   \[
   \frac{y_2 - y_1}{x_2 - x_1}
   \]

**Y-Intercept:**
1. Define what the y-intercept of a line is: vertical intercept

2. How do you find the y-intercept? First find the slope, then plug in one of the ordered pairs into the slope intercept form to find it.

Determine slope, y-intercept, slope-intercept form of a line, and graph the line for the following pairs of points.
1. (2,5) and (7,-10)

Slope: \( \frac{y_2 - y_1}{x_2 - x_1} = \frac{-10 - 5}{7 - 2} = \frac{-15}{5} = -3 \)

Y-intercept: \( y = mx + b \)
  
  \[ y = -3x + b \]
  
  \[ 5 = (-3)(2) + b \]
  
  \[ 5 = -6 + b \]
  
  \[ 11 = b \]

Slope-intercept form: \( y = -3x + 11 \)

Graph above
2. (-1,3) and (-3,-1)

Slope: \[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - 3}{(-3) - (-1)} = \frac{-4}{-2} = 2
\]

Y-intercept: \[y = mx + b\]

\[
y = 2x + b \\
3 = (-1)(2) + b \\
3 = -2 + b \\
5 = b
\]

Slope-intercept form: \[y = 2x + 5\]

Graph above
Resources:


http://www.netc.org/classrooms@work/classrooms/middleteam/assessing/equations.pdf