Solving Linear Equations

Textbook: McDougal Littell Algebra I

My primary teaching style for this unit is inquiry. Solving linear equations can become algorithmic, and this can be troublesome in the aspect that algorithms usually emphasize memorization and detached steps as opposed to understanding and conceptual knowledge. Since understanding solutions of linear equations naturally leads to solving quadratic equations and higher-degree equations, it is of utmost importance that students understand why the steps of the “algorithm” work; hence, the need to use questioning in order to build a sound foundation.

DPI Algebra I goals:

1.01 Write equivalent forms of algebraic expressions to solve problems.
1.02 Use formulas and algebraic expressions, including iterative and recursive forms, to model and solve problems.
4.01 Use linear functions or inequalities to model and solve problems; justify results.

The main objective of the lesson is to represent and analyze mathematical situations and structures using algebraic symbols. This essentially is what solving linear equations entails. I am particular concerned that students understand equivalent equations, for I use the concept to extend their knowledge of solving simpler equations to more complex equations (i.e. variable on one side to variable on both sides). In doing a variety of word problems, students learn and understand how mathematics can represent real life. In addition, students will develop problem-solving and reasoning skills. The latter will come from their discourse and the activities that ask them to examine multiple solutions to a linear equation. The way that students justify whether two equations are equivalent uses reasoning and proof skills to make an argument. Through cooperative learning activities, students communicate with each other.

I have not used technology much in my lesson plan. This is not a reflection on my belief of how technology should be used in education but rather that there are not many opportunities to use technology in this unit. In section 3.2 I used a powerpoint to explain the concept of solving equations using multiplication or division. Powerpoint is useful for being able to display visual aids in addition to showing work for a problem or concept. I could have used excel to create graphs and tables based on real-world data, but I think that it takes more time than it is worth. Besides, the textbook has some word problems that accomplish this nicely.

I believe that discourse is important because it provides valuable information about how students think about a topic. In many of my sections, students are asked to explain a solution in words. I think that doing so makes them aware of what they are actually doing when they solve an equation. Also, discourse is a form of communication, and it will appeal to some students. Granted, the discourse I use is mostly found in homework or in some cooperative learning. Again, discourse can be used to facilitate the conceptual learning that will encourage students to avoid thinking of solving linear equations solely as an algorithm. Generally, discourse and higher-level of understanding are synonymous. That is, if students can write well about a topic, then they probably understand it well.

My primary form of assessments is through the learning activities. I think it is more useful to do assessments as students first encounter new material as opposed to testing or
quizzing them. This way I am able to observe misconceptions and confusions as soon as possible. On the other hand, quizzes and tests are more helpful for seeing if students have mastered the material; still helpful, but more of a metaphorical end to mathematical education than a means to develop mathematics. As far as teaching goes, quizzes and tests are helpful for seeing where the problem spots of the students are, and thus this allows me to focus class time on the more confusing aspects of solving linear equations. I would give two quizzes: one after going over solving equations using multiplication/division and one after going over solving equations with variables on both sides. The test would be assigned after a day that the last section is covered. Homework will be graded by effort and will be corrected on the following day that it is assigned.

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Motivation: Linear equations are commonly used to model real-life situations. For example, cost and production of a company, unit conversions such as Fahrenheit to Celsius in temperature, can be represented with a linear function. Changes in the stock market can be modeled with a linear function.

Misconceptions: Students may think that checking solutions is redundant and useless. Reconciling this misconception by explaining that if a solution does not “check out”, then it is not a solution and it makes the equation false; therefore, it causes problems as opposed to solving one. Then, I can relate this to how linear equations are important in real life, and how a mistake could cause serious consequences.

Instructional Activities:
1. Write expressions (with no variables) on index cards so that it is possible to match up each index card with only one other index card. Then, give students the cards, and they have to find other students with an equivalent expression. When they match the expressions, the two students sit down and are partners for the next activity. After the activity, tell students that the equivalence of expressions is called an equation, and remind them that variables also can be used in expressions. (http://www.uen.org/Lessonplan/preview.cgi?LPid=23403)

2. Students are in groups of 2. Students are given a combination of red and blue tokens; it is possible to give all red or all blue. Red indicates a negative value, and blue indicates a positive value; each token is worth one unit. Students are to find all possible combinations of any size to match a given number; make sure to use zero, a positive number, a negative number, and a number that exceeds the amount of tokens that the students are given. Furthermore, students should write down relationships or patterns that they observe with the combinations. After the students are done with each number, ask the class, as a whole, what patterns they noticed. In particular, establish the concept of inverse operations through the realization that a red token cancels out a blue token.

Then, control the number of red tokens or the number of blue tokens so that the students are virtually finding solutions to a linear equation. For each “equation”, ask the students to write down the “equation” using mathematics. Tell students that finding the unknown quantity is finding the solution to the equation, the number that makes the equation true. Thus, we can check the solution to an equation by seeing if it makes the two sides of the equation equal.

Questions: What is a solution to a linear equation?

Why are inverse operations used to solve linear equations?
Can a linear equation have no solution? Why or why not?
Give an example of a linear equation with no solution.
How can two different equations have the same solution?
Can you think of an equation where its solution is all real numbers?

Leads to: Multiplying and dividing to solve linear equations. By considering linear equations, students may naturally wonder about equations where the variable is not raised to the first power.
Assessment: Observe student’s ability to find expressions through activity 1. Questions will provide input to how well the students understand solving linear equations as well as the content knowledge. In activity 2, students have the chance to write down mathematical patterns; thus, writing becomes a way of expressing their ability to think mathematically.

Homework:

1) Solve -8+x=2.
2) Solve 4-x=7.

Write and solve an equation for problems 3-5:

3) Sam has 3 more jellybeans than Sarah has. Sam has 25 jellybeans. Find the amount of jellybeans that Sarah has.

4) Maria runs 3 miles on every Monday. If she runs a total of 20 miles every week, how many miles does she run per week if she does not run on Monday anymore?

5) If Ben’s backyard is a square with sides of 3 meters, what is the perimeter of his backyard?

6) Convert the following sentences into equations, and then determine which two are equivalent:

   a) Five exceeds a number is 19.
   b) 19 less than 39 is 5 more than 16.
   c) Seven more than the sum of five and a number is 5 more than 19.

   3.2 Solving Equations Using Multiplication and Division

Prior Knowledge: Solving linear equations using addition/subtraction,

Objectives: Students will be able to solve linear equations using multiplication and division and will be able to use these linear equations to solve real-life and geometric problems.

Motivation: These kinds of linear equations can be used to solve problems involving similar triangles such as an architecture building a bridge using triangles as support. Similar ratios are used for unit conversions in science. Gardeners and landscapers use multiplication and addition to find the area or perimeter of a certain area. You can calculate how to split the cost of pizza by using linear equations.

Instructional Activities: 1. With graph paper, solve two (one involving multiplication and one involving division) linear equations (that has form bx=a) using the following:

   a) If any of the constants or coefficients is negative, make them positive i.e. multiply both sides by -1. Observe that if two negative constants are turned into positive numbers, then the solution doesn’t change. Thus, the final answer in (f) will be negative if there is only one negative constant. Show after the activity that this step is really transferring the negative sign to the variable by inverse operations.

   b) Identify the side of the equation that doesn’t have a variable. For the sake of describing the activity, call this constant a. a represents the area of some rectangle.
c) Draw a rectangle with length of 1 and width a to scale. Refer to this as the area rectangle; that is, it has the given area and given side but does not accurately model the equation.

d) Now consider the other side of the equation. It will have the form bx, where b is the coefficient. Think of b as the length of two opposite sides of the rectangle, and x as the other two sides of the rectangle. Moreover, we can think of x as the number that when multiplied by b gives the area a.

e) In particular, “divide both sides by b”, this is the inverse operation of multiplication. This means to divide the rectangle vertically b times so that virtually the width a is divided by b. Remind students that dividing by fractions is equivalent to multiplying by their reciprocal.

f) This will simplify the equation to \( x = \frac{a}{b} \); this is the solution to the equation. In short, the rectangle is now divided into equal sections. Thus, the area of each section is the solution to the linear equation.

While each person should have graph paper, the teacher should set up a powerpoint with these steps. Indeed, a powerpoint enables the teacher to show each step one at a time, and also allows the teacher to display work simultaneously with the picture of the rectangle. The purpose of this activity is mainly to teach students that solving linear equations is like working backwards. They need to understand that multiplying is the inverse operation of division and vice versa. In addition, they are given a practical use for this section.

2. Get into groups of 4. Using the following menu, create a problem that involves a linear equation which is solvable using multiplication or division. For example, solve this problem on the overhead or on the blackboard and explain it to the class: ‘Sally had $40, and she bought some number of lasagnas from Franklin St. Pizza & Pasta. After she paid for them, she had $18.25. How many lasagnas did she buy.’ Be sure to find the solution.

After creating the problem, trade problems with another group.

<table>
<thead>
<tr>
<th>Food Item</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baked Ziti</td>
<td>$5.75</td>
</tr>
<tr>
<td>Chicken Cutlet Parmigiana with Spaghetti</td>
<td>$8.50</td>
</tr>
<tr>
<td>Eggplant Parmigiana with Spaghetti</td>
<td>$7.25</td>
</tr>
<tr>
<td>Eggplant Rollentine with Spaghetti</td>
<td>$7.75</td>
</tr>
<tr>
<td>Lasagna</td>
<td>$7.25</td>
</tr>
<tr>
<td>Manicotti</td>
<td>$6.50</td>
</tr>
<tr>
<td>Ravioli (Cheese)</td>
<td>$6.50</td>
</tr>
<tr>
<td>Sausage and Peppers with Spaghetti</td>
<td>$7.75</td>
</tr>
<tr>
<td>Shrimp Parmigiana with Spaghetti</td>
<td>$10.25</td>
</tr>
<tr>
<td>Stuffed Shells (Cheese)</td>
<td>$6.50</td>
</tr>
<tr>
<td>Vegetable Lasagna</td>
<td>$7.25</td>
</tr>
</tbody>
</table>

(http://www.franklinstpizza.com/menu.html)

This activity serves as an introduction to word problems for this section. Having students create problems allows them to see how to translate verbal sentences into expressions or equations. Involving the restaurant menu makes the problem more realistic.

Questions: Why are multiplication and division inverse operations?
What does it mean in words to be a solution to $5x=10$?

Consider the equation $bx=10$. What numbers can $b$ be so that $x$ is a whole number?

Are the equations $5x=10$ and $20x=40$ equivalent? Justify your answer without finding their solutions explicitly.

For the same equations, now that they are proven to be equivalent, if the solution to $5x=10$ is $x=2$, what does this imply about the solution to $20x=40$?

Assessments: The questions test the student’s ability to understand inverse operations, solutions to linear equations, and equivalent equations. The group activity(2) allows the teacher to see how well the students can solve a word problem. Interactions among group members allow for observations on the ability of the students to understand and apply the section.

Homework:

1) $9x=81$
2) $x/6=42$
3) There are 56 students in the class, and $\frac{1}{2}$ of the students are boys. How many students are boys?
4) Chelsea runs 3 miles every day. How many miles does she run in a week?
5) Determine if $x/6=42$ and $x/5=40$ are equivalent equations. Justify your answer.
6) Bob has 10 meters of fencing. What is the greatest area that he can enclose in a rectangle, provided that he only uses whole numbers to measure his fencing?

3.3 Solving Multi-step Linear Equations

Prior Knowledge: Using multiplication/division and addition/subtraction to solve linear equations, order of operations.

Objectives: Use multiple operations to solve an equation.
Use multi-step linear equations to solve real-world problems.

Motivation: Calculating averages, scientific formulas in chemistry and physics(e.g. formula for average velocity: $v=\Delta d/\Delta t$ where $d=distance, t=time$), calculating profit(total revenue-total cost=$ax-b$ where $a=hours of work, x=salary in dollars/hour, and b=expenses$)

Instructional Activities:
1. Warmup(1-5) P.145 in Algebra I textbook: Simplify:
   1.2(3-9x)  2. 3x-7-8x  3.-7(2x-11)  4.7x-5x+3)  5.6x-2(4x-1)
   6.- (x-5)

A common misconception with this section is with using the distributive property correctly. While students may understand to distribute the coefficient, they sometimes will forget to distribute the negative sign. Thus, spending 5-10 minutes on this warmup is worth it for the misconception it clarifies.

2. Do two examples(examples 1 and 4 from pg. 145 and 146 in Algebra I respectively):
Before doing each step to solve equation, ask students what they think the correct step is. Emphasize that there are several possible steps and that the ultimate goal is to end up with the solution. A misconception here is that the distributive property may be the first step for all linear equations with parentheses; example 4 shows that not distributing makes the problem easy to solve. For the most part, this section is really synthesizing everything that the students have learned from the previous sections in chapter 3. Thus, it is not unreasonable to expect that the students should enter the classroom with some idea of how to solve multistep equations. Another misconception is that multistep equations look more complicated than they actually are. More practice and exposure with multistep equations should dissolve this misconception.

3. Divide class into groups of 4. Give each group a multistep equation. They should not only solve the equation but also find multiple ways to find the solution. Each group should present their problem on the blackboard with each possible way to find the solution. In addition, they should explain which way involves the least amount of work (i.e. the most efficient way).

Cooperative learning, here, helps to lessen the expected complexity of solving multistep equations. Also, it helps to facilitate the idea of solving problems in multiple perspectives, since different people will think in different ways.

Questions: Does solving an equation in a different way result in a different solution? Explain.

How do you solve a multi-step equation?

How do you simplify an equation where the coefficient of the variable is a fraction?

Assessment: #3 in homework helps to test the student’s ability to understand how solving for a solution works. The group activity(2) provides for an opportunity to observe how students solve equations and which method they prefer. The warm-up shows how much students remember from simplifying expressions. The word problems in the homework show how well the students are able to apply the section to translating sentences into mathematical equations.

Homework: 1) \( \frac{x}{2} + 5 = 10 \)

2) \( 3(x-4)=24 \)

3) Identify the error in these solution steps(*Algebra I*, p.149 #41):

\[
\begin{align*}
2(x-3) & = 5 \\
5-3x & = 10 \\
\frac{1}{4}x & = 2 \\
x-3 & = 7 \\
2x-3 & = 5 \\
2x & = 10 \\
x-2 & = 28 \\
x & = 30 \\
x & = 4
\end{align*}
\]

4) You have a 90-lb calf you are raising for a 4-H project. You expect the calf to gain 65 lbs per month. In how many months will the animal weigh 1000 lbs?(*Algebra I*, p.149 #52)
5) **Consecutive integers** are integers that follow each other in order (e.g. 5, 6, 7). You want to find 3 consecutive integers whose sum is 84. *(Algebra I, p.149 # 50)*

a) Why does \( n + (n+1) + (n+2) = 84 \) model the situation?

b) Solve the equation in part (a). Then, find the consecutive integers.

6) Explain in words how to solve the equation \( \frac{1}{4}x + 2 = 6 \). Then, find the solution and explain what it means to be a solution of \( \frac{1}{4}x + 2 = 6 \).

### 3.4 Solving Equations with Variables on Both Sides

**Prior Knowledge:** Solving multistep linear equations, identities, equations with no solutions

**Objectives:** Students will understand how to solve equations with variables on both sides.

Students will be able to use these equations to solve real-world problems.

**Motivation:** Architecture, economics (“membership clubs” like dvd companies that have a initial payment fee and then charge a small fee for each product ordered), sports (in basketball, finding out how many field goals are needed to tie the other team’s score),

**Instructional activities:**

1. a) Ask students to solve the equation \( 5x + 2 = 12 \), which has solution of \( x = 2 \).

   b) Now, ask students to consider \( 5x + 2 + 2x = 12 + 2x \). In particular, ask them to compare this equation to the equation in (a). They should note that \( 2x \) is added to both sides. If they don’t make the observations, compare the solutions of the two equations: they are equal. Thus, adding or subtracting variables to both sides does not change the equation. Hence, we can use this technique to solve equations with variables on both sides.

   c) Now ask them to solve \( 4x + 6 = 10x \), using what they’ve learned from the equivalent equations in (a) and (b).

   d) Then, similarly consider \( 2x + 4 = 6x - 3 \).

   e) Then consider equations like \( \frac{1}{4}(x + 2) = 6x - 3 \).

The teaching philosophy, here, is to encourage the students into extending the previous sections into this section. A misconception with this section is that students are unsure how to deal with the coefficients of the variables in the linear equations. Homework that asks students to identify errors in a solution step and further practice with linear equations should clarify this misconception.

2) **Tutoring Stations:** Set pairs of desks in a circle within a circle (so that each student is facing another student). Assign each pair a linear equation with multiple steps and that involves variables on both sides. In addition, use word problems from the textbook. Students on the inner circle are responsible for knowing how to solve the problem, and students in the outer circle will solve the problem. If the outer students need help, the inner students will tutor them. After about 1-2 minutes, the outer students will shift to a different problem. Make sure that inner circle students get to be in the outer circle and vice versa (take about 20 minutes).
This cooperative learning activity has several benefits. The first is that the activity will increase the students’ experience with linear equations, thereby increasing their confidence and skill. Having a student teach a problem to another student has mutual benefits: the tutor increases his ability to explain why solution steps to a problem are legitimate, and the outer circle student is able to learn how to do the problem.

3) Now each student will remain the circles and will think of a word problem regarding equations with variables on both sides. For example, use example 5 from the textbook (p. 156):

A video store charges $8 to rent a video game for five days. You must be a member to rent from the store, but the membership is free. A video game club in town charges only $3 to rent a game for five days, but membership in the club is $50 per year. Which rental plan is more economical?

Students will solve their own problem, and then share the problem and the solution with their partner.

Misconceptions: Again, equations with variables on both sides look intimidating, and students may think that they are necessarily more difficult to solve. Such is not the case, and practice and exposure with these kinds of equations should clarify this misconception. Another misconception is that students may think that they cannot add or subtract variables to both sides. The extension activity done in class should prove that equations like 2+5x=2 and 2+5x+2x=2+2x are equivalent.

Questions: Are 5x+2+2x=12+2x and 5x+2=12 equivalent?

Can an equation have more than one solution?

Can an equation have no solution?

Assessments: The inquiry activity indirectly tests student’s knowledge of equivalent equations. Most of the assessments in class are in the cooperative learning activity. **

Homework: Solve for x.

5x-4=16
6(x-2)=x-5
10x+7=-5(-x-7/5)+5x
5x+2=7x+2

A gym offers to packages for yearly membership. The first plan costs $50 to be a member. Then each visit to the gym is $5. The second plan costs $200 for a membership fee plus $2 per visit. Which membership is more economical? (Algebra I P.156 Extra Example 5)

3.5 Linear Equations and Problem Solving

Prior knowledge: Solving linear equations, graphs (line, bar, pie, etc.)

Objectives: Use diagrams to solve real-life problems. Use tables and graphs to facilitate problem-solving.

Motivation: Business (graphs/tables help people to visualize what an equation says), Physics (calculating position of an animal), economics (savings)
Instructional Activities: 1. Divide class into groups of 4. Ask them to solve the problem on pg. 161 example 2 in *Algebra I* without using a table:

At East High School, 579 students take Spanish. This number has been increasing at a rate of about 30 students per year. The number of students taking French is 217 and has been decreasing at a rate of about 2 students per year. At these rates, when will there be three times as many students taking Spanish as taking French?

Then, solve it using a table in front of class. They will write a short paragraph per each group on how and why tables are useful for problem-solving. Then they will share their paragraph. Do a similar activity with a diagram. Example 3(*Algebra I Pg.162*): A gazelle can run 73 ft/sec for several minutes. A cheetah can run 88 ft/sec but only for about 20 seconds. How far away from the cheetah does the gazelle need to stay for it to be safe?

Then, using example 3 students will draw a graph to model their solution.

A big part of this activity is promoting good problem-solving skills. Tables, diagrams, and any visual aid helps to organize information, which in turn can highlight patterns and spark ideas. Furthermore, students will write about which visual aid they prefer to use or if their usage depends on the word problem.

**Misconceptions:** There are not any misconceptions regarding the content of this section, but the problem-solving strategies in this section can help make word problems easier for people that either have a hard time with a certain real-world connection (e.g. physics) or for people that speak English as a second language.

**Questions:** Why should you use visual aids to solve a word problem?

- Compare and contrast a verbal model with visual aids.

**Assessments:** Questions will verify that the students understand the main point of the activity: the value of using tables and diagrams to solve word problems. The activity is a form of discourse, and the group’s paragraphs provide insight to their “philosophy” of problem-solving. The homework contains word problems that they are required to use diagrams or tables.

Homework: P.163 #6-8: 1. A rectangular package can have a combined length and girth of 108 inches. Suppose a package that is 36 inches long and as wide as it is high just meets the regulation. (#6 involves picking from 3 diagrams to best represent the relationships)

2. Choose the equation you would use to find the width of the package, and find the width.

\[
X + 36 = 108 \\
2x + 36 = 108 \\
4x + 36 = 108
\]

3. Make a table showing possible dimensions, girth, and combined length and girth for a “package that is 36 inches long and as wide as it is high.” Which package in your table just meets the postal regulation?

4(*Algebra I P.165 # 22*). A) Two friends are 60 miles apart. Sally starts from the college and heads east, riding at a rate of 21 miles/hr. At the same time Teresa starts from the river and heads west, riding at a rate of 15 mi./hr. Draw a diagram.

b) How far does each cyclist ride in t hours?
c) Write and solve an equation to find when they meet.

d) Would a park that is 26 miles west of the river be a good
meeting place? Explain, and if you used a visual aid or verbal model explain which model you
used and why in words.

3.6 Solving Decimal Equations

Prior Knowledge: Solving equations with variables on both sides, converting fractions to
decimals and decimals to fractions

Objectives: Students will learn how to solve equations with decimals and will be able to round
solutions. Additionally, students will be able to solve real-life problems using decimal equations.

Motivation: Economics( costs/income), physics (velocity of animals/humans),
arichitectures(bridge gaps), statistics(interpreting % of bar graphs)

Instructional activities: 1a. Each student will use a calculator to consider several
fractions/decimals:

Splitting the cost of a $12.89 pizza between 3 people: 4.296666…(Algebra I, P. 166
Example 1)

The area of a circular pool that has a radius of 14.3 meters($A = \pi r^2$): 642.424282

The tax on a $19.99 t-shirt if sales tax is 5%: $0.9995

Ask students if they can use “integer equations” to find these numbers. In particular, lead
their discussion to the need to have decimal coefficients and constants i.e. decimal
equations (in general, do whole numbers suffice to describe all quantities?).

1b. Rounding up or rounding down: Now, students will reconsider the pizza example in
1a. Ask them to write a decimal equation to describe it where x is the divided cost. They
should find that 4.29666… is the solution to 3x=12.89. Ask them if it is possible to have
exactly $4.29666… Since it is not the case, consider if they should round up or down.
Discussion should eventually lead to the point that rounding up gives a less round-off
error, but, more importantly, rounding down to 4.28 does not reflect the price of the
pizza. Thus, determining how to round depends on the problem.

1c. Rounding the “equation” vs. rounding the solution: Ask students if rounding before
the solution is more accurate than rounding the solution. Consider the second example:
$A = \pi(14.3)^2$. Since $\pi$ does not terminate, it can be rounded to 3.14. Doing so gives
642.0986 which rounds again to 642.10. On the other hand, “rounding the solution” gives
642.42. Obviously, 642.42 is more accurate than 642.10; examine why this is.
Discussion should lead to the fact that “rounding the equation” usually involves rounding
more than once. Thus, “rounding the solution” is more accurate and should be preferably
done over rounding in the intermediate steps of equation-solving.
1d. Let the class do the “round-off error analysis” for the other two problems (including changing sentence into equation).

2. Get students in groups of 3-4. Tell them to come up with a list of situations that involve rounding up, rounding down, or leaving the decimal. To facilitate this activity, here are some brainstorming situations: scientific experiments (e.g. measuring the length of a piece of string, the gravitational pull), sports (calculating points), statistics (averages of grades). After 5-10 minutes, compile group lists into a class list.

The point of this activity is to get students to understand how rounding affects the final answer. As a bonus, they get to compile this list of situations that help them in problem-solving.

Questions: Why is round-off error important?

Compare and contrast decimal equations with equations with integer coefficients and constants.

Assessments: The inquiry activity shows the students knowledge of how decimal equations represent the real world and also how they understand how round-off errors affect calculations. The questions verify the students’ understanding of the inquiry activity. The homework word problems show the student’s ability to solve decimal equations.

Homework: Round answers to the nearest hundredth.

3.5x + 4.2=10 2.2x+4.2=10x+4.2

3. You have $9.14 to spend on lunch. If you want to leave a 15% tip, then what is your price limit for the lunch with tax included? (Algebra I P.170 # 49)

4. Bridge sections expand as the temperature goes up, so a small expansion gap is left between sections when a bridge is built. As the sections expand, the width of the gap gets smaller. Suppose that for some bridge the expansion gap is 16.8 mm wide at 10°C and decreases by 0.37 mm for every 1°C rise in temperature.

If the temperature is 18°C, by how many degrees did the temperature rise? By how much would the width of the gap decrease? What would the new width of the gap be? Round to the nearest tenth of a millimeter (Algebra I, P. 171 # 56).

3.7 Formulas and Functions

Prior Knowledge: Solving decimal equations

Objectives: Students will learn to solve formulas for a variable. Also, students will learn how to rewrite equations in function form.
Motivation: Economics (interest), finding area and perimeter of objects, finding the time it takes for a vehicle (like a car) to move a certain distance if it has a fixed velocity

Instructional Activities: 1. Warmup (Algebra I pg. 174):

Find the value of each expression when a=3 and b=-4:

1. a - b  
2. 2ab  
3. 6a+12a-b  
4. b(5-a)  
5. (-3b)(-a)

In addition to being a warm-up, these 5 problems serve as an introduction to formulas. Students will consider the same problems, but now each solution will be equal to some variable c (e.g. #1 becomes a-b=c). Ask the students to treat a and c as constants and b as a “variable”; as such, they will solve for b (e.g. #1 becomes b=a-c). Tell them that this is called solving for the variable.

2. Divide class into groups of 3-4. Assign each group a formula problem:

1. Solve A=lw for l. Then, find L if A=25 m and w=5 m. Write in words what the solution means.

2. Solve \( C = \frac{5}{9} (F - 32) \) for F. Then, find C if F=25°F. Write in words what the solution means.

3. Solve \( d=rt \) for t. Then, find t if \( d=50 \) miles and \( r=25 \) miles/hr. Write in words what the solution means.

4. Solve \( A = \frac{1}{2}bh \) for h. Then, find h if \( A=10 \) inches and \( b=5 \) inches. Write in words what the solution means.

5. Solve \( C = 2\pi r \) for r. Then, find r if \( C=45.5\pi \) meters. Write in words what the solution means.

6. Solve \( I=prt \) for p. Then, find p if \( I=$1456, r=0.015, \) and \( t=1 \) year. Write in words what the solution means.

After all groups are done, each group will present their problem and its solution. (Algebra I Pg.174-175 Example 1,2,3)

This activity is to expose the students to different types of formulas, some of which they have encountered before. The discourse part of the activity emphasizes the fact that formulas are not just mathematical equations but equate some real-life quantities and rates.

3a. Consider this problem as a class: Bob has invested 100 dollars in a bank account, and he currently earns $20/hr as a waiter. When will he have $200?

Students should come up with \( P=20t+100 \) as the equation, which has \( t = \frac{p-100}{20} \), and t=5 hours as the answer. They will solve the same equation for when bob has $220 and $260. Ask them to explain in words how they came up with their answers. Discussion should lead to P as an input and t as an output, and furthermore the input and output can be organized as a table.

<table>
<thead>
<tr>
<th>Input(P)</th>
<th>Output(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$200</td>
<td>5 hrs</td>
</tr>
<tr>
<td>$220</td>
<td>6 hrs</td>
</tr>
<tr>
<td>$260</td>
<td>8 hrs</td>
</tr>
</tbody>
</table>

Explain that writing equations where one variable is isolated on one side of the equation is called rewriting equations in function form, and we write output variable is a function of input variable.
3b. Have students get into the same groups from activity 2, and they will use their “formula” to create input and output tables (groups with more than two variables in their equation need to fix variables until they have 2 “variables”). Also, they will write and share a paragraph on what real-life situation their table describes.

This activity introduces students to function form and also implies that all formulas are essentially functions. The discourse encourages the students to think how formulas are used in real-life and why function form is useful.

Questions: When is an equation in function form?

- Explain in words how to solve a formula for a variable.

Assessments: Discourse in the group activities act as the main assessment of the students’ understanding of this section’s concepts. The interactions in the group activities is also another measure of the student’s ability to problem-solve and to solve equations for a variable. The homework verifies the students’ knowledge of solving formulas for a variable and rewriting equations in function form.

Homework:

Solve each equation for x:

- \(5x+2=y\)
- \(3x+x=y+5\)
- \(2x+2=y\)

For each word problem write a formula and solve it for the unknown variable.

- John runs 12 mph. If he runs for 45 minutes, how many miles did he run?
- Dave works at a computers store and earns $27/hr. If he initially has $200, how much money does he have after he works from 9 a.m. to 5 p.m.?
- According to NASA, the earth has a diameter of about 8,000 miles. If \(C=\pi d\), then what is the circumference of the earth?

3.8 Rates, Ratios, and Percents

Prior Knowledge: Formulas and solving decimal equations, converting rates to ratios to percents, and knowing the definition of rates, ratios, and percents.

Objectives: Students will be able to use rates, ratios, and percents to model and solve real-life problems.

Motivation: Statistics (Ratios and percents are used in surveys), Science (Unit rates are used to convert units), physics (speed and acceleration are rates). Economics (elasticity of substitution and elasticity of demand are rates).

Misconceptions: Students may construct their proportions incorrectly. It is important that they write the proportions as words first before they use numbers. Students may forget to include the unit in the answer. Explain that 1 peso is not the same thing as 1 dollar i.e. the unit changes the answer.

Questions: What is a unit rate?

- How are unit rates related to proportions?
- Explain in words what a tip is, using percents.
Instructional activities: 1. Warm-up (Algebra I pg. 180)

Write each fraction or ratio as a decimal and as a percent.

\[
\frac{4}{5} \quad \frac{8}{4} \quad 14 \text{ to } 16 \quad 25:35
\]

7 students in the class are taking geometry; 5 students in the class are taking Algebra I; and 4 students in the class are taking Algebra II. What percent of kids are taking geometry?

What percent was the server’s tip if the customer left $1.75 for a $12.50 meal?

If John traded 30 red marbles and received 5 blue marbles, how many red marbles did he trade for one blue marble? Write the answer as a unit rate.

2. **Plan a Trip**: Divide class into groups of 3-4. Each group will plan a one-day trip to France by using the internet. They need not worry about details like what clothes they are bringing; they are primarily concerned with costs. They will consider all costs: airfare, taxi fare, food, hotel stay, tour costs, etc. Much of this information is available online, but if some cost is unknown, then the group is allowed to set the price themselves, within limits. Students will have 3 major cost variables: food and drink, tourism and transportation, and miscellaneous costs. Students will write down a summary of what their day entails, calculations, and a minimum of 4 equations: the 3 major cost variables and the sum of the major cost variables. In addition, students will first convert all costs to Euros, and then convert the total cost to U.S. dollars by using a website like [http://www.xe.com](http://www.xe.com), but they will strictly use the website to find the unit rate. That is, they will need to show their proportions. When they are finished, they will present their trip to the class.

Assessments: The warm-up serves to verify if students have the basic prior knowledge required for the activity. The Plan a Trip project provides the students with practice. The reflection in the homework serves to see if students understand the applications for proportions and rates. The problems in the homework show if the students are able to solve real-world problems using rates, ratios, and percents.

Homework: Reflect on the Plan a Trip activity. Why are proportions and rates so important for currency?

*Algebra I* Pg. 183

19. A store sells a box of 5 frozen yogurt bars for $1.20. The store also sells a box containing 7 of the same frozen yogurt bars for $1.59. Which is the better buy? Explain how you decided by using proportions.

*Algebra I* Pg. 184

Find the percent.

34. Tax of $.68 on an item priced at $11.29.

35. 292 people in favor out of 450 people surveyed.

**Conclusion**

Designing and creating my unit lesson plan has been a unique experience. Completing the assignment has made me more aware of how much work and time is put into a class period. The 30 hours observing high school mathematics has been helpful for creating the lesson plan, as
some of my activities are based on my reflections. For example, my teacher’s use of cooperative activities had especially inspired group activities in 3.1 and 3.4. Writing the unit lesson plan also has enhanced my understanding of the subject; perhaps not in content knowledge, but I have a better understanding of how linear equations are used in the real world. Finding motivation for my unit lessons was easier than I had expected from the motivation that we sometimes had to list in the problem sets, but that is an indication that the problem sets were useful. After finding motivation for a couple of lessons, it becomes easier to find real-world connections. In conclusion, the unit lesson plan truly felt like a final exam in the aspect that I had to essentially utilize and synthesize concepts and techniques from what we read, did, and observed in class.