

A Model of Endogenous Government Formation.*

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Abstract

Political parties bargain over the allocation of cabinet portfolios when forming coalition governments. Non-cooperative theories of legislative bargaining typically predict that the “formateur” enjoys a disproportionate share of government ministry positions. However, empirical evidence indicates that parties receive shares of portfolios proportional to their nominal voting weight, in support of Gamson’s Law of portfolio allocation. This paper examines government formation as a process in which the role of formateur is determined endogenously, or within, a coalition and parties have different preferences over cabinet positions. In equilibrium, if parties have similar preferences over cabinet portfolios, the share of seats they are allocated will be proportional to the parties’ sizes.

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1 Introduction

The defining feature of a parliamentary democracy is that the executive is not elected directly by the citizens but by the legislative body – the parliament. Under this system, unless a party wins a majority of seats, the emergence and the survival of a government is the result of a bargaining process among the parties represented in the parliament.

The key issue that parties must resolve when forming a coalition government is how to allocate the resources of government – cabinet portfolios – among themselves. Yet, the study of coalition formation is characterized by a striking paradox. On the one hand, a long and distinguished literature on the study of legislative bargaining predicts that the party that has the power to propose an allocation of cabinet portfolios is able to exploit its privileged position for its own benefit (e.g., Baron and Ferejohn 1989). On the other hand, a prominent empirical regularity about the formation of parliamentary governments suggests that the share of cabinet portfolios that each party in the government coalition receives is almost perfectly proportional to the share of legislative seats that it controls in the legislature (Laver 1998).

Even though this regularity is well documented empirically, no theoretical model exists whose predictions are consistent with it. Most of the theoretical work looks at the government formation as a bargaining process where one party proposes a distribution of resources that can be either accepted or rejected by the other parties (Baron and Ferejohn 1989). The main prediction is that the proposing party affects the bargaining process in a way that guarantees a larger share of the cabinet portfolios for itself.

In light of the significant disjunction between the theoretical predictions offered by the game-theoretical literature and the empirical evidence, this paper proposes a new non-cooperative theory of government formation where the role of proposer, or “formateur,” is not designated exogenously but is determined endogenously, or inside, the coalition. In fact, case studies (Muller and Strom 2000)

show that even where the power to nominate a prime minister (considered the formateur) is vested in the hands of a designated person (usually the monarch or the head of state), prime ministers are chosen, in practice, at the suggestions of the parties that have agreed to coalesce and have reached a previous agreement on the head of the government.

The case of the so-called “First Republic” in Italy is indicative. In all the coalition governments formed in this period (1948-1992), we can identify three distinct phases: a first phase of “talks” between the parties represented in Parliament, which continues from the general elections (or from the fall of the incumbent government) to the time the head of state asks the leaders of all the parties to consult; a second stage, in which the head of state nominates a prime minister-to-be; and a third stage, when the nominated prime minister-to-be bargains with the parties to compile a list of ministers.

In practice, in the first stage, parties agree to coalesce and designate a prime-minister to propose to the head of state, and in the second stage, the head of state ratifies the decision of the coalition. Of particular interest is the case of the VIII Legislature (1979), during which parties could not reach an agreement in the first phase of informal talks after the general elections, leading to repeated failures of all attempts to form a government. This case illustrates how a head of state alone has next to no discretionary powers in the process of government formation. Although the head of state may assume the role of a facilitator in the case of an impasse in the inter-party negotiations, coalition formation is the result of “free-style bargaining” between party leaders.

The paper posits an endogeneous formateur selection, as parties compete to become proposer. Parties that have previously agreed to coalesce are willing to offer their partners a share of cabinet portfolios in order to be vested of the role of formateur and be in the position to propose the distribution of resources.

Formally, government formation proceeds in three stages. In the first stage, parties simultaneously propose to form a coalition with a subset of other parties. The parties with identical matching proposals become partners of a “proto-

coalition” and proceed to the next stage. In the second stage, each party in the proto-coalition offers a share of the cabinet portfolios to each partner in the coalition in order to be the formateur. The formateur is the party who offers the highest share of cabinets to the partners. In the third stage, the coalition parties bargain over cabinet positions, such as occurs in the Baron-Ferejohn’s model (1989), with the formateur having been selected in the second stage.

This is an important departure from the legislative bargaining models in the literature. Allowing parties to compete for the role of formateur completely exhausts the “proposer advantage.” The main prediction of the model is that if parties are only office-motivated and the cabinet portfolio is a homogeneous and perfectly divisible good, then the share that each party in the government coalition is allocated in equilibrium is perfectly proportional to the number of seats that each controls in the legislature.

The paper extends the model to heterogeneous preferences over cabinet portfolios to examine the way equilibrium coalitions change when parties have different preferences among ministerial positions. The fact that political parties may care differently about the same ministries is not a novel idea in the literature, although its effect on government formation has not received much formal attention.

In equilibrium, if the coalition parties have similar preferences among the ministerial seats, then the share of ministries they receive is proportional to their legislative seat share. Otherwise, the allocation will not be proportional, but this imbalance would not be driven by a formateur advantage, as in the Baron-Ferejohn model, but by the Pareto efficient solution of the parties’ utility maximization problem.

A review of the literature follows in Section 2 of the paper. Section 3 introduces a baseline model in which cabinet positions are homogeneous. Section 4 then extends the model to heterogeneous cabinet portfolios. Section 5 summarizes the main findings and presents the next step of this research.

2 Literature review

The pioneering theories of coalition formation rely on the tools of cooperative game theory and spatial modeling to analyze the formation of coalitions as a function of parties' sizes. Assuming that the primary goal of parties is to gain office, the theories analyze government formation as a zero-sum game in which cabinet portfolios are the payoffs. The main prediction is that only coalitions that include parties whose support is needed in order to obtain the majority of the votes will form, and, as later refined by von Neumann and Morgenstern (1953) and Riker (1962), only those coalitions that are minimum winning – which contains “no surplus members” – will emerge.

At the same time, Gamson's Law (1961) – a sociological conjecture rather than a formal theory – focuses on the allocation of cabinet portfolios as the outcome of the government formation process. Gamson's Law suggests that parties seeking to form a coalition government demand a share of portfolios proportional to the amount of seats in the assembly that each contributed to the coalition. Although Gamson's prediction coincides with the game theoretic prediction about the size of the coalition (minimum winning), it diverges on the distribution of cabinet posts, which perfectly reflects the distribution of votes contributed by its members. Gamson's law owes its importance in the government formation literature to its intuitive nature, its independence from any specific government procedure, and its strong empirical evidence. As Laver (1998) notes, “Gamson's Law shows one of the highest non-trivial R-square in political science.”

More recently, formal theories shifted to non-cooperative game theoretic tools to emphasize the role of institutions, the set of rules and norms governing the process of government formation, as key variables of the coalition formation process (Baron and Ferejohn 1989; Austen-Smith and Banks 1988; Morelli 1999). Like the foundational theories, this new literature assumes that politicians pursue office benefits, predicting minimum winning coalitions. Yet, these theories are able to generate a number of additional implications that predict cabinet

composition as a function of the rules and norms of the coalition bargaining process. In particular, the power to propose the government coalition (*formateur*) and the power to control the timing of the government formation process (*agenda setting*) are the most studied procedural powers.

Most of the existing study of bargaining considers both finite or infinite-horizon games such that, during each period, one agent makes a proposal that can be either accepted, in which case the game ends with the proposed outcome; or rejected, in which case bargaining continues to the next period. Beginning with Rubinstein's 1982 work on two-person, alternating-offer bargaining, the study is modified by Binmore (1987) to allow for a randomly determined proposer. This model was extended to cover legislative politics by Baron and Ferejohn (1989), who allow for n legislators and assume a simple majority for a proposal to pass. As in Rubinstein's and Binmore's work, the subject of bargaining is the allocation of a fixed surplus, interpreted as cabinet portfolios¹.

Some of the literature, starting from the idea that the legislative policy-making game does not end once a proposal is accepted, analyzes the effects of "endogenizing" the status quo: each period begins with a status quo, then one agent makes a proposal that is either accepted, in which case it becomes the status quo for the next period, or rejected, in which case the current status quo remains in place².

All these theories reach the same conclusion: the formateur party (proposer) affects the bargaining process in a way to guarantee a larger share of the cabinet portfolios for itself. In the original Baron-Ferejohn model (1989), for example, the

¹A substantial literature has grown from these papers: Baron (1991) examines the case of a multi-dimensional set of alternatives; Eraslan (2002) proves uniqueness of stationary equilibrium in the original Baron-Ferejohn model; Banks and Duggan (2000) prove the existence, and examine connections to the core of the cooperative voting game in a version of the model with a general set of alternatives, preferences, and voting rules.

²Baron (1996) analyzes the one-dimensional version of the model with single-peaked utilities. Baron and Herron (2003) give a numerical calculation of equilibrium in a three-legislator, finite-horizon model. Fong (2005) considers a three-legislator model in which policies consist of locations in a two-dimensional space and allocations of surplus. Duggan and Kalandrakis (2007) prove existence of stationary Markov perfect equilibria in an infinite-horizon model with multidimensional policy space.

formateur party would enjoy a large bargaining advantage (with three players, none having a majority of seats, the proposer receives at least $2/3$ of the total payoff under simple majority rule), which is unaffected by nominal changes in voting weights, i.e., legislative seat shares.

Recently, the non-cooperative literature has moved in several new directions. On the one hand, new analysis of the procedural powers have resulted in models that relax the assumptions of exogenous recognition of the proposer and the agenda setter. Yildirim (2007) studies a sequential bargaining model in which players, before each round of bargaining, simultaneously choose their effort levels, which stochastically determine the proposer. The greater one's effort, the more likely one is to propose. As in Baron-Ferejohn (1989), the equilibrium coalition will always be "minimum winning" since the proposer wants to buy out the votes of the cheapest "winning coalition", which may vary across players. More efficient players (who face lower costs) have higher continuation values. But this makes their votes relatively more expensive, and thus less desirable to be bought out. Therefore, with sufficiently heterogeneous costs, the surplus between the proposer (the most efficient) and the partners in the coalition (the less efficient) may be potentially much larger than under a random recognition rule.

Čopič and Katz (2007) propose a bargaining model in which legislators are time-constrained in proposing possible bills to move the status quo, making the number of the proposals restricted. The proposal selection problem is analyzed as a special type of multi-good auction, where legislators bid for slots on a legislative agenda and an agent is vested with the power of choosing the proposals to put on the floor. Competition for access to the floor induces a final policy to move close to the policy preferences of the scheduling agent, even when the policy preferences of the proposers are contrary to the agent's preferences.

On the other hand, different hypotheses about the rules and institutions of the government formation process have resulted in innovative theories like the "alternating demand game" of Morelli (1999) or the bargaining models of Diermeier and Merlo (2000) and Baron and Diermeier (2001).

The bargaining protocol of the Morelli's model (1999) assumes that each player asks for a share of cabinet positions, leaving to subsequent players the possibility of asking for their respective shares. A coalition emerges between parties making compatible demands. The head of state chooses the first mover, who in turn chooses the order in which the parties formulate demands. In the case of three voting blocks, none of which by itself has a majority of votes, the unique sub-game perfect equilibrium outcome gives 1/2 of the payoffs to each of the first two movers who form a minimum winning coalition, regardless of the number of votes each voting block controls.

Diermeier and Merlo (2000) and Baron and Diermeier (2001) analyze the allocation of cabinet seats as only one side of the coin in the government formation process, where legislators bargain over office and policy. In these two models, the formateur party, in order to obtain the support in the legislature, can either make "policy concessions" by enacting a compromising policy, or propose "side payments" to other parties. The non-cooperative approach of these two models produce equilibrium governments that can be minority, minimum winning, or surplus. This is a major point of departure from the previous theoretical literature, which had consistently predicted minimum winning (or minimum winning-connected) coalitions in equilibrium.

None of these papers, however, is able to explain in equilibrium the strongly proportional relationship between the cabinet positions and the nominal voting weights each coalition member contributes to the coalition (Warwick and Druckman 2001). The following model analyzes a non-cooperative bargaining game where the formateur is determined endogenously within the winning coalition, and cabinet portfolios are heterogeneous bundles of ministerial positions.

3 A bargaining model with endogenous formateur

Consider a legislature with $N \equiv \{1, 2, \dots, n\}$ parties. Each party i has a legislative weight $w_i \in [0, 1]$, which we can think of these weights as being the proportion of

legislative seats held in the legislature. No party has a majority of votes necessary to pass a government proposal ($w_i < 1/2, \forall i \in N$).

The relationship between parties and legislators is one of mutual loyalty. Individual legislators $l \in \{1, 2, \dots, L\}$ are assumed to be loyal to the party to which they belong, meaning that they do not switch from one party to another. Parties maintain members' discipline through the use of patronage, meaning that loyalty is awarded with office positions that, correspondingly, are denied to renegade members. Yet, members can have different weights inside the party and, consequently, may enjoy different shares of the benefits. Thus, assuming party loyalty means that every member of a party enjoys a share of the benefits to which the party is entitled.

A.1 The utility function of party i is represented by the aggregate utility of each of its members: $U_i = \sum_{l \in i} u_l$.

The main difference between this model and most of the models discussed thus far is that the formateur (the proposer in the bargaining game) is not exogenously designated but is determined in equilibrium by the characteristics of the parties. Parties are rational players and, looking forward, foresee that the formateur would enjoy a first mover advantage in the bargaining game, thus they are willing to make an offer to the coalition partners in order to achieve this proposer role.

The bargaining game proceeds as follows. Parties behave non-cooperatively and decide how to allocate a perfectly divisible homogeneous bundle of ministerial cabinets (normalized to sum to 1) among themselves. Let $N \equiv \{1, 2, \dots, n\}$ denote the set of parties, and $S \equiv \{(s_1, s_2, \dots, s_n) | s_i \geq 0, \sum_i s_i \leq 1, \forall i \in N\}$ denote the set of feasible allocations, where s_i is the share party i receives. Assume that parties are risk neutral and that they discount the future by a discount factor $\delta \in (0, 1]$. Parties care only about their own consumption and have preferences that can be represented by a continuous and strongly monotonic – but not necessarily quasi-concave – utility function $U_i = f(s_i)$ ³.

³Since the bundle of cabinets is homogeneous, we can assume without further restrictions

Each party is assumed to know all other parties' preferences, and all actions are assumed to be observable (perfect information)⁴. The interaction between parties is modeled as a sequential game, where the proposing party needs the consent of a quota equal to the simple majority (50%+1) for its proposal to be agreed upon.

In each period, government formation proceeds in three stages. In the first stage, parties simultaneously propose to form a coalition with a subset of parties. Let $J \equiv \{1, 2, \dots, m\}$ with $m \leq n$ denote a subset of parties. Then party i proposes to form a coalition $c_J^i \in \mathcal{C}$ to a subset of parties $J \setminus i$. The parties with identical matching proposal ($c_J^j = c_J, \forall j \in J$) become partners of the proto-coalition c_J and proceed to the next stage.

In the second stage, each party $i \in c_J$ offers a share of the cabinet portfolios to each partner in the coalition in order to be the formateur ($\Lambda_j^i \in [0, 1], \forall j \in c_J, j \neq i$). The formateur, say m , is the party that offers the highest share of cabinets to the partners. The formateur and the coalition partners sign a government "proto-proposal" \tilde{S}^m , which assigns Λ_j^m to the coalition partners $j \in c_J$, the remaining share of cabinets ($1 - \sum_{j \in c_J} \Lambda_j^m$) to the formateur m , and 0 to the parties outside the proto-coalition $j \notin c_J$.

In the third stage, the coalition parties bargain over cabinet positions in accordance with standard alternating bargaining models. The main different is that if the receiving parties, that is, the parties in the proposed coalition which do not have proposal power, do not accept the proposer's proposal, the status quo is not a vector of zeros but the proto-proposal vector \tilde{S}^m that the coalition parties signed in the previous stage. That is, the formateur proposes an allocation $S^m = (s_1, s_2, \dots, s_n)$ to the coalition partners. The partners can either accept or reject the proposal. If the proposal is accepted, then the proposal S^m is put on

that the utility of each party is the share of the bundle that it obtains: $U_i = s_i$.

⁴This is not a completely natural assumption in all situations but provides a necessary preliminary to a more general analysis and may be reasonable in some circumstances. In the future, it may be interesting to relax this assumption and analyze how incomplete information would affect the qualitative properties of the equilibrium predictions.

the floor and voted by the legislature:

$$S^m \begin{cases} s_i & \text{if } i \in c_J, \quad i \neq m; \\ 1 - \sum_{i=1}^{m-1} s_i & \text{if } i \in c_J, \quad i = m; \text{ and} \\ 0 & \text{if } i \notin c_J. \end{cases}$$

If the formateur's proposal is rejected, the proto-proposal \tilde{S}^m is put on the floor and voted by the legislature:

$$\tilde{S}^m \begin{cases} \Lambda_i^m & \text{if } i \in c_J, \quad i \neq m; \\ 1 - \sum_{i=1}^{m-1} \Lambda_i^m & \text{if } i \in c_J, \quad i = m; \text{ and} \\ 0 & \text{if } i \notin c_J. \end{cases}$$

If neither the formateur's proposal nor the proto-proposal obtains the majority of votes required for passage, the game repeats itself at the first stage up to T periods. This process continues until an allocation generates the required number of votes. If at the end of period T no proposal has been passed, a "caretaker government," composed of non-partisan individuals, results, and parties' utility is $U_i = 0 \forall i \in N$.

I now describe the solution concept. Let $H_t \equiv \{h_1, h_2, h_3\}_{t=1}^T$ be the history of the game that contains the proposals that have been made, the identity of the proto-coalition, the identity of the proposer, and the actions taken at each of the three stages in period t .

In stage h_1 , party $i \in N$ takes action $a_i(h_1) = c_i$, that is, party i proposes a coalition with other parties; and a proto-coalition c_J with $J \subseteq N$ is selected.

In stage h_2 , only the parties in the proto-coalition $i \in c_J$ take action $a_i(h_2) = \Lambda^i = \{\Lambda_1^i, \Lambda_2^i, \dots, \Lambda_{i-1}^i, \Lambda_{i+1}^i, \dots, \Lambda_J^i\}$, that is, party i proposes an offer to the proto-coalition partners $j \in c_J$ to obtain the role of being formateur. The highest offer Λ^m is selected, and party m is designated to be the formateur.

$$a_i(c_J, h_2) \begin{cases} \Lambda_j^i & \text{if } i \in c_J \\ \emptyset & \text{if } i \notin c_J \end{cases}$$

In stage h_3 , only the parties in the proto-coalition take action, and a proposal (either S^m or \tilde{S}^m) is put on the floor.

$$a_{i \in c_J}(c_J, m, \Lambda^m, h_3) \begin{cases} S^m & \text{if } i \in c_J \quad \text{if } i = m \\ \{accept, reject\} & \text{if } i \in c_J \quad \text{if } i \neq m \\ \emptyset & \text{if } i \notin c_J \end{cases}$$

A strategy ξ_i for player i describes a sequence of actions, $a_i(H_t)_{t=1}^T$. A strategy profile $\xi_i \equiv \{\xi_1, \xi_2, \dots, \xi_n\}$ is a sub-game perfect equilibrium (SPE), henceforth termed “equilibrium” if it constitutes a Nash equilibrium in each period t and it is history dependent. Intuitively, a SPE calls for the same actions in each continuation game followed by rejection of an offer on the table. All moves are sequential, and there is perfect information, so the game will be solved by backward induction starting from the last period.

To understand the intuition behind the model’s main results, I examine the case in which members inside a party have the same weight.

A.2 Let u_i be the utility of each legislator belonging to party i and w_i party i ’s size, then the utility function of party i may be rewritten as $U_i = w_i u_i$

We solve the sub-game at period $t = T$ by backward induction, starting from the last stage in the next subsection.

3.1 Stage 3: Bargaining over cabinets

Once a formateur is selected, the bargaining process proceeds with the formateur as the proposing party and the coalition partners as the receiving parties. Now the status quo is the proto-proposal \tilde{S}^m that the formateur m proposed to the coalition partners in the previous stage. If the formateur proposal S^m is accepted

by the coalition partners, then S^m will be put on the floor and voted by the legislature, otherwise, the proto-proposal \tilde{S}^m will be put on the floor and voted by the legislature.

Assuming that each member of a party has equal weight inside the party, the share of benefits offered by the formateur to party $i \in c_J$ may be rewritten as $\Lambda_i^m = w_i \lambda^m$, where w_i is party i 's size and $\lambda^m \in [0, 1]$ is the benefit each member of party i enjoys. The proto-proposal vector may be rewritten as:

$$\tilde{S}^m = \left((\lambda^m w_1), (\lambda^m w_2), \dots, (\lambda^m w_{m-1}), \left(1 - \lambda^m \sum_{i=1}^{m-1} w_i\right), 0, \dots, 0 \right) \quad (1)$$

The proposer's maximization problem is to propose $S^m = (s_1, s_2, \dots, s_n)$, such that:

$$\begin{aligned} \max_{s_i \geq 0} \quad & U^m(s_m) \quad \text{subject to} \quad \sum_{i=1}^n s_i \leq 1 \\ & U^i(s_i) \geq U^i(\lambda^m w_i) \quad \forall i \in c_J, i \neq m \\ & U^i(s_i) \geq 0 \quad \forall i \notin c_J \end{aligned} \quad (2)$$

The first constraint ensures feasibility and the second constraint ensures that each partner will voluntarily choose s_i over $\lambda^m w_i$.⁵ Any solution must satisfy the last two constraints with equality, since the proposer maximizes its own utility by choosing an allocation (s_i) that makes the partners indifferent with the offer $\lambda^m w_i$, and that outcome makes the parties outside the proto-coalition indifferent with 0. Since cabinet portfolios are an homogeneous bundle normalized to 1, and $\lambda^m \in [0, 1]$, the equilibrium proposal is $S^* = \tilde{S}^m = (\lambda^m w_1, \dots, \lambda^m w_{m-1}, 1 - \lambda^m \sum_{i=1}^{m-1} w_i, 0, \dots, 0)$.

3.2 Stage 2: Formateur selection

The coalition parties make an offer to the partners⁵ in order to be the formateur of the government coalition. This process may be described as an auction in which each party i bids a share Λ^i to form a government coalition. This mechanism

⁵We assume that whenever a receiving party is indifferent, it accepts what the proposer prefers for it to select. The assumption is innocuous because a sufficiently small increase of s_i could induce the receiver to accept s_i without changing the utility of the players.

can be interpreted as a market in which each party pays a share of the cabinet positions to the partners in order to form a government coalition. The bid is distributed to the legislators belonging to the parties in the coalition according to their "weight." Assuming that each legislator has equal weight inside the party, parties will accept the offers that give them the highest "per capita" value. Hence, the winner of the auction is the party that bids the highest λ^i .

The winning party pays the auction price by signing a proto-proposal \tilde{S} , which allocates to the partners the winning bid.⁶

Each party has a well-defined reservation price for the role of formateur because they know the level of utility they would enjoy as formateur for every price λ^i . Furthermore, there exists a unique price that makes every party indifferent to adopting the role of formateur or receiving party. This means that the final outcome does not depend on the choice of auction.

Proposition 1 *Each party has a well-defined reservation price, and there exists a unique price that makes every party in the coalition indifferent to being either the formateur or receiving party.*

Proof. By application of the Intermediate Value Theorem⁷, there exists a unique price $(\hat{\lambda}_{i,j})$ that makes every party in the coalition $c_{i,j}$ indifferent to the possibility of winning and losing the auction. Thus, party m is indifferent to winning and losing the auction:

$$\begin{aligned}
 U^m(\lambda_m w_m) = \max_{s_i \geq 0} \quad & U^m(s_m) \quad \text{subject to} \quad \sum_{i=1}^n s_i \leq 1 \\
 & U^i(s_i) \geq U^i(\lambda^m w_i) \quad \forall i \in c_J, \quad i \neq m \\
 & U^i(s_i) \geq 0 \quad \forall i \notin c_J
 \end{aligned} \tag{3}$$

⁶This mechanism follows the procedure proposed by Crawford (1979), in which, by auctioning off the proposer role, the equilibrium allocations satisfy both the Pareto efficiency property and the Pazner's concept of equity.

⁷The Intermediate Value Theorem can be stated as the following: suppose that $f : [a, b] \rightarrow R$ is continuous and that u is a real number satisfying $f(a) < u < f(b)$ or $f(a) > u > f(b)$. Then for some $c \in [a, b]$, $f(c) = u$.

The problem for the other j (with $\in c^*$) parties is similar:

$$\begin{aligned}
U^j(\lambda^j w_j) = \max_{s_i \geq 0} \quad & U^j(s_j) \quad \text{subject to} \quad \sum_{i=1}^n s_i \leq 1 \\
& U^i(s_i) \geq U^i(\lambda^j w_i) \quad \forall i \in c_J, \quad i \neq j \\
& U^i(s_i) \geq 0 \quad \forall i \notin c_J
\end{aligned} \tag{4}$$

But, from (3) and (4), party m is indifferent about whether it makes the government proposal or agrees to the proposal made by the partner.

$$\max_{s_i \geq 0} U^m(s_m) = U^m(\lambda^m w_m) = U^m(\lambda^j w_m)$$

Therefore, $\lambda^m = \lambda^j = \widehat{\lambda}_{c_J}$ for every $j \in c_J$. ■

Proposition 2 *The equilibrium bid for each proto-coalition is unique and identical for every party ($\widehat{\lambda}_{c_J}$). In equilibrium, therefore, the formateur is selected inside the proto-coalition by a tie-breaking rule that has been agreed upon by the parties.*

This is one of the innovative results of the model with respect to the extant literature: because parties compete for the role of formateur, the “first-mover” advantage traditionally found in other legislative bargaining models is completely eliminated.

From the solution of the third stage, we know that the allocation S^* generated by this process gives each party i in the proto-coalition c_J the same utility of \widetilde{S} :

$$\begin{aligned}
U(S^*) &= (u_1(S), \dots, u_{m-1}(S), u_m(S), u_{m+1}(S), \dots, u_n(S)) \\
&= ((\widehat{\lambda}_{c_J} w_1), \dots, (\widehat{\lambda}_{c_J} w_{m-1}), (\widehat{\lambda}_{c_J} w_m), 0, \dots, 0)
\end{aligned} \tag{5}$$

Therefore, the equilibrium allocation is unique no matter which party has the charge to form the government. This leads us to our next theoretical result, Proposition 3:

Proposition 3 *If parties are only office-motivated and cabinet portfolios are homogeneous and perfectly divisible goods, then the share that each party in the government coalition is allocated in equilibrium is perfectly proportional to the nominal*

voting weight.

Proof. Notice that for the feasibility constraint in the third-stage solution (which holds with equality) $\sum_{i=1}^n s_i = 1$. From Equation 5, we derive that

$$\begin{aligned} \sum_{i \in c_J} \hat{\lambda}_{c_J} w_i &= 1 \\ \hat{\lambda}_{c_J} \sum_{i \in c_J} w_i &= 1 \\ \hat{\lambda}_{c_J} &= 1 / \sum_{i \in c_J} w_i \end{aligned} \tag{6}$$

where $\sum_{i \in c_J} w_i$ is the size of the proto-coalition. We can then rewrite S^* as:

$$S^* = \left(\frac{w_1}{\sum_{i \in c_J} w_i}, \dots, \frac{w_{m-1}}{\sum_{i \in c_J} w_i}, \frac{w_m}{\sum_{i \in c_J} w_i}, 0, \dots, 0 \right) \tag{7}$$

where $w_i / \sum_{i \in c_J} w_i$ is the nominal voting weight of party i in the proto-coalition. ■

3.3 Stage 1: Coalition selection

We solve party i 's maximization problem by backward induction for every possible proto-coalition c_i :

$$\begin{aligned} \max_{c_i \in \mathcal{C}} U^i(s_i | c_i, \lambda_i, \lambda_{-i}) &= \max_{c_i \in \mathcal{C}} U^i \left(w_i / \sum_{j \in c_J} w_j \right) \\ \text{subject to} & \sum_{j \in c_i} w_j \geq L/2 + 1 \end{aligned} \tag{8}$$

Proposition 4 *Suppose that the cabinet portfolios is a homogeneous and perfectly divisible good, that the utility function of the parties satisfy **A.2**, and that $w_i \neq w_j \forall i, j$. Then, there exists a unique equilibrium coalition c^* , such that:*

- c^* has the smallest winning size.

- Every party $i \in c^*$ offers to the partners $j \in c^*$ a share equal to $\hat{\lambda}w_j = \frac{w_j}{\sum_{k \in c^*} w_k}$.
- The formateur's proposal allocates $s_j = \hat{\lambda}w_j$ to $j \in c^*$ and 0 to $j \notin c^*$.

The equilibrium of the game is unique as long as there exists a unique smallest minimum winning coalition, otherwise at least one party will be indifferent to the choice between coalitions, which will be equally likely to be proposed.

In contrast with Carrol and Cox (2007), who argued that the proportional trend in coalitional bargaining over portfolios is a focal point, a solution to which agents are naturally drawn, the model presented here explains the proportional allocation as a rational equilibrium point.

3.4 Multi-period SPE

Assuming that the number of periods is finite, the game may be solved by backward induction starting from the last period. There exists a sub-game perfect equilibrium in which each party makes the same coalition proposal, the same bid to be formateur, and the same proposal to allocate the cabinet portfolio. The equilibrium strategies are thus unique.

Proposition 5 *A strategy configuration is a sub-game perfect equilibrium for a three-stage, T -period, n -party legislature, with endogenous formateur, if and only if the following properties attain:*

1. *In every period, each party i proposes the smallest minimum winning coalition, which includes itself: c_i such that $\sum_{j \in c_i} w_j = \min \sum_{j \in \mathcal{C}} w_j, \forall \mathcal{C} \geq L/2 + 1$.*
2. *In every period, each party i in the proto-coalition bids (offers to the members of the partners $j \in c^*$) a share of ministries equal to $\hat{\lambda} = 1 / \sum_{j \in c^*} w_j$;*
3. *In every period, every party i selected as formateur makes a proposal that allocates $s_j = \hat{\lambda}w_j$ to every partner in the proto-coalition ($j \in c^*$), $s_i = 1 - \sum_{j \in c^*} \hat{\lambda}w_j = \hat{\lambda}w_i$ to itself, and 0 to every party outside the proto-coalition.*

4. In every period $t < T$, every party in the proto-coalition c^* votes for any proposal in which all parties receive at least $\delta\hat{\lambda}w_j$, and every party outside the coalition is indifferent to voting for the proposal or going to the next period.
5. At period T , every party is indifferent about whether to accept the proposal or to call for a new election at the end of the legislature.
6. In the first period, the formateur's proposal allocates to every party in the proto-coalition $s_j = \hat{\lambda}w_j \quad \forall j \in c^*$, which is greater or equal than $\delta\hat{\lambda}w_j$. The proposal in the first period is thus passed, and the legislature adjourns in the first period.

Proof. Since the game ends after period T , the continuation value for all null sub-games and for all parties is $v_i = 0$. In the third stage of period T , the formateur proposes an allocation that must make the parties in the proto-coalition better off or at least indifferent within the proto-proposal, and all the other parties at least indifferent to their continuation value (i.e., $v_i = 0$). As proved in Proposition 3, the proto-proposal allocates $w_i / \sum_{j \in c^*} w_j$ ($\forall i \in c^*$) to the parties in the proto-coalition and 0 to the parties outside the proto-coalition. As in Proposition 4, the proto-coalition is the smallest minimum winning coalition. Thus, the continuation v_i at period $T - 1$ is the expectation of party i -th allocation in the T -th period.

$$v_i \begin{cases} w_i / \sum_{j \in c^*} w_j & \text{if } i \in c^*; \\ 0 & \text{if } i \notin c^*. \end{cases}$$

At time $T - 1$, the formateur party has to offer to the other parties at least the continuation value δv_i in order to obtain a majority vote. However, as it happens in period $T - 1$, the formateur has to propose an allocation that must make the parties in the proto-coalition better off or at least indifferent within the proto-proposal. The constraint imposed by the future utilities on what the partners can receive, then, is not binding ($\delta w_i / \sum_{j \in c^*} w_j \leq w_i / \sum_{j \in c^*} w_j \quad \forall \delta \in (0, 1]$).

Therefore, the game ends at the first period, with the formateur proposing an allocation that makes indifferent the partners in the smallest minimum winning coalition with the proto-proposal $(v_i = w_i / \sum_{j \in c^*} w_j, \quad \forall i \in c^*)$ and the other parties with their continuation value $(v_i = 0, \quad \forall i \notin c^*)$ ■

In such a framework, at every period of the game each party proposes the same proto-coalition; each party in the proto-coalition offers the same share $\hat{\lambda}$ to be the formateur; and each formateur proposes the same equilibrium allocation S^* . The equilibrium strategies are, thus, unique⁸.

If the number of periods is infinite, the qualitative properties of the equilibria in the finite-period case hold for sub-game perfect equilibria that are stationary, i.e., if the continuation values for each structurally equivalent sub-games are the same.

4 Heterogenous cabinets positions

In the previous section we have seen how the baseline model with a perfectly divisible homogeneous good predicts equilibrium allocations that are perfectly proportional to the nominal voting weight of the coalition parties. However, cabinet portfolios are not homogenous, and parties have different preferences over different ministerial positions. As an illustration, consider the “Green” parties in several parliamentary legislatures. Because of stated interests, one may assume that these parties would obtain a greater utility from holding an “environmental ministry” than a “postal ministry.”

In this section we relax the assumption of homogeneity, allowing parties to extract different utilities from different positions. As in the last section, parties behave non-cooperatively and decide how to allocate a perfectly divisible bundle composed of k ministerial positions. Units are chosen so that the m -vector of

⁸The equilibrium specification of the government coalition is unique unless a unique smallest minimum winning coalition does not exist.

goods to be allocated is 1.

Let $N \equiv \{1, 2, \dots, n\}$ denote the set of parties, and $Z \equiv \{(z_1, z_2, \dots, z_n)\}$ denote the set of feasible allocations, where z_i is the m -vector consumption bundle of party i . Let $U_i = f_i(z_i)$ denote party i 's continuous and strongly monotonic utility function $i = 1, \dots, n$. Notice that $U_i \neq U_j \forall i, j \in N$.

Government formation proceeds in three stages. The first stage is perfectly equal to the homogenous case: each party i simultaneously proposes to form a coalition $c_J^i \in \mathcal{C}$ with a subset of parties $J \setminus i$. The parties having identical matching proposals ($c_J^i = c_J, \forall j \in J$) become partners of the proto-coalition c_J and proceed to the next stage.

In the second stage, each party $i = \{1, \dots, m\}$ in the coalition c_J bids some scalar multiple Λ^i of a *numéraire* $\tau \geq 0^9$. The highest bidder, says m , wins the privilege of being formateur and signing a proto-proposal \tilde{Z}^m which assigns $(\Lambda_i^m \tau)$ to the coalition partners $i \in c_J$, the remaining share of cabinets $(1 - \sum_{i \in c_J} \Lambda_i^m \tau)$ to the formateur m , and (0) go the parties outside the proto-coalition $i \notin c_J$. The *numéraire* $\tau \geq 0$ may be any bundle of goods. The only assumption that it must satisfy is a “desirability” property: every party must prefer some finite multiple of it to the bundle 1.¹⁰

In the third stage, the coalition parties bargain over cabinets positions much as they had in section 3, the only difference being that the proto-proposal vector is:

$$\tilde{Z}^m = \{\Lambda_1^m \tau, \dots, \Lambda_{m-1}^m \tau, 1 - \sum_{i=1}^{m-1} \Lambda_i^m \tau, 0, \dots, 0\}.$$

4.1 Stage 3: Bargaining over cabinets

Assuming again that each member of a party has equal weight inside the party, the proto-proposal \tilde{Z}^m maybe re-written as:

⁹The whole value of what the players bid upon is normalized to 1, equivalently we will refer to it as the numéraire.

¹⁰The desirability assumption is satisfied if the utility function U_i is strongly monotonic for every strictly positive component of τ .

$$\tilde{Z}^m = ((\lambda^m \tau w_1), (\lambda^m \tau w_2), \dots, (\lambda^m \tau w_{m-1}), (1 - \lambda^m \sum_{i=1}^{m-1} \tau w_i), 0, \dots, 0)$$

Thus, the proposer maximization problem will be to propose $Z^m = (z_1, z_2, \dots, z_n)$, such that:

$$\begin{aligned} \max_{z_i \geq 0} \quad & U^m(z_m) \quad \text{subject to} \quad \sum_{i=1}^n z_i \leq 1 \\ & U^i(z_i) \geq U^i(\lambda^m \tau w_i) \quad \forall i \in c_J, i \neq m \\ & U^i(z_i) \geq 0 \quad \forall i \notin c_J \end{aligned}$$

Any solution must satisfy the last two constraints with equality, since the proposer maximizes its own utility by choosing an allocation (z_i) that makes its partners indifferent with the offer $\lambda^m \tau w_i$ and makes the parties outside the proto-coalition indifferent with 0. However, the equilibrium proposal $Z^m = (z_1, z_2, \dots, z_n)$ is not necessarily equal to $\tilde{Z}^m = ((\lambda^m \tau w_1), \dots, (\lambda^m \tau w_{m-1}), (1 - \lambda^m \sum_{i=1}^{m-1} \tau w_i), 0, \dots, 0)$ because the bundle is heterogeneous, and parties have different preferences on the m goods ($U^i(z_i) \neq z_i$).

4.2 Stage 2: Formateur selection

Each party has a well-defined reservation price for the role of formateur. Furthermore, if the numeraire is *desirable*, there exists a unique price that makes every player indifferent to holding the role of proposer or receiver.¹¹

Proposition 6 *For any numeraire bundle there exists a unique price $(\hat{\lambda})$ that makes every coalition party indifferent to being the formateur or a receiving party.*

Proof. No party would strictly prefer the role as a receiving party if $\hat{\lambda} = 0$. The desirability assumption of τ ensures that for any party there exists a

¹¹The two parties must offer a scalar multiple of a numeraire bundle on which they must agree. The numeraire bundle is not necessarily the bundle of goods to be divided, but it may be any bundle that satisfies the property of desirability for the two parties.

finite value of $\hat{\lambda}$ at which the party would prefer the role as a receiving party. Assuming that each party's utility as proposer is a continuous function of $\hat{\lambda}$, then, by the Intermediate Value Theorem, there exists a unique price ($\hat{\lambda}_{i,j}$) that makes every party in the coalition $c_{i,j}$ indifferent to the outcome of winning or losing the auction. The rest of the proof follows Proposition 1. ■

The equilibrium bid for each proto-coalition is unique and identical for every party ($\hat{\lambda}_{c_j}$). However, from the solution of the third stage, the allocation Z is no longer equal to the proto-proposal $\Lambda\tau$.

Proposition 7 *If the bundle of ministries is heterogeneous and parties have differing utilities with regard to different cabinet positions, the vector of equilibrium allocation is no longer perfectly proportional to the size vector.*

Example

In order to understand how preferences of the parties for heterogeneous bundles of cabinets affect the allocation of cabinets, assume a legislature composed of three parties, none with a majority of seats. Assume that the cabinet positions may be of two different types: “domestic” (x_d) and “foreign affairs” (x_f) and that parties have Cobb Douglas utility functions over these two types of ministerial seats. Assume that party i and party j are in the proto-coalition and that their preference parameter over the “domestic affair” ministry are α_i and α_j , respectively. Assume that the numeraire is a vector of the two cabinet positions (x_d, x_f).¹² Thus, in the second stage of the game, party i and j solve the following maximization problems:¹³

¹²This assures desirability and avoids the introduction of a third good.

¹³See Appendix for solution algorithm.

$$\begin{aligned}
& \max_{x_d \geq 0; x_f \geq 0} U_i = (x_d)^{\alpha_i} (x_f)^{1-\alpha_i} \\
& \text{subject to } (1 - x_d)^{\alpha_j} (1 - x_f)^{1-\alpha_j} \geq (\lambda_i w_j)^{\alpha_j} (\lambda_i w_j)^{1-\alpha_j} \\
& \max_{x_d \geq 0; x_f \geq 0} U_j = (x_d)^{\alpha_j} (x_f)^{1-\alpha_j} \\
& \text{subject to } (1 - x_d)^{\alpha_i} (1 - x_f)^{1-\alpha_i} \geq (\lambda_j w_i)^{\alpha_i} (\lambda_j w_i)^{1-\alpha_i}
\end{aligned}$$

In order to see how the allocations change as a function of the size of the two parties and of the preference parameters α_i and α_j , let party j 's size fix to 5 ($w_j = 5$) and $\alpha_j = 0.5$, and let party i 's parameters vary ($w_i \in [1, 10]$ and $\alpha_i \in [0, 1]$).

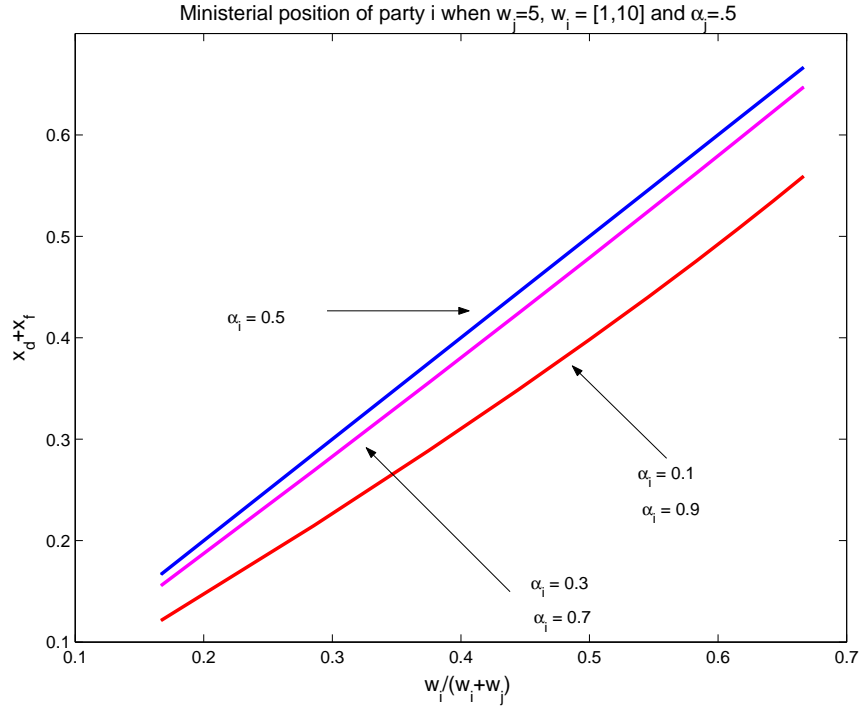


Figure 1: Number of cabinets as function of parties' size and preference

Figure 1 shows how party i 's equilibrium number of cabinets (sum of “domestic” and “foreign affairs” ministries) changes as a function of the relative weight of the parties in the coalition and the preference parameter over the two types of ministerial seats. For every parameter α_i , the number of cabinets allocated to party i increases the number of seats controlled in the legislature (w_i). How-

ever, an exact proportionality exists only if both parties in the coalition value the ministerial positions in the same way ($\alpha_i = \alpha_j$).

4.3 Stage 1: Coalition selection

As in the homogeneous bundle case, parties solve by backward induction the maximization problem for every possible proto-coalition and propose the coalition that secures the highest utility.

$$\max_{c_i \in \mathcal{C}} U^i(z_i | c_i, \hat{\lambda}_{c_i}) \quad \text{subject to} \quad \sum_{j \in c_i} w_j \geq \frac{L}{2} + 1 \quad (9)$$

However, since the equilibrium bid $\hat{\lambda}_{c_i}$ for every party i is no longer equal to party i 's nominal voting weight, it is not necessarily the case that parties maximize their utility by choosing the smallest minimum coalition.

Proposition 8 *Suppose that cabinet portfolios are an heterogeneous and perfectly divisible bundle of goods, and that the utility functions of the parties satisfy **A.2**. Then, there exists an equilibrium coalition c^* that is minimum winning.*

Proof. In a proof by contradiction, let us assume to have a three-party legislature; the set of proto-coalition is: $\{c_{ij}, c_{ik}, c_{jk}\}$. If an equilibrium does not exist, then we have a cycle of preferences: party i prefers coalition c_{ij} to c_{ik} , party j prefers coalition c_{jk} to c_{ij} , and party k prefers coalition c_{ik} to c_{jk} . If this is the case, then: $(\Lambda_{ij} > \Lambda_{ik})$, $(\Lambda_{jk} > \Lambda_{ij})$, $(\Lambda_{ik} > \Lambda_{jk})$. The first two inequalities imply that $\Lambda_{jk} > \Lambda_{ij} > \Lambda_{ik}$, and it contradicts the third inequality $\Lambda_{ik} > \Lambda_{jk}$. ■

However, as in the homogeneous case, the equilibrium may not be unique.

Example

In order to understand how size and preferences of the parties for heterogeneous cabinets affect the equilibrium coalition, assume again the three-party legislature of the previous example. Assume the three parties i , j , and k to have Cobb Douglas utility functions over “domestic” (x_d) and “foreign affairs” (x_f)

Table 1: Winning coalitions: Equilibrium allocations and party' utilities.

	$(x_d)_i$	$(x_d)_j$	$(x_d)_k$	$(x_f)_i$	$(x_f)_j$	$(x_f)_k$	λ	u_i	u_j	u_k
i+j	0.43	0.57	0.00	0.43	0.57	0.00	0.14	0.43	0.57	0.00
i+k	0.60	0.00	0.40	0.60	0.00	0.40	0.20	0.60	0.00	0.40
j+k	0.00	0.67	0.33	0.00	0.67	0.33	0.17	0.00	0.67	0.33

Notes - The table reports for each winning coalition (first column), the equilibrium allocation of x_d and x_f , the equilibrium offer to be the formateur, and the utility for each party. The size of the three parties is: $w_i = 3$, $w_j = 4$, $w_k = 2$. The office parameters are: $\alpha_i = \alpha_j = \alpha_k = 0.5$.

Table 2: Winning coalitions: Equilibrium allocations and party' utilities.

	$(x_d)_i$	$(x_d)_j$	$(x_d)_k$	$(x_f)_i$	$(x_f)_j$	$(x_f)_k$	λ	u_i	u_j	u_k
i+j	0.22	0.78	0.00	0.54	0.46	0.00	0.15	0.45	0.60	0.00
i+k	0.38	0.00	0.62	0.71	0.00	0.29	0.21	0.63	0.00	0.42
j+k	0.00	0.67	0.33	0.00	0.67	0.33	0.17	0.00	0.67	0.33

Notes - The table reports for each winning coalition (first column), the equilibrium allocation of x_d and x_f , the equilibrium offer to be the formateur, and the utility for each party. The size of the three parties is: $w_i = 3$, $w_j = 4$, $w_k = 2$. The office parameters are: $\alpha_i = 0.2$, $\alpha_j = 0.5$, $\alpha_k = 0.5$.

ministries. Assume that the preference parameters of the three parties over the “domestic affair” ministry are α_i , α_j , and α_k respectively, and that party i controls three legislative seats ($w_i = 3$), party j controls four seats ($w_j = 4$), and party k controls two seats ($w_k = 2$).

If all the three parties have the same preferences over the two ministries, the solution of the game is equivalent to the homogeneous case. As shown in Table 1, the allocation of cabinets and the utility of the three parties in all the possible coalitions is perfectly proportional to their nominal voting weight inside the coalition. As Proposition 4 states, the equilibrium coalition is the smallest minimum winning coalition.

Table 2 shows what happens when the parties' preference parameters are no longer identical. Party i displays more extreme preferences, exploiting more utility by obtaining the “foreign affairs” ministry rather than the “domestic affairs” one. In such a case, party i would obtain more utility than before in a coalition with both party j and party k . This happens because by having complementary

Table 3: Winning coalitions: Equilibrium allocations and party' utilities.

	$(x_d)_i$	$(x_d)_j$	$(x_d)_k$	$(x_f)_i$	$(x_f)_j$	$(x_f)_k$	λ	u_i	u_j	u_k
i+j	0.12	0.88	0.00	0.83	0.17	0.00	0.18	0.56	0.75	0.00
i+k	0.60	0.00	0.40	0.60	0.00	0.40	0.20	0.60	0.00	0.40
j+k	0.00	0.94	0.06	0.00	0.31	0.69	0.21	0.00	0.84	0.42

Notes - The table reports for each winning coalition (first column), the equilibrium allocation of x_d and x_f , the equilibrium offer to be the formateur, and the utility for each party. The size of the three parties is: $w_i = 3$, $w_j = 4$, $w_k = 2$. The office parameters are: $\alpha_i = 0.2$, $\alpha_j = 0.9$, $\alpha_k = 0.2$.

preferences, parties are able to divide the cabinet portfolios in a more efficient way. Notice that the global allocation ($\frac{x_d+x_f}{2}$) for the two parties is no longer perfectly proportional to their nominal voting weight: party i receives 54.5% of the ministerial seats, while party k receives 45.5%; their nominal voting weights being 0.6 and 0.4, respectively.

Table 3 shows that, when the preferences of the three parties are sufficiently different from each other, the complementarity's benefit may lead to an equilibrium coalition that is not the smallest winning coalition (but still minimum winning). In this case, the complementarity's benefit is exploited by the coalition composed by party j and k . Party k receives a larger utility by forming a coalition with a larger party (party j controls one seat more than party i) but with more complementary preferences.

5 Concluding remarks

Previous empirical studies on government formation have shown that portfolio allocations can be well explained by a "rule of thumb" that assigns offices in proportion to the votes each party contributes to the coalitions total. This empirical evidence is known in the literature as Gamson's Law. As Laver and Schofield (1990) note, the fact that cabinet seats seem to be allotted roughly in proportion to seat shares is one of the strongest empirical regularities of coalition governments.

Formal theorists have generally analyzed the process as a bargaining game where parties allocate the resources of government – cabinet portfolios – among themselves. Yet, these models predict that proposers should be able to exploit their privileged position for their own benefit, contradicting the empirical evidence.

The model presented here analyzes how resources are allocated between parties with conflicting preferences in a framework where the proposer is not designated exogenously but is determined endogenously, or inside, the coalition. As case studies show, the government formation process proceeds, in practice, in three stages: in the first stage, parties agree to coalesce; in the second stage, coalition partners designate a prime minister to propose a portfolio to either the head of state, the monarch, or the speaker of the house (who ratifies this decision formally); and in the third stage, parties bargain over the allocation of ministries.

The model is able to explain in equilibrium both proportional and not perfectly proportional allocations as functions of parties' preferences with regard to cabinet portfolios. In this respect, the present study represents a contribution that improves on the theoretical non-cooperative literature. Moreover, the model is able to explain how the composition of the equilibrium government coalition changes as the preferences of the parties change.

The model may be fruitfully extended in several ways. First, the introduction of policy-pursuing parties would lead the way to a more complete understanding of government formation as a function of parties' size and policy preferences. In fact, it seems reasonable to build a model that accounts for both the cabinet allocation and the government platform, as it is the cabinet that implements the joint policy program of the winning coalition. Ministerial positions are important in the policymaking activity of the government because individual ministers have a significant impact on policy in areas that fall under their jurisdiction. Hence, the larger the minister's jurisdiction area is, the bigger the impact of that minister on the government policy.

Second, assuming that parties are not only interested in dividing up the spoils

of office but that they have also substantive interests in policy may help understanding why parties form smaller or wider coalitions than minimum winning to obtain the confidence vote in the legislature. Parties whose utility is a function of the distance between the joint policy platform of the government coalition and their own ideal policies are said to be “policy concerned”. They may prefer to give external support to a minority government, whose joint policy is closer to their ideal policy, rather than forming a majority coalition, whose policy platform is farther. Thus, minority governments may be explained in equilibrium for sufficiently large “policy concerns”. Furthermore, risk adverse parties may prefer to form wider coalitions (super-majority governments) to reduce the variance of the expected joint policy for a sufficiently high degree of uncertainty.

6 Appendix

6.1 Heterogeneous cabinet positions' Solution Algorithm

Compute the equilibrium of the following simultaneous game:

$$\text{choose } x_i, y_i, \lambda_i \in [0, 1] \quad (10)$$

$$\text{to } \max(x_i^{\alpha_i} y_i^{1-\alpha_i})$$

$$\text{s.t.: } (1 - x_i)^{\alpha_j} (1 - y_i)^{1-\alpha_j} \geq k_j \lambda_i$$

and

$$\text{choose } x_j, y_j, \lambda_j \in [0, 1] \quad (11)$$

$$\text{to } \max(x_j^{\alpha_j} y_j^{1-\alpha_j})$$

$$\text{s.t.: } (1 - x_j)^{\alpha_i} (1 - y_j)^{1-\alpha_i} \geq k_i \lambda_j$$

Pick ε and ζ small enough and iterate on the following five steps:

1. Guess a value for $\lambda_i^{(1)}$ and $\lambda_j^{(1)}$.
2. Solve (10) and (11) for $\{x_i, x_j, y_i, y_j\}$.
3. Compute $\lambda_i^\#$ and $\lambda_j^\#$ using the indifference conditions:

$$(x_i^{\alpha_i} y_i^{1-\alpha_i}) = k_i \lambda_j^\#$$

$$(x_j^{\alpha_j} y_j^{1-\alpha_j}) = k_j \lambda_i^\#$$

4. Update $\lambda_i^{(1)}$ and $\lambda_j^{(1)}$

$$\lambda_i^{(2)} = \begin{cases} \lambda_i^{(1)} - \varepsilon, & \text{if } \lambda_i^\# \leq \lambda_i^{(1)} \\ \lambda_i^{(1)} + \varepsilon, & \text{if } \lambda_i^\# > \lambda_i^{(1)} \end{cases}$$

$$\lambda_j^{(2)} = \begin{cases} \lambda_j^{(1)} - \varepsilon, & \text{if } \lambda_j^\# \leq \lambda_j^{(1)} \\ \lambda_j^{(1)} + \varepsilon, & \text{if } \lambda_j^\# > \lambda_j^{(1)} \end{cases}$$

5. Iterate until $|\lambda_i^{(1)} - \lambda_i^{(2)}| < \zeta$ and $|\lambda_j^{(1)} - \lambda_j^{(2)}| < \zeta$.

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