Voting Systems and Strategic Manipulation: an Experimental Study [*]

Anna Bassi[†]

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Abstract

This paper presents experiments analyzing the strategic behavior of voters under three voting systems: plurality rule, approval voting, and the Borda count. Strategic behavior is significantly different under each treatment (voting system). Plurality rule leads voters to play in a more sophisticated manner, but not necessarily insincerely, displaying the lowest levels of manipulation. The opposite holds for the Borda count games, where players are the least sophisticated but the most insincere. Approval voting shows intermediate levels of strategic behavior. In terms of social efficiency, plurality rule unexpectedly performs better than both approval voting and the Borda count. Yet, plurality rule is the weakest performer under Condorcet efficiency, whereas approval voting and the Borda count perform remarkably well even with a small electorate.

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[†] NYU, Department of Politics, New York, NY, 10012. E-mail address: anna.bassi@nyu.edu.

1 Introduction

Voting is one of the chief foundations on which democracy and its political institutions are constructed and developed. Hence, understanding how citizens vote is key both to understanding democratic processes and to constructing formal models of political institutions.

An issue in voters' behavior that has long been of interest to political science scholars is whether voters vote "sincerely" for their most preferred alternative or "strategically", casting their vote for a different alternative in order to induce a better outcome. Let us suppose a voter believes that her most preferred candidate has little chance of competing for the lead in the election. Voting for such a candidate may be "wasted." The voter may decide to switch her vote to the expected leading candidate she most prefers in order to make her vote "pivotal" in determining a better outcome for herself. This is the trade-off a rational voter faces in an election. She must balance her relative preference for the different candidates against the relative likelihood of influencing the outcome of the election.

Gibbard (1973) proved that any preference aggregation method is vulnerable to strategic manipulation by voters. However it is not clear whether the strategic behavior of the voters leads to outcomes that are less desirable than the more representative sincere outcomes. On the one hand, a voting procedure should be thought of as a tool to aggregate the preferences of the society and the more accurately that voting represents the true preferences, the better the elected political institutions serve the society's objectives. On the other hand, sophisticated voting outcomes may be more efficient or superior to sincere outcomes, electing more often the Condorcet alternative. As Palfrey (1989) claims, when voters behave in a strategic way and expect others to do the same, they end up voting for one of the two leading candidates, making the Condorcet alternative more likely to be elected.

One can think that some rules are more vulnerable than others and that the degree to which they are empirically affected by strategic manipulation by voters in the elections provides an interesting criterion for evaluating alternative voting rules.

This paper explores the extent that voters engage in such manipulation through the use of laboratory experiments. The experiments are designed to evaluate the effect of different voting systems and voter experience on the emergence of strategic behavior in committee elections and how strategic manipulation affects the efficiency of the elections outcomes. Strategic behavior is studied in a series of complete-information elections in which subjects know both their own preferences and the preferences of each member of the committee. The committee is composed of five subjects who are asked to vote to elect one of four candidates according to either plurality voting, approval voting, or the Borda count. Abstention is not allowed, which is justified by the small size of the electorate which makes each voter highly likely to be pivotal, and consequently abstention a dominated strategy.

The experimental literature on voting in committees is wide and increasing. This line of research, beginning with the landmark articles by Plott (1967), Fiorina and Plott (1978), McKelvey (1976, 1979), and Schofield (1983) explores the cooperative and non-cooperative theory predictions on voting. The experimental works on multi-candidate elections focus on the fact that almost anything can happen in equilibrium. The reason is that there are many Nash equilibrium voting strategies (Palfrey 1989, Myerson and Weber, 1993). This work has been extended in a number of directions. For example, Gerber, Morton, and Rietz (1998) analyze cumulative voting to see if it can improve minority representation. Forsythe et al. (1996) explore the multi-candidate elections in a series of experiments. They compare plurality rule, Borda Count (BC) and Approval Voting (AV), finding that BC and AV produce better outcomes than plurality rule, in the sense that the Condorcet loser won the election less frequently.

This paper departs from the existing literature, and in particular from the Forsythe et al. (1996) paper with which it shares many of the same goals, by predicting unique equilibrium strategies and by allowing for more than three candidates. The problem of multiplicity of Nash equilibrium voting strategies is eliminated by adopting the iterated elimination of weakly dominated strategies equilibrium refinement that allow for a unique sophisticated strategy. The number of alternatives is increased to four to allow for a non-trivial analysis of voters' strategic behavior under AV (with three candidates every non-sincere vote is dominated by a sincere one, therefore there is no rational reason for a voter to move from a sincere behavior to a sophisticated one).[1]

2 Strategic behavior

Strategy-proofness has received a great deal of attention in social choice theory, where it is frequently discussed in the context of elections. The first impossibility result comes from Gibbard (1973) and

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[1] According to the definition of Brams and Fishburn (1978, 1983), when the number of candidates is less or equal to three, the set of approval voting strategies may be partitioned in two subsets: "dominated" and "admissible sincere" strategies. This implies that any Nash Equilibrium strategy is also admissible sincere. Extending the number of candidates to more than three generates a third subset of strategies that are neither admissible sincere nor dominated.

Satterthwaite (1975): given some basic conditions on the number of available alternatives and the size of the electorate, they show that every election procedure which is non-dictatorial cannot be strategy-proof.

Then, the question turns in designing a voting procedure that is the least manipulable. Manipulation occurs when a voter behaves strategically (insincerely) and he/she determines a change in the outcome (i.e., being pivotal). Strategic behavior may be studied in two settings: on the one hand strategic voting is decision-theoretic when individuals act optimally assuming that others in the electorate will vote sincerely; on the other hand, a non-myopic voter might account for the possibility of strategic voting behavior by others. To cope with this, a game-theoretic perspective is required, such as that taken by Cox (1994) and Myerson and Weber (1993).

When an outcome is strategy proof - from individual deviations - it is a Nash equilibrium in sincere strategies: the sincere strategy is the best reply to the other players' action for every player. As Gibbard pointed out, no voting system is able to produce a unique Nash equilibrium with strategic voters using only sincere strategies. There may exist multiple Nash equilibria with strategic voting. If the number of voters is greater or equal to three, any candidate may win in a Nash equilibrium.

A way to deal with this problem of multiplicity of Nash equilibria is to look for equilibrium refinements, such as ruling out weakly dominated voting strategies. Eliminating weakly dominated strategies, as Besley and Coate (1997) argue, is a reasonable approach in voting games. The first step of elimination simply amounts to no player voting for the worst-ranked alternative. But this does not prevent voters from going one step further and recalculating which strategies are weakly dominated for them given that other voters will not use the already deleted weakly dominated strategies (to vote for their worst-ranked candidate).

Allowing players to be strategic and to anticipate that the other voters will not use the weakly dominated strategies either (iterative elimination of weakly dominated strategies), pins down a set of possible outcomes in a voting game. This allows the computation of the sophisticated strategies each voter should use to maximize his own utility for any of the different voting systems. When the game is dominance-solvable, a unique sophisticated strategy for each player survives the elimination process, leading to a unique equilibrium.[2]

As Rajan (1998) shows, any strategy chosen under the iterated weak dominance solution concept

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[2] The order of deletion of weakly dominated strategies doesn't matter here because we assume strict preferences and maximal simultaneous deletion of weakly dominated strategies as elimination algorithm (see Gale, 1953 and Luce and Raiffa, 1957).

can be justified by beliefs that are consistent with the equilibrium concepts, which requires the assumption that players know each others' beliefs. Common knowledge of rationality leads to iterated weak dominance which results in perfect equilibria.

Moreno and Wooders (1996) prove that if a game is dominance solvable, then the unique NE is also coalition-proof. The iterated elimination refinement produces an equilibrium that is not only resistent to individual deviations but also to deviations by a coalition of players.

The literature about iterative elimination of weakly dominated strategies for predicting outcomes in voting games dates back to the seminal contribution of Farquharson (1969). He calls a voter "sophisticated" if, after considering all possible votes by others and seeing in which cases his vote makes a difference, he/she eliminates strategies that are dominated and assumes all the other voters do the same. This process tells the voter which strategies are best in all possible circumstances. He called this procedure "sophisticated voting" and defined a voting game "determinate" if sophisticated voting led to a unique outcome.

The power of the dominance solvability as solution concept has been studied in several experiments. The general finding (Nagel, 1995 and 1999) is that most of the subjects engage in up to four steps of eliminations. However, the depth of reasoning and rate of learning seem to depend on group size, sophistication of the players, and previous experience with the game.

The following will briefly characterize the three voting procedures analyzed in this paper. Assume that there are $k \geq 3$ candidates and n voters; both k and n are finite. Each voter i has a strict preference order over the set of candidates. When necessary, assume that the tie breaking rule is to pick a candidate randomly among those that are receiving the most votes.

Definition 1 (Plurality Voting (PV)) *Each voter casts a vote for one of the k candidates. The winner is the candidate with the most votes.*

As shown by De Sinopoli and Turrini (2002), in a four (or more) candidate PV game, the iterated weak dominance equilibrium refinement eliminates all the Nash equilibria except for one, leading to a unique NE strategy.

Definition 2 (Borda count (BC)) *Each voter ranks k candidates giving $k - s$ points to the s^{th} candidate. The winner is the candidate with the most points.*

It has been proved that this system, above all others, is the one which forces voters to make strategic decisions instead of encouraging sincere voting (Smith, 1999). Giving a high ranking to a candidate who is the most viable challenger of a voter's most preferred candidate hurts the chances of the preferred candidate winning. For example, a voter might not want to risk giving points to other candidates if his top choice is in a tight race. Then, given the number and strength of candidates, a strategic voter may give a high rank to candidates unlikely to win and a low rank to candidates who are in strong competition with a voter's favorite candidate.

Definition 3 (Approval Voting (AV)) *Each voter casts a vote for as many of the k candidates as she wants. The winner is the candidate with the highest total number of votes.*

The higher degree of freedom that this system allow to the voters (voters may choose how many candidates to vote for) leads to non-uniqueness or non-definiteness of sincere strategy. Voting for the most preferred alternative is undoubtedly sincere, but voting for both the most preferred and the second most preferred alternative may be considered sincere as well. Since under AV a voter is not limited to cast or to rank a defined number of alternatives, a sincere strategy is indeterminate.

Brams and Fishburn (1978, 1983) define admissible and sincere strategies under AV. They define a strategy "admissible" if it is not dominated, always involves voting for a most-preferred candidate, and never involves voting for a least preferred candidate. They define a strategy as "sincere" if, for any given candidate that a voter casts a vote, the voter also casts a vote for all candidates the voter ranks higher than that candidate.

From the definition of admissibility it follows that any weakly undominated strategy must belong to either the set of "admissible sincere" or to the set of "admissible non-sincere" strategies. Notice that when the number of alternatives is less than or equal to three, all the "admissible strategies" are also "sincere," meaning that the best reply strategies are always sincere as well.

Even if AV is subject to strategic manipulation, Brams and Fishburn (1978) argue that approval voting is subject to a milder type of strategic manipulation: voters react to strategic considerations by truncating their preference scale in a different way (setting their approval cutoff point higher or lower in their scale), without any need to desert their first choice.

Yet, sincere voting typically implies non-strategic behavior. Even if the AV sophisticated strategies belong to the set of the "admissible sincere" strategies, these strategies cannot be considered as sincere, since they are the result of strategic considerations. As Niemi (1984) points out: "...The fact the AV fails to yield a unique sincere strategy is of major significance: it means that, under AV, to say voters use admissible sincere strategies is not equivalent to say that they make no strategic calculation. The existence of multiple, equally sincere admissible strategies, suggests that AV encourage strategic (albeit admissible) voting...".

3 Experimental design

The experimental design focuses on the case in which information is complete and common to all players. Information is one of the main determinants of strategic behavior: the incentive to vote strategically increases with the precision and the type of information (private or common) available to the voter. Not surprisingly, when voters are not perfectly or privately informed, strategic voting exhibits negative rather than positive patterns (Feddersen & Pesendorfer, 1997). The intuition is that the less or more private the information about preferences of other voters, the more a voter is concerned about switching her vote in the wrong direction. This does not happen when voters base their decisions on sources of information that are complete and common across voters. Each voter can be assured that all others are observing the same information, and hence can coordinate on an appropriate candidate that is believed to be the trailing candidates by all the voters.

The game-theoretic perspective adopted here may lead to the possibility of self-reinforcing strategic voting. An initial perceived bias in favor of one particular candidate may lead to some strategic voting away from less favored candidates. But this strategic switching increases the incentive for others to vote strategically. This process continues until every subject plays a Nash equilibrium “sophisticated strategy.”

In order to allow for learning, every subject participated in twenty, one-shot election rounds. In order to eliminate repeated effects, every subject was randomly matched in a different group at every round.

3.1 Recruitment and Organization

I conducted three sessions of computerized experiments using undergraduate students at New York University during the Fall of 2005.[3] The experiment was conducted at the Center of Experimental and Social Sciences(C.E.S.S.) computer lab.

Each experimental session was played by students who enrolled in the NYU e-recruit subject pool. Students had joined the subject pool voluntarily by completing a form online indicating their interest in participating in experiments. When a student enrolled for participation in the experiment, he or she was told only that she would participate in a “voting experiment” and that “...the experiment will last about one hour. Subjects should earn between $10 and $25...”.

The experiment required each participant to cast one or more votes (abstention was not allowed)

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[3] The experiment was programmed and conducted with the software z-Tree, (Fischbacher 1999).

Table 1: Number of Observations.

Session	First 10 rounds	Second 10 rounds	# subjects	# obs.
1	Plurality (CW)	Plurality (Cycle)	10	200
2	Plurality (Cycle)	Plurality (CW)	10	200
3	Approval Voting (CW)	Approval Voting (Cycle)	10	200
4	Approval Voting (Cycle)	Approval Voting (CW)	10	200
5	Borda Count (CW)	Borda Count (Cycle)	10	200
6	Borda Count (Cycle)	Borda Count (CW)	10	200

Notes - The table reports for each voting systems and each sequence of treatments (in parentheses), the total number of subjects who participated in the experiment and the total number of observations.

over the set of candidates. Every subject was asked to vote according to one of the three different voting rules: PV, AV, and BC.

The experimental design employed a 3x2 design with three sessions and two treatment per session. I ran the experiment on 60 subjects, 20 per session. The 20 subjects formed four groups, each comprised of five subjects. To minimize any possible repeated-game influences on the outcomes of the experiments, the participants were allowed to participate in only one session. Sessions involved 20 rounds. Each subject was randomly grouped with different members in each round in such a way that a subject was not in the same group of subjects more than once in each treatment. Table 1 reports some summary statistics about the experiment.

Asking all the players in the same session to vote according to only one voting rule allows them to learn how the voting system works during the experiment. This seems reasonable because elections are usually run with a constant voting system over a long time interval. If a voting system turns out to be relatively non-manipulable when voters know it well, this is a stronger argument for its non-manipulability than if voters have only a brief experience with it.

Even if the group within which each subject was asked to play changes from round to round, the preference profile of the group did not change for ten consecutive rounds. This facilitates learning especially in small committees. Furthermore, if a voting system did turn out to be less manipulable in small committees, the result is stronger than if it did so in large committees.

At the beginning of the 11th round, each subject was asked to play according to a different preference ordering, which remained the same from that round until 20 rounds were completed. This allowed two different treatments (associated with two different preference profiles) per session, which provides a check for robustness of each voting system. In one treatment the preference profile of the group allows for the existence of the Condorcet Winner; in the other there was a cycle of

Table 2: Payoffs Schedule.

Period 1-10	A	B	C	D	Period 11-20	A	B	C	D
Type 1	7	5	1	3	Type 1	3	7	1	5
Type 2	7	5	1	3	Type 2	3	7	1	5
Type 3	3	1	7	5	Type 3	7	1	5	3
Type 4	3	1	7	5	Type 4	7	1	5	3
Type 5	5	1	3	7	Type 5	3	5	7	1

Notes - The table reports the number of experimental points each subject would receive if any of the four candidates won the election. In this session, subjects played the "Condorcet Winner treatment" in the first ten periods and the "Cycle treatment" in the last ten periods.

preferences. The former treatment is referred to as the "Condorcet" (CW) treatment and the latter as the "Cycle" treatment.

After all subjects had arrived, they were assigned to separate cubicles, each with a PC and a desk. The cubicles were arranged so that the subjects could not see anyone except the experimenters, nor could they been seen by anyone except the experimenters. Subjects were given instructions along with tables (Table 2) with the experimental points attached to every potential winning candidate. The instructions specified that each earned experimental point would have been converted into $, at a rate of 20 cents per point, at the end of the experiment.

Within each group, the payoff schedules were identical for every subject. Thus, each subject knew his or her own payoffs, the payoffs of the other types in the group, and the number of subjects of each type. However, subjects did not know the specific assignments of types nor the identities of the other subjects with whom they were grouped.

The instructions were read aloud so that subjects had common knowledge of the voting rules. After that, subjects were given an opportunity to ask questions. After the instructions, each subject was randomly assigned the role of one of six types. Types stayed the same throughout the experiment.

In each election each subject was asked to cast one or more ballots (according to the specific voting system/treatment), and the candidate with the most votes was declared the winner. Subjects were informed of the vote totals received by each candidate, how the different types cast their votes, the winner of the election, and the payoffs each type received from the election (if a tie occurred between two or more candidates, subjects received a payoff equal to the average value of the tied winners).

3.2 Voting Equilibria

Preferences are induced in the experiment by a vector of payoffs attached to the alternatives $(u_1^i, ...u_k^i)$, where u_K^i is the payoff (in terms of experimental points) that voter i would receive if candidate k won the election. The vectors of payoffs are shown in Table 2.

A social choice analysis of the preference profiles shows that candidate "D" is the Condorcet alternative (the candidate who wins against every other candidate in pairwise comparisons) in the "CW" treatment; that the Condorcet loser (the candidate who loses against every other candidate in pairwise comparisons) is candidate "B" in the "CW" treatment and candidate "D" in the"Cycle" treatment; and that candidate A is the social optimum (the candidate who maximizes the aggregate payoff of the voting group) in both the treatments.

In order to make both the voting game and the analysis of the results simpler and more intuitive, only symmetric voting strategies are considered (the strategies of the players depend only on their preferences and not on payoff-irrelevant factors such as the particular identity of the voter). Three comments are in order at this point.

First, I do not allow voters to abstain: abstention is always a weakly dominated strategy for any voter (Brams, 1994) and so will be deleted at the first round of the iterated deletion process.

Second, since the strategy of voting for one's worst alternative is always weakly dominated as well as the strategy of not voting for one's best alternative, this will be set as the first round of the elimination process for each voting system.

Third, the voting games determined by the preference profiles in Table **??** are all dominance solvable, but they have different characterizations: the number of strategies for each player increases moving from PV (k) to AV ($\sum_{i=1}^{k} \binom{k}{i}$) and to BC ($k!$). At the first step of elimination, the number of undominated strategies under PV, AV, and BC are three, seven, and fourteen, respectively. Given a larger set of strategies under BC, a greater number of steps of elimination of weakly dominated strategies is required. PV and AV require three steps of eliminations in the Condorcet Winner treatment and four steps in the Cycle treatment. BC requires six and eight steps respectively.

Under PV and BC, the sets of sincere and sophisticated strategies are singletons. Under AV, the strategy that is the best reply after three steps of iterated elimination of weakly dominated strategies belongs to the set of "admissible sincere" strategies. The "admissible non-sincere" strategy is always weakly dominated in the iterative process of elimination. The Nash Equilibria of each election, after n rounds of iterate elimination of weakly dominated strategies, are described in Table 3.

Table 3: Strategies and Equilibria.

	Voting System	Voter type	Sincere Strategy	Sophisticated Strategy	NE Outcome
CW treatment	PV	1 & 2	A	A	A
		3 & 4	C	C	
		5	D	A	
	AV	1 & 2	A, AB, ABD	AB	D
		3 & 4	C, CD, CDA	CD	
		5	D, DA, DAC	D	
	BC	1 & 2	ABDC	ABDC	D
		3 & 4	CDAB	DCBA	
		5	DACB	DACB	
Cycle treatment	PV	1 & 2	B	B	C
		3 & 4	A	C	
		5	C	C	
	AV	1 & 2	B, BD, BDA	BDA	A
		3 & 4	A, AC, ACD	AC	
		5	C, CB, CBA	C	
	BC	1 & 2	BDAC	BDAC	C
		3 & 4	ACDB	CADB	
		5	CBAD	CBAD	

Notes - The table reports for each voting systems (first column) and each voter's type (second column), the sincere and sophisticated (Nash Equilibrium) strategies. The last column tells the Nash Equilibria. The upper panel refers to the 'Condorcet winner treatment,' the lower panel refers to the 'Cycle' treatment.

4 Experimental Results

In this section a summary of the election results and individual voter behavior is presented. First, the election results are summarized for each of the three sessions and two treatments (section 4.1), and the frequency with which particular candidates, like the Condorcet Winner wins the race is compared. Second, the occurrence of the Duvergerian races is analyzed in the PV treatment (section 4.2). Third, the observed voters' behavior is analyzed, and sophisticated voting (section 4.3) and manipulation (section 4.4) is compared across the three voting systems and the two treatments. Last, the Condorcet efficiency (section 4.5) and the social efficiency (section 4.6) of the election results are discussed.

4.1 Summary of results

All voting systems may produce some undesirable paradoxes like electing the Condorcet loser or failing to elect the Condorcet winner. Among different voting systems AV and BC are expected to

Table 4: Summary of Election Results.

Treatment	A	B	C	D
Plurality (CW)	61.3%	0.0%	10.0%	28.7%
Approval (CW)	38.1%	6.0%	15.2%	40.6%
Borda (CW)	30.8%	6.7%	18.8%	43.8%
Plurality (Cycle)	11.3%	52.5%	36.2%	0.0%
Approval (Cycle)	31.0%	32.7%	30.2%	6.0%
Borda (Cycle)	39.2%	21.7%	32.1%	7.1%

Notes - The table reports the frequency with which each candidate (columns) wins the election under different voting systems (rows) and Treatments (the first three rows refer to the Condorcet Winner treatment; the last three ones refer to the Cycle treatment).

reduce the frequency of wins by Condorcet losers and to increase the frequency of wins by Condorcet winners compared to PV (Saari, 1989). In this section, election results are compared for each voting system in order to see how frequently each produces these paradoxes, and how repetition changes the election results.

Table 4 shows the frequencies with which each candidate won the elections. The upper panel shows that when the Condorcet winner exists, it is most likely to be elected by BC and least likely by PV, which instead elects the social optimum candidate with the largest frequency. The Condorcet loser is never elected by PV but is elected about 6-7% of the time by AV and BC. The lower panel illustrates that when there is a cycle in the preference profile of the electorate, all the candidates but the Condorcet loser are all almost equally likely to win the election (only PV shows a preponderance for electing two out of the four candidates).

Looking at these summary statistics, it appears that PV performs better compared to the other voting systems. First of all, the Condorcet loser alternative is never elected under this rule; second, even if the Condorcet Winner does not win so frequently (30% of the time) as under the other voting systems, this is mitigated by the fact that more than 60% of the time the winner is the alternative that maximizes the social welfare.

4.2 Plurality and Duverger's law

The voting outcomes showed in the previous section seems to suggest PV producing the regularity called Duverger's law, making only two candidates viable and the other candidates permanently weak.

Duverger (1954) claims that “the single-majority single ballot system favors the two party-system,” suggesting a causality relationship between plurality voting and the structure of the

Table 5: Duverger's law.

	'CW' treatment		'Cycle' treatment	
Period	Inexperienced	Experienced	Inexperienced	Experienced
1	-	√	-	√
2	√	√	√	√
3	-	√	-	√
4	-	√	√	√
5	-	√	√	-
6	√	√	-	-
7	√	√	√	-
8	-	√	-	-
9	-	√	-	√
10	√	-	-	√

Notes - The table shows for which elections Duverger's law is satisfied (√) or not (-). The satisfaction of Duverger's law is reported for each round (rows) and each treatment for both the unexperienced subjects and the experienced one (columns).

party system. More recent works reformulate Duverger's Law predicting strict bipartism as the outcome of any plurality rule election (Cox, 1994; Palfrey, 1989; and Myerson and Weber, 1993). Palfrey (1989) claims that: "[...] with instrumentally rational voters and fulfilled expectations, multicandidate contests under the plurality rule should result in only two candidates getting any votes." The uniquely stable equilibrium outcome - that is the result of strategic voting - involves positive support for only two candidates.

In order to capture the occurrence of a Duvergerian election, two criteria are used:

i) Duverger's law predicts that the spread between the candidates who obtain the second and the third highest vote total is larger than the spread between the candidates who obtain the first and the second highest vote total:

$$(|2'| - |3'|) > 2 \cdot (|1'| - |2'|)$$

ii) Duverger's law predicts that the third candidate, compared with the second one, obtains a smaller number of votes than expected:

$$|2'| > 2 \cdot |3'|$$

Table 5 shows the elections in which Duverger's Law is supported (satisfying either of the two criteria above). The first and the third column refer to the voting subjects who are inexperienced (i.e., that play the relative voting treatment as first), while the second and the fourth column refer to the voting subjects who are experienced (i.e., that played the relative voting treatment as second, after having experienced the other voting treatment).

The number of times that Duverger's law is satisfied seems to be sensible to the preference profile of the electorate, as data shows that turning from the "Condorcet Winner Treatment" to the "Cycle treatment" reduces the Duverger's law occurrence. Furthermore learning affects the frequency of Duvergerian races, as Table 5 shows, when the subjects had experienced a former treatment, the elections are more likely to be driven by two candidates.

4.3 Strategic behavior

As Farquharson (1969) suggests, if a voter doesn't have a strategy which dominates every other strategy, it is necessary for him/her to attempt to predict how the other voters are likely to vote to make the best use of his/her vote. If a voter has complete information of the other voters' strategies, this is sufficient to determine which strategies will not be chosen by the other voters (being dominated), pinning down the number of possible contingencies that may be arise. But at this point, the voter may reconsider his position on the basis of a smaller set of contingencies. This method reaches conclusions by successively eliminating contingencies that will not happen.

However, the process of successively eliminating dominated strategies may require a substantial amount of time, especially when the number of players increases. The voting games in our experimental design are difficult to solve, and the experimental subjects are time constrained. Thus it would not be surprising to find that subjects do not immediately play the sophisticated strategy, but rather play different strategies over the course of the session, converging ultimately toward the sophisticated strategy (a QRE analysis which takes into consideration noise behavior in experimental settings is discussed in detail in section 5). Furthermore, the preference profile of the electorate and the voting procedure affects the players' ability to converge or coordinate on particular equilibria.

The analysis of the voters' behavior is summarized along the following hypotheses (the frequencies with which each type played sophisticated, sincere, and dominated strategies are shown in Table A1 and Table A2 in the Appendix).

Hypothesis 1: "Voting system" treatment effect. *Sophisticated behavior is most likely under PV and less likely under BC.*

Sophisticated behavior increases as the number of iterated eliminations and the number of strategies decreases. Since BC involves a number of eliminations greater than both PV and AV, it will be likely to display the lowest level of sophisticated behavior. Furthermore, since the set of strategies is smaller under PV than AV, then PV will be likely to display sophisticated behavior

Table 6: Sophisticated behavior.

	'CW treatment'			'Cycle treatment'		
	PV	AV	BC	PV	AV	BC
Pooled	0.69	0.45	0.27	0.71	0.40	0.35
Inexperienced	0.71	0.47	0.21	0.61	0.38	0.35
Experienced	0.66	0.42	0.32	0.81	0.42	0.35

Notes - The table reports the frequency with which subjects played the sophisticated strategy. For each voting system and each treatment, three statistics are given: the first row for each type reports the frequency for the pooled subjects, the second row for the non experienced subjects, and the third row for the experienced subjects (who experienced a early treatment).

more often than AV.

Hypothesis 2: "Preference Profile" Treatment effect. *Sophisticated behavior is more likely in the CW treatment than in the Cycle treatment.*

Since the "Cycle treatment" implies a greater number of eliminations than the "CW treatment," it will be less likely to display sophisticated behavior.

Hypothesis 3: Learning. *Sophisticated behavior increases as the number of voting games experienced by the subjects increases.*

Hypothesis 1.

Table 6 summarizes the frequency with which the experimental subjects chose the sophisticated strategy in each preference profile and voting system treatment. From this first descriptive analysis, it seems clear that there is a distinction between the ability to behave sophisticatedly under the three voting systems. Plurality appears to be the system that induce the players to play the sophisticated strategy the most (70%), while the Borda Count appears to be the one that is less likely to lead players to behave in a sophisticated manner. Players under AV play the sophisticated strategy 40%-45% of the time, which is a fairly standard result in a game involving three steps of iterative eliminations.

I test the null hypothesis that the sophisticated behavior for the three voting system treatments comes from populations with the same median using a Wilcoxon non-parametric rank sum test. Comparing pairwise all three voting treatments (pooling data across CW and Cycle treatments), the equality of the median is rejected at a 5% significance level (P-value=5.62e-007 for PV vs AV, P-value=5.36e-013 for PV vs BC, P-value=0.0374 for AV vs BC). When comparing the three voting systems according to the preference profile treatments, the results do not change but for the comparison between AV and BC, for which the equality cannot be rejected at a 5% significance level (P-value=0.88).

In addition, I also used a Kolmogorov-Smirnov test to examine whether the distribution of the observed payoffs was the same across the two treatments. Consistent with the results of the Wilcoxon test, the equality of distributions is rejected at a 5% significance level for all the comparing voting systems (P-value=9.36e-005 for PV vs AV, P-value=6.02e-011 for PV vs BC, P-value=0.0446 for AV vs BC). Even here, comparing the three voting systems according to the preference treatments, the results do not change but for the comparison between AV and BC, for which the equality cannot be rejected at a 5% significance level (P-value=0.63).

Both the descriptive and the statistical analyses suggest that sophisticated behavior under different voting systems is significantly different.

Hypothesis 2.

From Table 6 it is not clear if the preference profile has a significative effect on the sophisticated behavior. Wilcoxon and Kolmogorov-Smirnov tests are used again to test the effect of the preference profile treatment.

According to Wilcoxon non-parametric rank sum test, the equality of the median under the "CW treatment" and the "Cycle treatment" cannot be rejected at a 5% significance level for PV and AV, while it is rejected for BC (P-value=0.8522 for PV, P-value=0.3881 for AV, and P-value=0.0592 for BC).

A Kolmogorov-Smirnov test confirms the above tests for PV and AV but not for BC: the equality of distributions cannot be rejected at a 5% significance level for all the comparing voting systems (P-value=0.6289 for PV, P-value=0.9808 for AV, P-value=0.6289 for BC). When the distribution of frequency of sophisticated choices is compared, instead of the median, the equality of distributions cannot be rejected.

The sophisticated behavior of the players, controlling for the voting system treatment, is not affected by the preference profiles of the voting group. As shown by the data, the fact that the preference profile allows for the existence of a Condorcet alternative does not lead to the voters choosing more sophisticated strategies.

Hypothesis 3.

The experimental results do not show any general learning effect in the later rounds. However, as Table A1 and A2 report, the data show a significant experience effect: subjects who experienced an early treatment played less randomly than subjects who did not (the standard deviation for all strategies is significantly lower).

4.4 Manipulation

In order for a scheme to be manipulable, there must be at least one voter whose insincere voting can cause a change in the set of the winning alternatives.

The most used indices of manipulability like the “tainted”, the “F1”, and the “expected” index (see Smith, 1999) measure the susceptibility of a voting rule to strategic voting by computing the benefit a voter would enjoy by misrepresenting her true preference ordering, requiring the definiteness of the voters’ sincere strategy. Since AV sincere voting is not definite, an alternative index of manipulability must be used to compare the manipulability of the three voting systems.

Manipulation occurs when a voter votes insincerely and he/she is pivotal. Then, in order to have a measure of the manipulation which occurred in the experiment, we compute the number of times the experimental subjects cast an insincere vote that induced a change in the outcome. As discussed in section 2, under PV and BC, there is a unique sincere strategy, while under AV, all the strategies in the set of “admissible sincere” set are equally potentially sincere.

We adopt here three definitions of sincerity under AV:

1. *Admissible sincerity: every strategy in the “admissible sincere” set is sincere (Brams and Fishburn, 1978, 1983).*

2. *Non-sophisticated sincerity: every strategy in the “admissible sincere” set but the sophisticated strategy is sincere.* Since voters can play strategically by deviating to the “admissible sincere” sophisticated strategy; only non-sophisticated admissible sincere strategies are considered as sincere.

3. *Pure sincerity: an AV strategy is sincere if a vote is casted for all the candidates whose utility exceeds the expected utility of the election (Merrill, 1988).* Since voters know with certainty the preferences of every other voter, the expected utility is calculated by assuming that the other voters play the sophisticated strategy.

$s_{i,t}$ is defined as the sincere vote and $n_{i,t}$ the insincere vote cast by voter i in round t, where N is the number of voters and T is the number of rounds. The manipulability index of a voting system is defined as the frequency with which voters cast pivotal (with probability $p_{i,t}$) insincere votes.

$$M = \sum_{t}^{T} \sum_{i}^{N} \frac{n_{i,t} p_{i,t}}{NT} \tag{1}$$

Table 7 reports the score of the different voting systems. BC shows a higher manipulability with respect to PV in every statistic. Furthermore, as Table A1 and A2 show, most of the insincere votes are dominated strategies, meaning that voters manipulate the voting outcome in a non-rational way (choosing weakly dominated strategies).

Table 7 shows how different definitions of AV sincere strategy are critical in determining the level of manipulation to which AV is vulnerable.

According to the definition of Brams and Fishburn (1978, 1983), with four alternatives, there are three admissible sincere strategies. Since proposition 4 shows that all the strategies that do not belong to the "admissible sincere" set are dominated, according to this definition, manipulation happens only when voters play dominated strategies. Data show that this type of manipulation occurs 15%-20% of the time.

Yet, this definition doesn't take into account the possibility of manipulating the outcome deviating to "admissible sincere" strategies, which is shown in section 4.3 to occur 40%-45% of the time. If voters manipulate the outcome by deviating to "admissible sincere" strategies (Niemi, 1984), then this strategy cannot be considered as sincere. Data show that this type of manipulation occurs between 50% and 60% of the times.

According to the definition of pure sincerity given by Merrill (1984), data shows that manipulation occurs between 60% and 80% of the times.

As one turns from the first definition to the second, and then to the third, the set of sincere strategies becomes smaller and smaller (with four alternatives there are three, two and one sincere strategies respectively). Not surprisingly, the level of manipulability increases monotonically as the definition of AV sincere strategies becomes more restrictive (i.e., as the sincere strategy set shrinks).

Table 7: Individual manipulation.

		PV	BC	AV (1)	AV (2)	AV (3)
‘CW treatment’	Pooled	0.25	0.63	0.13	0.58	0.60
	Inexperienced	0.27	0.68	0.10	0.57	0.65
	Experienced	0.24	0.58	0.16	0.58	0.55
‘Cycle treatment’	Pooled	0.30	0.48	0.18	0.58	0.81
	Inexperienced	0.25	0.46	0.29	0.67	0.76
	Experienced	0.35	0.56	0.06	0.48	0.85

Notes - the table reports the manipulability score for the three voting systems under the two treatments (upper and lower panel). The three AV scores refer to different definition of sincerity. For each voting system three statistics are given: the first row for each voting system reports the score for the pooled subjects, the second row for the non-experienced subjects, and the third row for the experienced subjects (who experienced a early treatment).

4.5 Condorcet-Efficiency

Numerous criteria have been suggested to help select which voting systems, among various systems for choosing winners from profiles, best reflect the cumulative will of the electorate. One of the most common of these is the Condorcet criterion. A candidate is the Condorcet winner if she defeats each candidate in pairwise majority elections. It is well known that a Condorcet winner need not exist. However, many feel that a reasonable voting system should elect the Condorcet alternative whenever such a candidate exists; this is called “Condorcet efficiency”.

For each voting system, Condorcet Efficiency is measured as the percentage of a given class of elections for which the Condorcet candidate is chosen. Table 8 reports the Condorcet efficiency score of each voting system. Consistent with the findings of other authors, even in small electorates, PV is the weakest performer, whereas AV and BC perform quite well (with the latter being the most likely to produce efficient outcomes consistent with Merrill’s work (1984)).

However, the last three rounds of each voting treatment highlight that under PV voters learn how to manipulate the outcome in a way to elect the Condorcet alternative, while the frequency with which it is elected under the other voting systems decreases in the last rounds.

None of the voting systems considered in the experimental analysis guarantees the selection of the Condorcet alternative, but we can see how the sophisticated outcomes are superior to the sincere outcomes. Recall what the sincere and the sophisticated outcomes would be for each voting system. The theoretical Condorcet efficiency if every player behaved sincerely would be 0% for PV, 23% for AV, and 50% for BC. Instead, the theoretical Condorcet efficiency if every player behaved sophisticatedly would be 0% for PV and 100% for AV and BC, since the NE candidate for the

Table 8: Condorcet Efficiency.

Period	Plurality	Approval	Borda
1	12.5%	33%	50%
2	25%	50%	62.5%
3	12.5%	37.5%	37.5%
4	37.5%	25%	83%
5	37.5%	37.5%	58%
6	25%	65%	25%
7	25%	54%	0%
8	25%	54%	33%
9	50%	25%	62.5%
10	37.5%	25%	25%
Average	29%	41%	44%
Last 3 Per. Av	37.5%	34.6%	40.16%
Sincere Outcome	0%	23%	50%
Sophisticated Outcome	0%	100%	100%

Notes - The Condorcet Efficiency is defined as the percentage of a given class of elections for which the Condorcet candidate is chosen. The Condorcet efficiency is reported for each round (rows) and each voting system (columns). After the tenth period row, the overall average and the average of the last three periods is reported. The last two rows indicate what the Condorcet Efficiency would be for the sincere and the sophisticated outcomes (breaking the ties with a fair coin and assuming that under approval voting voters randomizes between sincere strategies).

latter ones coincides with the Condorcet winner.

4.6 Social Welfare Efficiency

The second measure of efficiency considers the value of each candidate for the entire electorate as compared with other candidates. As distinct from the notion of Condorcet efficiency, the intensity of the electorate's preferences are weighted. Since the intensities of the preferences is usually not revealed and difficult to discover by a researcher, this measure of efficiency is used less frequently than the first one. An experimental analysis allows an analysis on intensity of preferences, since subjects' preferences are functions of the payoffs they receive at the end of the experimental session.

Following Harsanyi (1977), the "social optimum" is the candidate who maximizes the sum of all voters' utilities. Therefore "social efficiency" of a voting system is the normalized ratio between the expected social utilities of the candidate selected by the system and the candidate maximizing social utility.

Table 9 reports the performance of each voting system. Unexpectedly the PV System performs

Table 9: Social Efficiency.

	Plurality		Approval		Borda	
Period	CW	Cycle	CW	Cycle	CW	Cycle
1	0.93	0.90	0.92	0.88	0.96	0.96
2	0.95	0.91	0.96	0.91	0.89	0.91
3	0.96	0.87	0.91	0.87	0.94	0.83
4	0.94	0.88	0.95	0.87	0.87	0.90
5	0.95	0.87	0.81	0.92	0.93	0.83
6	0.95	0.89	0.89	0.88	0.80	0.87
7	0.98	0.88	0.87	0.93	0.82	0.91
8	0.95	0.92	0.89	0.95	0.79	1.00
9	0.96	0.89	0.95	0.89	0.92	0.86
10	0.94	0.89	0.92	0.88	0.95	0.93
Average	0.95	0.89	0.91	0.90	0.89	0.90
Sincere Outcome	0.88	0.96	0.96	0.87	0.96	0.91
Sophisticated Outcome	1	0.83	0.92	1	0.92	0.83

Notes - The Social Efficiency of a voting system is calculated as the ratio between the expected social utilities of the elected candidate and the social optimum. The social efficiency is reported for each round (rows) and each voting system in both the CW treatment and Cycle treatment (columns). After the tenth period row, the overall average and the average of the last three periods is reported. The last two rows indicated what the social efficiency would be for the sincere and the sophisticated outcomes (breaking the ties with a fair coin and assuming that under approval voting voters randomize between sincere strategies).

significantly better than both AV and BC in the Condorcet winner treatment and approximately the same as the other two voting systems in the cycle treatment. The explanation is simple: in the first treatment, the PV system's Nash Equilibrium, that is the actual outcome in 61% of experimental elections, is the social optimum candidate. Besides this case, the measure of social efficiency is approximately the same across voting systems and treatments.

Note that the social optimum candidate is not always the Condorcet candidate. That is, the Condorcet criterion and the criterion of maximizing social utility may be different. However, the Condorcet candidate yields a high level of social efficiency, even if not the highest one.

Recalling again the sincere and the sophisticated outcomes of each voting system, we can now analyze if the sophisticated outcomes are superior to the sincere outcomes in terms of social welfare. Under BC, sincerity always has a positive effect on the social welfare of the society, but under PV and AV this effect is not clear, depending on the preference profile of the voting group.

As Table 9 shows, sophisticated behavior may not necessarily lead to more socially efficient outcomes: when players act on the basis of individual self-interest alone, their behavior could lead to the degradation of the social welfare of the electorate.

5 Quantal Response Equilibrium analysis

Iterated elimination of dominated strategies relies extensively on the assumption of perfect rationality. However, choice behavior in laboratory is shown to be noisy, revealing mistakes and inconsistencies over time. In order to evaluate the iterated elimination procedure using data from laboratory experiments, it is convenient to model a type of noisy behavior that includes the rational-choice Nash predictions as a limit case.

One way to relax the assumption of noise-free, perfectly rational behavior is to specify a utility function with a stochastic component. Probabilistic choice models (e.g., logit, probit) have long been used to incorporate stochastic elements into the analysis of individual decisions, and the quantal response equilibrium (McKelvey and Palfrey, 1995) is the analogous way to model games with noisy players. The quantal response equilibrium (QRE) is a statistical version of Nash equilibrium, where every strategy is played with positive probability. Players' beliefs about other players' actions determines players' expected payoffs from different strategies, which in turn, generates choice probabilities according to some quantal response function.

An identical and independently distributed stochastic term λ, accounting for the noise in the best replies, is added to the expected payoff of each strategy. In the absence of noise (λ=0), the QRE equilibrium reduces to a Nash equilibrium. Hence, the equilibria described in section 3.2 are limit cases of the quantal response equilibria described here. In order to provide parametric estimates, a logit specification of the QRE is analyzed, where the quantal response functions are logit curves, and λ is the response parameter.

The Quantal response functions for the PV and AV games, which describe the voting probability as "noisy best responses" to the expected payoff of different strategies, are presented in figure 1 and 2 respectively.

Figure 1 presents the relationship between the probability of playing different strategies under PV and the equilibrium values of λ along with the estimated values for both the Condorcet winner treatment and the Cycle treatment. The curves in the plots show the equilibrium probability of playing each of the three strategies associated with given values of λ. For $\lambda = 0$, the QRE predicted probability is equal to 0.33. As λ increases, the equilibrium probability of playing the equilibrium strategy (section 3.2) approaches one, while the equilibrium probability of playing the

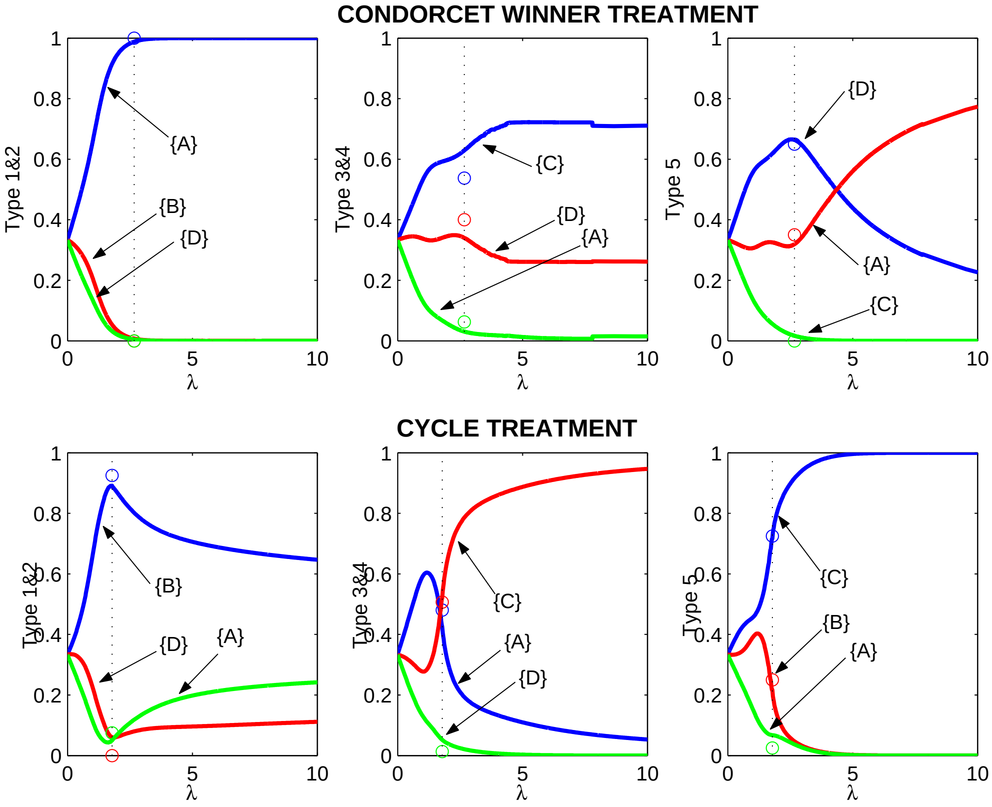

Figure 1: Quantal Response Equilibrium functions for PV Games

other strategies approach zero. The vertical lines denote unconstrained estimated value of λ, and the small circles denote the observed frequency with which each strategy is played.

Figure 2 presents the quantal response functions for AV games. The curves in the plots show the equilibrium probability of playing each of the four strategies (three "admissible" and one "not admissible") associated with given values of λ. For $\lambda = 0$, the QRE predicted probability is equal to 0.25. As λ increases, the equilibrium probability to play the equilibria strategy (section 3.2) approaches one. The vertical lines denote the unconstrained estimated value of λ, and the small circles denote the observed frequency with which each strategy is played.

Table 10 reports the estimates of the QRE for the plurality and approval voting games. The table reports two estimated values of the error term λ_{CW} and λ_{Cycle} for the two voting treatments. The hypothesis that a unique parameter explain the behavior of the subjects for each voting systems has been tested ($H0 : \lambda_{CW} - \lambda_{Cycle} = 0$). The estimated values for λ in the two voting treatments are $\lambda_{PV} = 1.84$ and $\lambda_{AV} = 1.28$ and the value of the Log Likelihood function is -219.96 and -404.76 respectively. For both PV and AV, using a likelihood ratio test, the difference between

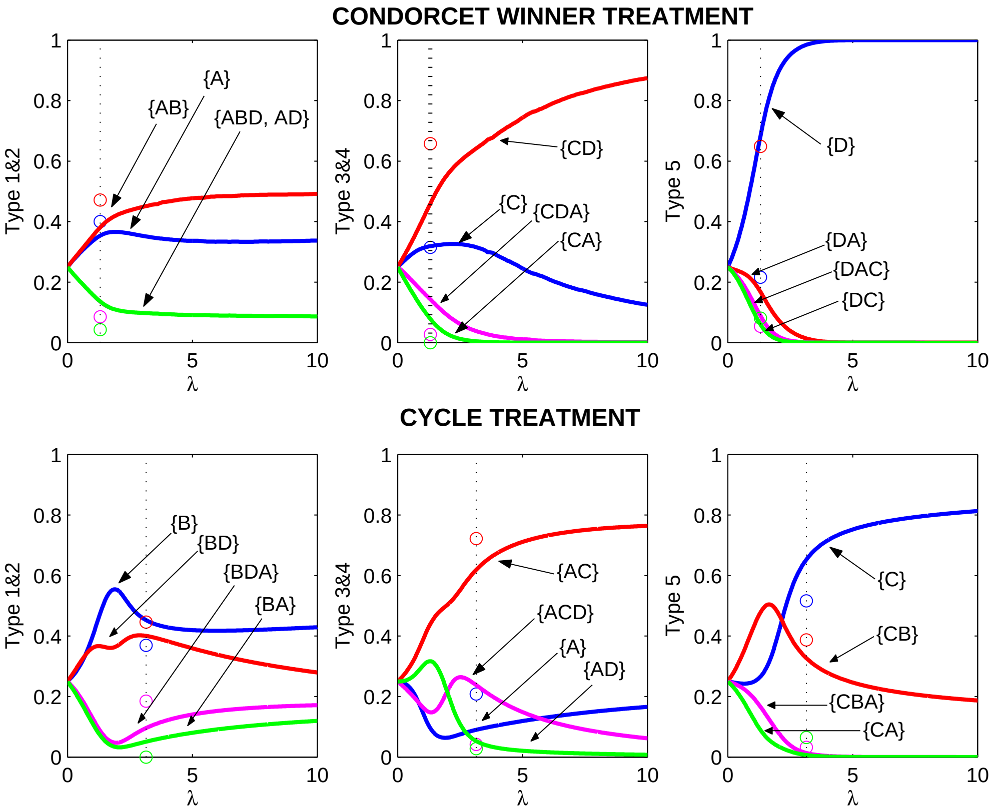

Figure 2: Quantal Response Equilibrium functions for AV Games

λ_{CW} and λ_{Cycle} is significant (the $\chi 2$ equals 12.58 and 22.94 respectively), suggesting that a unique parameter cannot explain the behavior of the subjects in different strategic environments.

As shown by table 10, the QRE estimated probability fits the data better than the Nash predictions, especially under PV which shows a larger log likelihood value with respect to AV. The likelihood ratio test suggests that a same estimation for the "noise" does not explain the behavior under the two treatments: in the cycle treatment, players' best responses are much more noisy than under the Condorcet winner treatment.

By combining behavior across voting systems, one can analyze whether the effect of different preference aggregation methods on overall voting behavior is different. The estimations of lambda, constrained to be equal across voting treatments, is $\lambda_{CW} = 1.70$ (LogL=318.31) and $\lambda_{Cycle} = 1.81$ (LogL=313.73) for the Condorcet winner and cycle treatments respectively. For both treatments, using a likelihood ratio test, the difference between λ_{CW} and λ_{Cycle} is significant (the $\chi 2$ equals 26.00 and 24.15 respectively), suggesting that a unique parameter cannot explain the behavior of the subjects under different voting systems.

Table 10: QRE Estimates

		Type 1 & 2			Type 3 & 4			Type 5			λ
Treatment	Votes	Data	NE	QRE	Data	NE	QRE	Data	NE	QRE	(-LogL)
	1’	1.00	1.00	0.99	0.54	1.00	0.63	0.65	0.00	0.66	
Plurality	2’	0.00	0.00	0.01	0.40	0.00	0.34	0.35	1.00	0.32	2.67
CW	3’	0.00	0.00	0.00	0.06	0.00	0.03	0.00	0.00	0.02	(-99.44)
	1’	0.93	1.00	0.89	0.46	0.00	0.43	0.73	1.00	0.72	
Plurality	2’	0.00	0.00	0.06	0.49	1.00	0.52	0.25	0.00	0.22	1.78
Cycle	3’	0.07	0.00	0.05	0.05	0.00	0.05	0.02	0.00	0.06	(-114.23)
	1’	0.35	0.00	0.35	0.30	0.00	0.32	0.20	1.00	0.68	
Approval	1’-2’	0.41	1.00	0.38	0.60	1.00	0.46	0.60	0.00	0.17	1.30
CW	1’-2’-3’	0.08	0.00	0.13	0.04	0.00	0.14	0.05	0.00	0.09	(-205.87)
	1’-3’	0.04	0.00	0.14	0.00	0.00	0.08	0.07	0.00	0.06	
	1’	0.30	0.00	0.44	0.19	0.00	0.10	0.40	1.00	0.70	
Approval	1’-2’	0.36	1.00	0.39	0.65	0.00	0.66	0.30	0.00	0.29	3.67
Cycle	1’-2’-3’	0.15	0.00	0.11	0.04	1.00	0.21	0.03	0.00	0.01	(-178.42)
	1’-3’	0.00	0.00	0.06	0.03	0.00	0.04	0.06	0.00	0.00	

Notes - The table reports, for each voter type (column) and each strategy (rows), the observed frequency and the forecasted probability of casting a vote by NE and QRE under the four treatments. Under PV, three strategies are reported: casting the vote for the first choice (sincere strategy), the second and the third choice. Under AV, the three admissible sincere strategies are reported (casting the vote for the first choice, for the first two choices, and for the first three choices) along with the dominated strategy of casting a vote for the first and the third choice.

These results are consistent with the test of hypothesis in section 4.3: the sophisticated behavior of the players is a function of the different voting system treatments, and of the preference profiles of the voting groups.

6 Conclusions

This paper reports the results from experiments designed to test the effects of different voting systems on strategic behavior and election outcomes. Laboratory experiments allow for control of intensity of preferences and voter information. From the analysis, one of the main results is that voters appear to be playing the best reply strategy a significantly high number of times. Sophisticated voting is common: across all 240 elections, 70% of the subjects voted the Nash Equilibrium strategy under the PV system, 42% of the subjects under AV, and 31% under BC (sophisticated behavior under different voting systems is significantly different controlling for both preference profiles and subjects’ experience).

Surprisingly, subjects play the unique equilibrium strategy more often than the existing literature on iterated solvable games would suggest, and subjects also engage in three or more steps of elimination significantly more often.

It is worth noting that dominated (and insincere) behavior is quite frequent, especially when the voting system allows for a higher degree of freedom (i.e., when the voter may cast more than a single vote or rank the alternatives). In fact subjects casted dominated ballots between 10% and 18% of the time under either PV or AV, but these percentages increase to 40%-50% under BC. Though such actions are labelled "dominated," they can be interpreted as strategic attempts to manipulate the elections results, therefore providing greater support to the evidence that subjects are not "naive."

Under PV, the experimental outcomes provide support for Duverger's law, in creating two-candidate elections races, especially when the subjects are experienced. This result is consistent with Gerber et al. (1998) and Forsythe et al. (1996), who find support for Duverger's law creating two-candidate races. The frequency of Duvergerian races seems to be sensible to the preference profile of the electorate, as data show that the absence of a Condorcer candidate reduces the Duverger's law occurrence.

To analyze the manipulability of the three voting systems, the number of times the experimental subjects cast an insincere vote that induced a change in the outcome is compared. Three definitions of sincerity under AV are adopted: the Brams and Fishburn's (1978, 1983) definition of "admissible sincerity," the Merrill's (1988) definition of "pure sincerity," and an intermediate definition that considers all the strategies in the "admissible sincere" set as sincere except the sophisticated one. Data show how the three different definitions of AV sincere strategy are critical in determining the level of manipulation to which AV is vulnerable. Under the first definition, AV is almost always the least manipulated, but when a more restrictive definition is adopted, the level of manipulation dramatically increases, showing levels of manipulation sometimes larger than BC (which is on average the most manipulated one).

According to the mild definition of AV "admissible sincerity," AV displays by far the lowest level of manipulation. Elsewhere, PV is the system which displays the lowest level of individual manipulation. This is because under PV, it is not optimal for all the voters to vote insincerely, but only for the voters who do not support the trailing candidates. Under AV and BC, this is not the case.

In terms of social efficiency, unexpectedly the PV System performs significantly better than both AV and BC (producing an outcome 5% more efficient) in the Condorcet Winner treatment, while all voting systems perform equally well in the Cycle treatment (producing a social welfare equal to 90% of the optimum social welfare). Yet, consistent with the literature, PV is the weakest performer under Condorcet efficiency, whereas AV and BC perform remarkably well even with a small electorate. However, in the last three rounds, PV's performance improves significantly while the performances of AV and BC decrease.

A Quantal Response analysis shows that the QRE concept fits the experimental data better than the Nash equilibrium. The estimated value of λ for PV is fairly high under the Condorcet Winner treatment, meaning that players play the Nash equilibrium strategies. However in the Cycle treatment, players' best responses are more affected by noise. This implies that the analysis of sophisticated behavior is not robust across different preference profiles. The value of the Log likelihood for AV games is not as high as for PV, meaning that a Quantal response interpretation does not fit the data as well as under PV. This implies that under AV, voters' best responses are more affected by noise. Even here, the difference under the two treatments is significant suggesting that different strategic environments do matter in the the strategic analysis of voting games.

7 Appendix

7.1 Instructions

You are about to participate in an experimental session on voting procedures and you will be paid for your participation with a cash voucher privately at the end of the session. What you earn depends partly on your own decisions, partly on the decisions of others, and partly on chance. Please turn off pagers and cellular phones now. Please close any programs you may have open on the computer.

The entire session will take place through computer terminals and all interaction between you and other session participants will take place through the computers. Please do not attempt to directly communicate with other participants during the session. If you have any question during the experiment, ask the experimenter and she will answer them for you. Other than these questions, you must keep silent until the experiment is completed. If you break silence while the experiment is in progress, you will be asked to leave the experiment.

7.1.1 Voting groups

This experiment will last for 20 periods. In each period you will be placed into groups of 5 people. You will not be told who else is in your group. You and the other members of your group will be asked to make a decision in that period. Then the next period you will be placed into new groups and again asked to make decisions, and so forth, for 20 periods. Thus, in each period the group memberships will be different from the memberships in previous periods and you will not belong to the same group more than once.

In each period, the group is asked to choose one alternative among a set of four: A, B, C, and D. Every member of the group is asked to vote accordingly to the rule discussed in the following screens. The votes cast will determine the winning alternative in that period.

In each period you may earn some ”experimental points” which will be converted into $, at a rate of 20 cents for every point, at the end of the experiment.

7.1.2 Types

At the beginning of this experiment each of you will be randomly assigned a type (1, 2, 3, 4 or 5). The type you are assigned to will remain the same for the entire experiment. Each group will be composed of one person of each type.

7.1.3 Payoffs

In each round, the payoff you will receive will depend upon the winning alternative in your group. The Payoff Tables show the payoffs you will receive if any of the four alternatives wins. In the Payoff Tables you will also find the payoffs that other types of voters will receive depending on which alternative wins. This means that every member of the group knows the payoffs that the other members of the group will receive if each of the four alternatives wins.

The payoff schedule will stay the same for 10 periods. At the beginning of the 11th period, the payoffs will change for all types of voters as shown in the Payoff Table. -Please look at the Payoff Tables now-

In the Payoff Table you will find two tables. The first one tells you the payoffs that every type will gain in the first 10 periods (periods: 1 - 10). The second one tells you the payoffs that every member will get in the last 10 periods (periods: 11 - 20).

Period 1-10	A	B	C	D	Period 11-20	A	B	C	D
Type 1	7	5	1	3	Type 1	3	7	1	5
Type 2	7	5	1	3	Type 2	3	7	1	5
Type 3	3	1	7	5	Type 3	7	1	5	3
Type 4	3	1	7	5	Type 4	7	1	5	3
Type 5	5	1	3	7	Type 5	3	5	7	1

7.1.4 Example: How to read the table

Let's take the first table. It tells you the payoffs you and the other members of the group would receive for every potential winning alternative.

For example, in each of the first ten periods, if alternative C wins, members of type 1 and 2 will get 1 experimental point, members of type 3 and 4 will get 7 points, and members of type 5 will get 3 points. After the 10th round, the payoffs attached to each alternative changes. Now if alternative C wins, members of types 1 and 2 will get 5 experimental points, members of types 3 and 4 will get 1 point, and members of type 5 will get 7 points.

Whenever a tie occurs between 2 alternatives, each member gets the average of the experimental points attached to each alternative. For example, in each of the first ten periods, if alternative A and C tie, members of type 1 and 2 will get 4 experimental points, members of type 3 and 4 will get 5 points, and members of type 5 will get 4 points.

7.1.5 Voting Rule

If PV treatment.

You may cast only 1 vote for an alternative of your choice. You can change your ballots until you press the button "Continue", but you cannot change the ballot after that. After all the members of the group have casts their ballots, the number of votes for every alternative will be summed up. You will be told the total number of votes each alternative receives in your group. The alternative with the highest number of votes in your group will be the winner of the election in your group. If two or more alternatives tie with equal (highest) vote totals, your payoff will be the average of the payoffs you attach to each tying alternative.

If AV treatment.

You may cast only 1 vote for one or more alternatives of your choice. You can change your ballots until you press the button "Continue", but you cannot change the ballot after that. After all the members of the group have casts their ballots, the number of votes for every alternative will be summed up. You will be told the total number of votes each alternative receives in your group. The alternative with the highest number of votes in your group will be the winner of the election in your group. If two or more alternatives tie with equal (highest) vote totals, your payoff will be the average of the payoffs you attach to each tying alternative.

If BC treatment.

You may cast your ballot by ranking the 4 alternatives from the most preferred to the least preferred one. A decreasing number of score points will be attached to them starting from 3 points for your most preferred to 0 points for the least preferred. You can change your ballots until you press the button "Continue", but you cannot change the ballot after that. After all the members of the group have casts their ballots, the number of score points for every alternative will be summed up. You will be told the total number of points each alternative receives in your group. The alternative with the highest number of points in your group will be the winner of the election in your group. If two or more alternatives tie with equal (highest) points totals, your payoff will be the average of the payoffs you attach to each tying alternative.

7.1.6 Example: How the voting rule works

If PV treatment.

Let's suppose that 1 member voted for A; 1 member for B; 1 member for C; and 2 members for D. D would win, since D received the highest number of votes. The payoffs would be: types 1 and 2 would get 3 experimental points; types 3 and 4 would get 5 points; type 5 would get 7 points

7.2 Tables of experimental results

Table A1: Voters' behavior in CW treatment.

		Plurality			Approval Voting			Borda		
		Sinc	Soph	Dom	Sinc	Soph	Dom	Sinc	Soph	Dom
	Pooled	1.00		0.00	0.43	0.41	0.16	0.39		0.61
		(0.00)		(0.00)	(0.22)	(0.23)	(0.19)	(0.13)		(0.13)
1&2	Non Exp	1.00		0.00	0.53	0.43	0.04	0.35		0.65
		(0.00)		(0.00)	(0.25)	(0.29)	(0.16)	(0.13)		(0.13)
	Exp	1.00		0.00	0.33	0.39	0.28	0.43		0.57
		(0.00)		(0.00)	(0.12)	(0.17)	(0.14)	(0.12)		(0.12)
	Pooled	0.54		0.46	0.34	0.60	0.06	0.34	0.08	0.58
		(0.26)		(0.26)	(0.23)	(0.25)	(0.11)	(0.23)	(0.12)	(0.25)
3&4	Non Exp	0.55		0.45	0.28	0.63	0.09	0.30	0.03	0.67
		(0.26)		(0.26)	(0.25)	(0.32)	(0.13)	(0.28)	(0.08)	(0.29)
	Exp	0.53		0.47	0.40	0.57	0.03	0.38	0.13	0.49
		(0.28)		(0.28)	(0.21)	(0.17)	(0.08)	(0.18)	(0.13)	(0.17)
	Pooled	0.65	0.35	0.00	0.65	0.20	0.15	0.40		0.60
		(0.40)	(0.40)	(0.00)	(0.37)	(0.25)	(0.24)	(0.21)		(0.21)
5	Non Exp	0.55	0.45	0.00	0.55	0.25	0.20	0.30		0.70
		(0.37)	(0.37)	(0.00)	(0.37)	(0.26)	(0.26)	(0.26)		(0.26)
	Exp	0.75	0.25	0.00	0.75	0.15	0.10	0.50		0.50
		(0.42)	(0.42)	(0.00)	(0.35)	(0.24)	(0.21)	(0.00)		(0.00)

Notes - The table reports the average frequency with which voters play sincere, sophisticated, and dominated strategies under different voting systems (columns). Standard deviations are in parenthesis. For each type of voter (row), three statistics are given respectively for pooled subjects, for the non experienced subjects, and for the experienced subjects (who experienced a early treatment).

Table A2: Voters' behavior in Cycle treatment.

		Plurality			Approval Voting			Borda		
		Sinc	Soph	Dom	Sinc	Soph	Dom	Sinc	Soph	Dom
	Pooled	0.93		0.07	0.66	0.15	0.19	0.48		0.52
		(0.14)		(0.14)	(0.365)	(0.19)	(0.21)	(0.16)		(0.16)
1&2	Inexp	1.00		0.00	0.43	0.22	0.35	0.43		0.57
		(0.00)		(0.00)	(0.37)	(0.22)	(0.175)	(0.17)		(0.17)
	Exp	0.85		0.15	0.89	0.08	0.03	0.53		0.47
		(0.18)		(0.18)	(0.13)	(0.12)	(0.08)	(0.14)		(0.14)
	Pooled	0.46	0.49	0.05	0.23	0.65	0.12	0.61	0.19	0.20
		(0.31)	(0.29)	(0.10)	(0.27)	(0.29)	(0.13)	(0.17)	(0.16)	(0.17)
3&4	Inexp	0.65	0.30	0.05	0.26	0.57	0.16	0.68	0.20	0.12
		(0.27)	(0.20)	(0.11)	(0.24)	(0.17)	(0.12)	(0.21)	(0.20)	(0.18)
	Exp	0.28	0.67	0.05	0.20	0.73	0.08	0.54	0.18	0.28
		(0.22)	(0.24)	(0.11)	(0.31)	(0.37)	(0.12)	(0.11)	(0.12)	(0.14)
	Pooled	0.73		0.27	0.33	0.40	0.27	0.43		0.57
		(0.38)		(0.38)	(0.30)	(0.31)	(0.34)	(0.29)		(0.29)
5	Inexp	0.45		0.55	0.30	0.30	0.40	0.50		0.50
		(0.37)		(0.37)	(0.26)	(0.26)	(0.32)	(0.24)		(0.24)
	Exp	1.00		0.00	0.35	0.50	0.15	0.35		0.65
		(0.00)		(0.00)	(0.34)	(0.33)	(0.34)	(0.34)		(0.34)

Notes - The table reports the average frequency with which voters play sincere, sophisticated, and dominated strategies under different voting systems (columns). Standard deviations are in parenthesis. For each type of voter (row), three statistics are given respectively for pooled subjects, for the non experienced subjects, and for the experienced subjects (who experienced a early treatment).

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