Durable Goods Monopoly with Vertical Product Differentiation*

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Abstract

This study analyzes a vertically differentiated market for an imperfectly durable good served by a monopolist in an infinite horizon, discrete time game. I characterize Markov perfect equilibria of this game as a function of the common discount rate, the common depreciation rate of the goods, the length of the time period between successive price changes, and the quality levels of the goods. I establish that quality differentiation may alleviate the commitment problem a durable goods monopolist faces. This study suggests that when the innate durability of a good is high, the monopolist will damage a portion of the goods and produce a lower quality good to credibly commit to high future prices for the higher quality good.

1 Introduction

Durable goods markets have received much attention since Coase (1972) argued that a durable goods monopolist cannot exercise his monopoly power when

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the good is perfectly durable. Upon sale of a unit, the monopolist has incentives to sell more by lowering the price of the good. Thus, a rational consumer, anticipating that the monopolist will cut the price as long as it is no less than the marginal cost of the good, prefers deferring consumption so long as the price is not close to the competitive level. Hence, in the absence of the ability to commit to the future prices, the monopoly power of a durable goods monopolist is largely deteriorated, and the monopolist is unable to extract as much consumer surplus as a monopolist selling a perishable good could. Moreover, when the monopolist becomes extremely flexible in adjusting prices, market power vanishes and the competitive outcome is immediately achieved.

The existing theories on durable goods monopolies find the commitment problem that Coase conjectures extremely robust and show that in Markov perfect equilibria, a durable goods monopolist cannot exercise his market power.[1] Casual empirical evidence, however, suggests that durable goods monopolists charge prices much higher than the marginal cost of production and make considerable profits (e.g. Microsoft, Apple, etc.).

This article studies an unexplored motive, the role of quality differentiation, for solving the commitment problem a durable goods monopolist faces. I consider a market for an imperfectly durable good served by a monopolist in an infinite horizon, discrete time game. In each period, the monopolist can sell two versions, high and low quality, of the durable good that depreciate stochastically. I analyze whether the simultaneous introduction of vertically differentiated goods enables the monopolist to maintain his market power and to credibly commit to deferring sales to low valuation consumers.

In an interesting study, Deneckere and McAfee (1996) analyze damaged goods in a static market and show that manufacturers may intentionally damage a portion of their goods in order to price discriminate. For example, Intel first produced a fully functioning 486DX processor, then by disabling the math coprocessor produced the 486SX that is inferior to the 486DX. Even though the 486SX was more expensive to produce, in 1991 the price of the 486SX was almost twice as low as the price of

[1] See Bulow (1982), Stokey (1981), Gul, Sonnenschein and Wilson (1986), and Sobel (1991).

the 486DX. Intel, however, might use the 486SX not only to price discriminate but also to maintain his market power by credibly deferring sales of the 486DX to low valuation consumers by selling the 486SX to low valuation consumers instead. The issue that the current article studies is whether a monopolist will damage a portion of the goods and produce a lower quality version of it in an attempt to mitigate the commitment problem and enhance his market power.

Earlier studies on durable goods monopolies suggest that there are many responses a durable goods monopolist can adopt to restore the profitability, such as reducing the durability of the good (Coase, 1972), leasing (Bulow, 1986), contractual provisions (Butz, 1990), or using an inferior high cost technology (Karp and Perloff, 1996). Such responses, however, are not extensively observed in several durable goods markets. This implies that the commitment problem that Coase conjectures does not decrease profits of durable goods monopolists as much as theory claims. Deneckere and Liang (2008) provide a comprehensive understanding of Coase's insight by establishing that a durable goods monopolist can commit to high future prices when the good is sufficiently perishable.

Recent studies extend the seminal analyses on durable goods monopolies by considering new product introductions. By offering new products for sale the monopolist seller can increase the economic depreciation of the initial version of the good and regain his profitability.[2] Even though we have a clearer understanding of why the Coase Conjecture may not hold, our exact understanding of Coase's insight remains incomplete, since a glance at durable goods markets suggests that many durable goods are characterized by menus of multiple quality levels and prices: Dell's Inspiron laptop vs its XPS laptop, Mathematica's student version vs its full version, hardcover textbooks vs paperback textbooks, etc.

From a theoretical point of view, this study follows Deneckere and Liang (2008). I extend their single good setting into a setting of a vertically differentiated market to analyze the effect of quality differentiation on the commitment problem a durable

[2]See Levinthal and Purohit (1989), Waldman (1993, 1996), Choi (1994), Fudenberg and Tirole (1998), Lee and Lee (1998), Fishman and Rob (2000), Kumar (2002, 2006), and Anton and Biglaiser (2009).

goods monopolist faces. Similar to Deneckere and Liang (2008), I establish that there exist three types of Markov perfect equilibria: a Coase Conjecture equilibrium, a monopoly equilibrium, and a reputational equilibrium. *For sufficiently low depreciation rates*, the unique equilibrium is the Coase Conjecture equilibrium. The Coase Conjecture equilibrium has a unique steady state equal to the competitive quantity. At the steady state, the monopolist serves the high quality good to all consumers. *For sufficiently high depreciation rates,* the unique equilibrium is the monopoly equilibrium. The intuition is that when the durable good is sufficiently perishable, replacement sales at high prices compensate the desire to penetrate the market further by lowering the price of one or both versions of the durable good. This equilibrium has two monopoly steady states one of which is equal to the static monopoly quantity. At this steady state, the monopolist only serves the high quality good to the high type consumers. The market at the other monopoly steady state is segmented into two: the monopolist serves the high quality good to the high type consumers and serves the low quality good to the low type consumers. Upon deviation from a monopoly steady state, if the good is sufficiently perishable, the monopolist returns to one of these two steady states from any state. Otherwise, the Coase Conjecture steady state coexists with the monopoly steady states. *For intermediate values of the depreciation rate*, all three types of equilibria exist. In the reputational equilibrium, the monopolist cuts the production of the high quality good to create a reputation of pricing high. The steady state quantity of the high quality good falls short of the monopoly quantity of the high quality good.

I prove that the set of parameters supporting the Coase Conjecture equilibrium is smaller and the set of the parameters supporting the monopoly equilibrium is larger when the monopolist can produce a lower quality good as well. Hence, this study establishes that quality differentiation may enhance market power of a durable goods monopolist and alleviate the commitment problem, and suggests that when the innate durability of a good is high, the monopolist will damage a portion of the goods and produce a lower quality good to credibly commit to monopoly prices.

The rest of the paper proceeds as follows. Section 2 introduces the model. Section 3 characterizes the dynamic optimization problem of the agents. Section 4 discusses the characterization of steady states. Section 5 describes Markov perfect equilibria and the construction of stationary paths for each type of equilibrium. Section 6 studies the coexistence of the equilibria and compares the results with the single good version of this game with an example. Section 7 discusses the related literature. Section 8 concludes. All proofs are relegated to the Appendix.[3]

2 The Model

Consider a market for an indivisible and imperfectly durable good. The market is served by a monopolist who can produce two versions of the durable good that differ in quality: high quality q_H and low quality q_L, where $q_H > q_L$. The durable good with quality level q_i may be referred as durable good i. The monopolist offers both versions of the durable good for sale simultaneously at discrete points in time. The monopolist is risk neutral and has discount rate r. The objective of the monopolist is to maximize the present value of his expected profits.

There exists a continuum of infinitely-lived buyers indexed by $b \in [0, 1]$. Buyers are segmented into two groups: high valuation buyers and low valuation buyers. Buyer b's reservation price for acquiring one unit of durable good i is represented by

$$f^i(b) = \begin{cases} \overline{\theta} q_i \text{ for } b \in [0, \widehat{b}] \\ \underline{\theta} q_i \text{ for } b \in (\widehat{b}, 1] \end{cases}.$$

All buyers are risk neutral and have the same discount rate r. Buyer b derives a net surplus of $e^{-rt}(f^i(b) - p^i)$ if she purchases durable good i at price p^i at time t. Each buyer wishes to possess at most one unit of the durable good.[4] A buyer is allowed to access the markets as often as she wishes and seeks to maximize the present value of her expected payoff.

The length of the time period between successive price changes is $z > 0$. Both ver-

[3] Appendix is available on http://www.unc.edu/~altan/papers.html.
[4] We may assume that the storage costs of the second unit is infinite.

sions of the durable good depreciate stochastically at the same rate. The probability that a good is still working after a length of time t is $e^{-\lambda t}$. Hence, with probability $\mu = 1 - e^{-\lambda z}$, the good fails between successive price changes. The marginal cost of production is assumed to be independent of the quality of the good and lower than $\underline{\theta}(q_H - q_L)$. Hence, without loss of generality, it is set to zero.

There exists a perfectly competitive second-hand market in which buyers can trade with each other. Sales occur only at $t = 0, z, 2z, \ldots, nz, \ldots$ in all markets. The time nz is referred as period n. In each period, the game runs as follows. First, the monopolist sets the price of each version of the good before trade occurs. Then, buyers choose whether or not to hold a good and which version to hold. After a time interval of z passes, the game repeats itself.

Markov perfect equilibria of this game in which agent strategies only depend on the current state are sought to derive. A strategy of the monopolist specifies the price he charges for each version of the good, and a strategy of a buyer specifies whether or not to hold a durable good and which version to hold. Formally, let $G(z, r, \lambda, q_H, q_L)$ denote the game. Let σ be a pure strategy for the monopolist where σ is a sequence of functions $\{\sigma^n\}_{n=0}^{\infty}$. The function σ^n determines the price charged for each version of the good by the monopolist in period n as a function of the prices charged in the previous period and the actions chosen by the buyers in the previous period. Let the set of buyers' acceptances of good i in period n be denoted by Q_i^n. Since there exists an active perfectly competitive second-hand market, strategies of a buyer are independent of her holding status and depend only on the current prices.[5] Let a buyer's strategy in such equilibria be described by acceptance functions, $V^H(\cdot)$ and $V^L(\cdot)$, where buyer b chooses to hold good i in the current period if and only if the current price of good i satisfies $p^i \leq V^i(b)$.[6] Hence, the set of buyers holding good i after trade is an interval of the form $[0, b^H]$ for the high quality good and $(b^H, b^L]$

[5] If there were no perfectly competitive second-hand market, the strategies of the monopolist would depend on the distribution of the current holdings of each version of the good rather than the size of the current stock of goods, and the strategies of a buyer would depend on not only the current prices, but the current holding status as well.

[6] If there exists b such that $p^H \leq V^H(b)$ and $p^L \leq V^L(b)$ then b accepts the offer that gives her the highest payoff.

for the low quality good. It follows that σ^n can be represented as a function of the current stock of goods in the market. Let Ω be the Borel sigma algebra on $[0,1]$ and $Y \equiv [0, \overline{\theta}q_H] \times [0, \overline{\theta}q_L]$. Then $\sigma^n : \Omega \times \Omega \to Y$. A strategy combination of a buyer, τ, is a sequence of functions $\{\tau^n\}_{n=0}^{\infty}$ where $\tau^n : Y \times Y \to \{0,1,2\}$. Decision 0 indicates that the buyer chooses not to hold any good in the current period. Decision 1 indicates that the buyer chooses to hold the low quality good in the current period. Decision 2 indicates that the buyer chooses to hold the high quality good in the current period. Since weak Markov strategies are considered, $\sigma^i(x) = \sigma^j(x)$ and $\tau^i(y) = \tau^j(y)$ hold for all $i, j \in \{0, 1, \ldots\}$. The pure strategy profile $\{\sigma, \tau\}$ generates a stationary path of prices and sales that can be derived recursively. The monopolist is also allowed to mix. Later, however, it is established that the monopolist never randomizes in any period of the game unless it is the initial period. As in Gul, Sonnenschein, and Wilson (1986), the attention is restricted to equilibria in which deviations by sets of measure zero buyers change neither the actions of the monopolist nor the actions of the other buyers. Hence, in such equilibria buyers behave as price takers. From now on, I refer to *Markov perfect equilibrium* simply as *equilibrium*.

3 Characterization of Dynamic Optimization

In this section, first, the dynamic optimization problems of the buyers and the monopolist are represented. Then, the characteristics of equilibrium strategies of the monopolist are discussed.

The acceptance function $V^i(b)$ must be consistent with buyer b's intertemporal optimization which requires that buyer b is indifferent between purchasing the good today at the price $V^i(b)$ and waiting one period to purchase it. Hence, $V^i(b)$ is derived from

$$f^i(b) - V^i(b) = \rho(f^i(b) - \acute{p}^i) \tag{1}$$

where $\rho = \delta(1-\mu)$ and $\acute{p}^i$ is the expected price of good i in the next period. Since $f^i(\cdot)$ is monotone and deviations of measure zero buyers do not affect the equilibrium, $V^i(\cdot)$ is a non-increasing left-continuous function. The acceptance function $V^i(b)$

acts as the static demand function that the monopolist faces for good i. Since both price and quantity are not contractible if the monopolist sets the quantity, the market price must be determined by demand.

Let x^i denote the stock of the durable good i before trade. The value function of the monopolist is $R(x^H, x^L)$ and must satisfy

$$R(x) = \max_{y^i \in [x^i,1]} \left\{ P^H(y)\left(y^H - x^H\right) + P^L(y)(y^L - x^L) + \delta R((1-\mu)y) \right\} \quad (2)$$

where $\delta = e^{-rz}$ is the discount factor and $P^i(\cdot)$ is the price of durable good i. The price of the high quality good must be consistent with the incentive compatibility constraint of the marginal buyer of the high quality good, and the price of the low quality good must be consistent with the participation constraint of the marginal buyer of the low quality good. Hence, if the marginal buyer holding the high quality good is buyer y^H, we must have

$$P^H(y) = V^H\left(y^H\right) - V^L\left(y^H\right) + P^L(y) \quad (3)$$

and if the marginal buyer holding the low quality good is buyer $y^H + y^L$, we must have

$$P^L(y) = V^L(y^H + y^L) \quad (4)$$

where $V^L(y^H) - P^L(y)$ is the payoff of buyer y^H if she purchases the low quality good at the price $P^L(y)$.

Let $T(\cdot)$ denote the argmax correspondence of the objective function. By the generalized theorem of the maximum and the contraction mapping theorem, there exists a unique continuous function $R(\cdot)$, and $T(\cdot)$ is a non-empty and compact valued correspondence.[7] Moreover, the supermodularity of the objective function implies that $T(\cdot)$ is non-decreasing. It follows that there exist at most countable number of points for which $T(\cdot)$ is multi-valued. Even though the monopolist is allowed to use behavioral strategies, it is established that

[7] See Ausubel and Deneckere (1989) for the generalized theorem of the maximum.

Proposition 1 *The monopolist does not randomize along any equilibrium path and chooses the minimum of the argmax correspondence with probability one unless it is the initial period.*

Hence, given a state (x^H, x^L), the equilibrium output choice of the monopolist is $t(x) = \min T(x)$, and since $T(x)$ is a monotone correspondence, the equilibrium output function $t^i(\cdot) : [0, 1-\mu] \times [0, 1-\mu] \to \mathbb{R}_+$ for all $i = H, L$ is nondecreasing.

An equilibrium is represented by $\{P^H(\cdot), P^L(\cdot), t^H(\cdot), t^L(\cdot), R(\cdot)\}$. The structure of a stationary path is as follows. In the initial period, the monopolist selects prices: $P^H(y_0)$ and $P^L(y_0)$. All buyers $b \leq y_0^H$ purchase the high quality good and all buyers $y_0^H < b \leq y_0^H + y_0^L$ purchase the low quality good.[8] At the beginning of the next period, the stock of the high quality good is $x_1^H = (1-\mu)y_0^H$ and the stock of the low quality good is $x_1^L = (1-\mu)y_0^L$. The monopolist selects prices, $P^H(y_1) = P^H(t(x_1))$ and $P^L(y_1) = P^L(t(x_1))$, and all buyers $b \leq y_1^H$ choose to hold the high quality good and all buyers $y_1^H < b \leq y_1^H + y_1^L$ choose to hold the low quality good. This continues until a steady state is reached. Once a steady state is reached, the monopolist continues by selling to replacement demands.

In order to construct equilibria of this game, the solution method introduced by Deneckere and Liang (2008) is followed. First, the existence of a steady state in any equilibrium is proved. Then, all possible steady states are characterized. Finally, stationary paths that reach a steady state by using backward induction from the steady state are derived.

The analysis of this paper focuses on the nontrivial case where $\widehat{b}\,\overline{\theta} > \underline{\theta}$. Otherwise, the static monopoly prices would be $\underline{\theta} q_H$ and $\underline{\theta} q_L$.[9] Since a durable goods monopolist who does not have any commitment power can achieve this outcome, the unique stationary steady state of this game when $\widehat{b\overline{\theta}} < \underline{\theta}$ is the static monopoly outcome.[10]

[8] If $T(0,0)$ is multi-valued, the monopolist may select the price randomly from $P(T(0,0))$.

[9] Indeed, any price for the low quality good would be an equilibrium price as long as all buyers purchase the high quality good.

[10] For the hairline case $\widehat{b\overline{\theta}} = \underline{\theta}$, one can use the limit of $\widehat{b\overline{\theta}} > \underline{\theta}$.

4 Characterization of Steady States

In this section, after proving the existence of a steady state in any equilibrium, all possible steady states that may coexist are characterized.

Let a steady state (y_s^H, y_s^L) be defined as the stock levels of durable goods satisfying $t^H((1-\mu)y_s) = y_s^H$ and $t^L((1-\mu)y_s) = y_s^L$. The following proposition establishes the existence of a steady state by showing that in any equilibrium there exists at least one corresponding steady state.

Proposition 2 *Any equilibrium has at least one steady state, and the steady state prices satisfy* $P^H\left(y_s^H, y_s^L\right) = f^H\left(y_s^H\right) - f^L\left(y_s^H\right) + f^L\left(y_s^H + y_s^L\right)$ *and* $P^L\left(y_s^H, y_s^L\right) = f^L\left(y_s^H + y_s^L\right)$.

The economic intuition behind the steady state prices is as follows. At a steady state (y_s^H, y_s^L), the marginal buyer of the high quality good is y_s^H and the marginal buyer of the low quality good is $y_s^H + y_s^L$ in each period. It follows that buyer $y_s^H + y_s^L$ is indifferent between today's and tomorrow's offer for the low quality good when the price of the low quality good is $f^L(y_s^H + y_s^L)$. Similarly, buyer y_s^H is indifferent between today's and tomorrow's offer for the high quality good when the price of the high quality good is $f^H(y_s^H)$. However, when the price of the low quality good is $f^L\left(y_s^H + y_s^L\right)$, buyer y_s^H's net surplus from the low quality good is $f^L(y_s^H) - f^L(y_s^H + y_s^L)$. It follows that in order to sell the high quality good to buyer y^H, the monopolist has to leave an information rent no less than $f^L(y_s^H) - f^L(y_s^H + y_s^L)$. Therefore, at the steady state, the price of the high quality good is $f^H(y_s^H) - f^L(y_s^H) + f^L(y_s^H + y_s^L)$ and the price of the low quality good is $f^L(y_s^H + y_s^L)$.

Let us consider a market for *a perfectly durable good* served by a monopolist. The monopolist cannot credibly commit to a static monopoly output since he has an irresistible temptation to cut the price to sell the good to the remaining buyers. Thus, in this setting, the static monopoly output would never be a steady state. Deneckere and Liang (2008) point out that when the good depreciates the monopolist may prefer serving to the replacement demand of the high type buyers at a higher price rather than cutting the price in an attempt to increase sales. They show that there exist three types of steady states: a Coase Conjecture, a monopoly, and a reputational steady state.

When a monopolist produces two versions of a durable good, there exist five possible steady states: $(1, 0)$, $(\widehat{b}, 0)$, $(\widehat{b}, 1-\widehat{b})$, $(\hat{y}^H, 0)$, and $(\check{y}^H, 1-\check{y}^H)$ where $\hat{y}^H, \check{y}^H \in (0, \widehat{b})$. When the steady state is $(1, 0)$, all buyers hold the high quality good after trade and the monopolist serves their replacement demand in each period. The state $(1, 0)$ is called the *Coase Conjecture steady state* and the equilibrium having $(1, 0)$ as the unique steady state is called the Coase Conjecture equilibrium. When the steady state is $(\widehat{b}, 0)$, all high type buyers hold the high quality good after trade and the monopolist sells to the replacement demand of the high type buyers for the high quality good in each period. When the steady state is $(\widehat{b}, 1-\widehat{b})$, all high type buyers hold the high quality good and all low type buyers hold the low quality good after trade, and the monopolist serves the replacement demands in each period. In the one-period version of this game, due to the assumption that $\widehat{b\overline{\theta}} > \underline{\theta}$, the monopolist sells the high quality good to the high type buyers at the price $\overline{\theta} q_H$ and sets the price of the low quality good high enough so that none of the buyers purchase it. Hence, the state $(\widehat{b}, 0)$ is called *the static monopoly steady state* and $(\widehat{b}, 1-\widehat{b})$ is called *the segmented monopoly steady state*. The static monopoly steady state $(\widehat{b}, 0)$ always coexists with the segmented monopoly steady state $(\widehat{b}, 1-\widehat{b})$. Depending on the magnitude of the depreciation rate, μ, the Coase Conjecture steady state $(1, 0)$ may coexist with the monopoly steady states. The equilibrium with a monopoly steady state is called the monopoly equilibrium. Finally, the states $(\hat{y}^H, 0)$ and $(\check{y}^H, 1-\check{y}^H)$ are called *reputational steady states* and the equilibrium corresponds with them is called the reputational equilibrium. At a reputational steady state, the monopolist limits the production of the high quality good and sells it to some of the high type buyers. The Coase Conjecture steady state $(1, 0)$ always coexist with the reputational steady states. The results are summarized by

Proposition 3 *Let S denote the set of steady states. In any equilibrium one of the followings holds:*

1. $S = \{(1,0)\}$;

2. $S = \{(\widehat{b}, 1-\widehat{b}), (\widehat{b}, 0)\}$ *or* $S = \{(\widehat{b}, 1-\widehat{b}), (\widehat{b}, 0), (1,0)\}$ *or* $S = \{(\widehat{b}, 1-\widehat{b}), (1,0)\}$;

3. $S = \{(\hat{y}^H, 0), (\hat{y}^H, 1-\hat{y}^H), (1,0)\}$ *or* $S = \{(\check{y}^H, 1-\check{y}^H), (1,0)\}$ *where* $\check{y}^H, \hat{y}^H \in (0, \widehat{b})$.

The intuition behind Proposition 2 comes from the following two observation. First, given the expectations of the buyers, some states cannot be a steady state as the monopolist can profitably deviate from these states. Second, since the number of the steps in f^i is two, if the marginal buyer of the high quality good is a high type then the marginal buyer of the low quality good can be either a high type or a low type, whereas if the marginal buyer of the high quality good is a low type, then the marginal buyer of the low quality good must be a low type as well. Hence, at most three steady states can coexist in an equilibrium.

5 Characterization of Equilibria

In this section, by constructing the stationary paths that lead us to a steady state from any state all possible equilibria are derived. The uniqueness of the equilibrium of each type is showed, and the effects of quality differentiation on each type of equilibrium is analyzed.

5.1 The Coase Conjecture Equilibrium

Consider equilibria with a unique steady state at which all buyers, after trade, hold the high quality good. By conducting backward induction from the Coase Conjecture steady state $(1,0)$, the stationary paths for all (x^H, x^L) are constructed.

First, stationary paths when stock of the high quality good is greater than $(1-\mu)\widehat{b}$ are described. Since the stock of the high quality good has to increase in each period in such equilibria, the marginal buyer of both versions of the good is a low type thereafter. The low type buyers, anticipating that the monopolist will saturate the entire market with the high quality good, do not accept any price greater than $\underline{\theta} q_i$ for durable good i.[11] Hence, for $x^H > (1-\mu)\widehat{b}$ the stationary paths are defined as follows. When none of the buyers hold the low quality good, the monopolist sets the price of the high quality good as $\underline{\theta} q_H$ and the price of the low quality good as $\underline{\theta} q_L$. All buyers hold the high quality good after trade and the monopolist continues by selling the high quality good to the replacement demand μ at the price $\underline{\theta} q_H$ thereafter. When some buyers hold the low quality good, if the stock of the low quality good is low enough, the monopolist prefers selling the high quality good to buyers who do not currently hold any good. Hence, the monopolist sets the price of the high quality good as $\underline{\theta} q_H$ and the price of the low quality good as $\underline{\theta} q_L$ and continues by selling the high quality good to the replacement demand μ at the price $\underline{\theta} q_H$. However, if the stock of the low quality good falls above this threshold, the monopolist prefers selling the high quality good to all buyers. Due to an excess supply of the low quality good in the second-hand market, the low quality good will be available for free. Hence, the monopolist sets the price of the high quality good as $\underline{\theta}(q_H - q_L)$.[12] All buyers hold the high quality good after trade, and the monopolist continues by selling the high quality good to the replacement demand μ at the price $\underline{\theta}\Delta q$.

Second, the stationary paths when stock of the high quality good is less than $(1-\mu)\widehat{b}$ are described. There exist four paths depending on the state of the low quality good. On path 1, none of the buyers hold the low quality good. On path 2, the marginal buyer of the low quality good is a high type. On path 3, the marginal buyer of the low quality good is a low type. On path 4, there exists an excess supply of the low quality good. The sequence of states $\{(\overline{x}^H_{k,j}, \overline{x}^L_{k,j})\}_{k=2}^{m_j+1}$ for each path $j = 1, \ldots, 4$ is constructed such that when the state is $(\overline{x}^H_{k,j}, \overline{x}^L_{k,j})$, the monopolist is indifferent between bringing the next period's state to $(\overline{x}^H_{k-1,j}, \overline{x}^L_{k-1,j})$ by charging $(\overline{p}^H_{k-1,j}, \overline{p}^L_{k-1,j})$ and bringing the next period's state to $(\overline{x}^H_{k-2,j}, \overline{x}^L_{k-2,j})$ by charging $(\overline{p}^H_{k-2,j}, \overline{p}^L_{k-2,j})$. The sequence $\{(\overline{p}^H_{k,j}, \overline{p}^L_{k,j})\}_{k=0}^{m_j+1}$ is set such that the incentive compatibility constraint

[11] If buyer $b \in (\widehat{b}, 1]$ is willing to pay more than $\underline{\theta} q_i$ for durable good i, so is buyer $b \in (\widehat{b}, \widehat{b} + \varepsilon)$, since $V^i(.)$ is non-increasing. This implies that b is expecting to make a capital gain by purchasing it. Namely, buyer b expects that the price of good i will increase next period. However, since neither $(\widehat{b}, 1 - \widehat{b})$ nor $(\widehat{b}, 0)$ is a steady state, this is not possible.

[12] For ease of exposition from now on $q_H - q_L$ is referred as Δq.

of the marginal buyer of the high quality (3) and the participation constraint of the marginal buyer of the low quality good (4) hold.

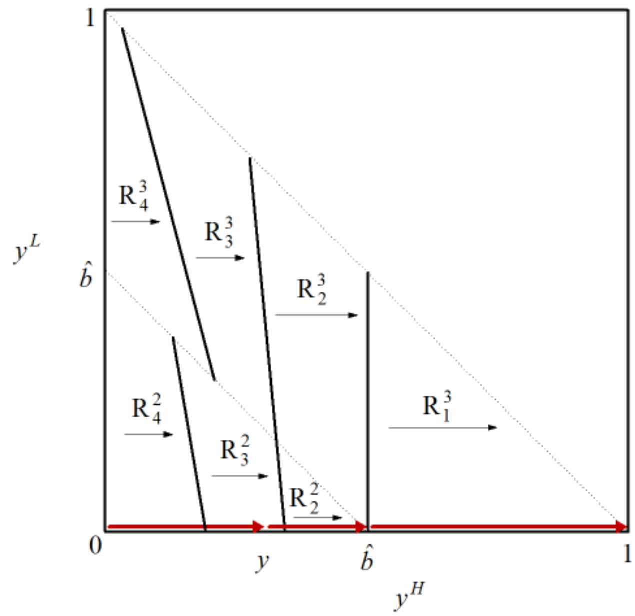

Figure 1.1: The Coase Conjecture Equilibrium

Figure 1.1 illustrates how states move towards the Coase Conjecture steady state. The arrows indicate the direction of movement of the state at any state (y^H, y^L). When there exists no low quality good in the market, the states move towards the Coase Conjecture steady state along path 1 as follows. For $y^H \leq y$, the stock of the high quality good after trade in the next period will be $\widehat{b}$. For $y^H > y$, the monopolist will penetrate the entire market with the high quality good in the next period and reach the Coase Conjecture steady state, and continue by serving the high quality good to the replacement demand of all buyers thereafter. When there exists the low quality good in the market and the marginal buyer holding the low quality good is a high type, the states move towards the Coase Conjecture steady state along path 2 as follows. If (y^H, y^L) is in region R_4^2, the state after trade in the next period will be on the line that separates regions R_3^2 and R_2^2. If (y^H, y^L) is in region R_3^2, the state after trade in the next period will be $(\widehat{b}, 1 - \widehat{b})$. If (y^H, y^L) is in region R_2^2, the monopolist will charge the Coase Conjecture steady state prices in the next period and continue by serving the high quality good to the replacement demand of all buyers thereafter. When the marginal buyer holding the low quality good is a low type, the states move towards the Coase Conjecture steady state along path 3 as follows. If (y^H, y^L) is in region R_i^3 where $i = 4, 3$, the state after trade in the next period will be on the line that separates regions R_{i-1}^3 and R_{i-2}^3. If (y^H, y^L) is in region R_2^3 or in region R_1, the monopolist will charge the Coase Conjecture steady state prices in the next period and continue by serving the high quality good to the replacement demand of all buyers thereafter. Figure 1.1 shows that as the stock of the low quality good increases, the real time that passes to reach the Coase Conjecture steady state increases as well.

Figure 1.2 illustrates the movement of the high quality good towards the Coase Conjecture steady state and the corresponding prices. For expositional purposes for $x^H \geq (1-\mu)\widehat{b}$ and $x^L > 0$, the monopolist is assumed to immediately penetrate the market with the high quality good by charging $\underline{\theta}\Delta q$ for the high quality good. Suppose that the marginal buyer holding the low quality good is a high type. This implies that the monopolist will be on path 2. If $x^H \in (\overline{x}_{k+2,2}^H, \overline{x}_{k+1,2}^H]$ for $k \geq 1$, the monopolist sets the price of good i as $\overline{p}_{k,2}^i$ so that after trade the stock of good i will be $\overline{y}_{k,2}^i$. If $x^H \in (\overline{x}_{1,2}^H, 1-\mu]$, the monopolist sets the price of the high quality good as $\underline{\theta}\Delta q$ to penetrate the entire market with the high quality good. All buyers will hold the high quality good and the low quality good will be available for free. Figure 1.1 illustrates how the price and the stock of the high quality good evolves along path 2. For ease of exposition $\overline{y}_{k,2}^H$ is referred as y_k and refer $\overline{p}_{k,2}^H$ as p_k. The arrows indicate the direction of the movement of a state.

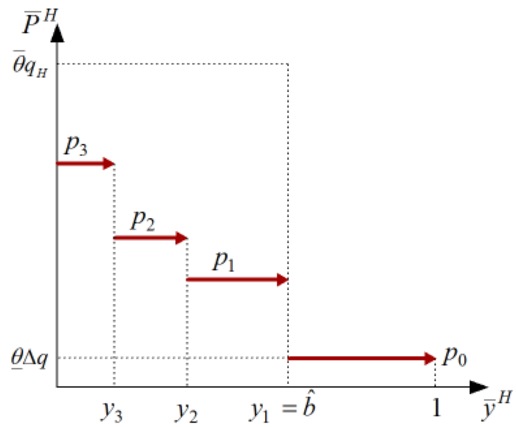

Figure 1.2: Path 2

It is established that existence of a Coase Conjecture equilibrium requires that $\{\overline{x}^H_{k,j}\}_{k=0}^{m_j+1}$ must be strictly decreasing and must satisfy $\overline{x}^H_{m_j+1,j} \leq 0 < \overline{x}^H_{m_j,j}$. The set of parameters supporting this condition is derived and it is proved that the equilibrium is unique.

Theorem 1 *There exists at most one Coase Conjecture equilibrium if and only if $\mu < \overline{\mu}$ for some $\overline{\mu} \in (0,1)$.*

When depreciation rate μ was large, rather than fully penetrating the market with high quality good at the price $\overline{p}^H_{0,j}$, the monopolist would sell to the replacement demand $\mu\overline{y}^H_{1,j}$ for the high quality good at the price $\overline{p}^H_{1,j}$ and to the replacement demand $\mu\overline{y}^L_{1,j}$ for the low quality good at the price $\overline{p}^L_{1,j}$. Hence, the Coase Conjecture equilibrium does not exist for sufficiently perishable goods.

The differences among paths with respect to the high quality good is illustrated in order to analyze how the production of a low quality good affects the stock and the price of the high quality good and analyze whether the production of the low quality good helps the monopolist maintain his market power. For ease of exposition $\overline{y}^H_{k,j}$ is referred as $y_{k,j}$ and $\overline{p}^H_{k,j}$ is referred as $p_{k,j}$.

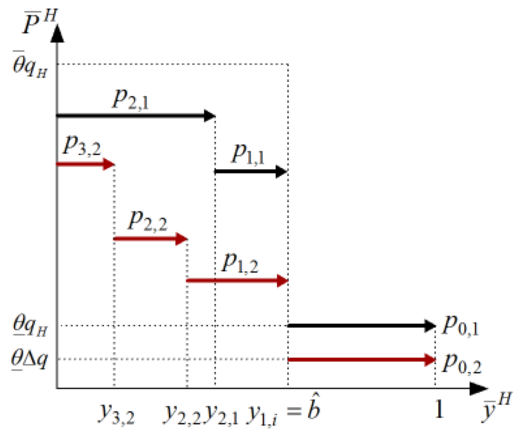

Figure 1.3: Path 1 vs Path 2

First, the marginal effect of the low quality good when the initial stock of the low quality good is zero is studied. We must compare path 1 on which none of the buyers hold the low quality good with path 2 on which the marginal buyer of the low quality good is a high type. Figure 1.3 illustrates path 1 and path 2. We observe that when some high types hold the low quality good, the price of the high quality good diminishes. For all $x^H < (1-\mu)\widehat{b}$, on path 2 the marginal buyer of the low quality good is a high type. Hence, information rent is not an issue and cannot explain the price difference between these two paths. However, while moving towards the steady state along path 2, once all high type buyers hold the high quality good, the marginal buyer of the low quality good becomes a low type. Then, in order to make high type buyers hold the high quality good rather than the low quality good, the monopolist has to leave some information rent to the high type buyers. Hence, the high type buyers, anticipating that the monopolist will eventually lower the price of the high quality good, are willing accept a price for the high quality good that is significantly lower than the price when none of the buyers hold the low quality good. In addition to the price difference, we also observe that when some high types hold the low quality good, the time elapses to reach the Coase Conjecture steady state is longer.

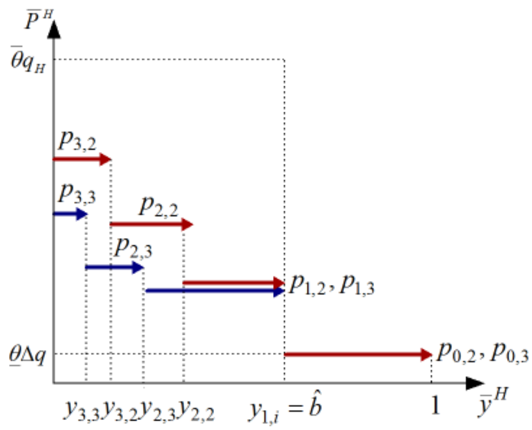

Figure 1.4: Path 2 vs Path 3

Second, the marginal effect of the low quality good when some high types hold the low quality good is studied. We must compare path 2 on which the marginal buyer of the low quality good is a high type with path 3 on which the marginal buyer of the low quality good is a low type. Figure 1.4 illustrates path 2 and path 3. We observe that the price of the high quality good is lower on path 3. In addition to the rent that the monopolist has to leave to the high type buyers since they anticipate that the monopolist will eventually lower the price of the high quality good, on path 3 the monopolist has to leave some information rent to the high type buyers to eliminate their incentive to buy the low quality good rather than the high quality good. In addition to the price difference, we also observe from Figure 1.4 that the time elapses to reach the Coase Conjecture steady state is longer on path 3.

Last, the marginal effect of the low quality good when there is an excess supply of the low quality good is studied. The low quality good is available for free when there exists excess supply of the low quality good. Since, the monopolist does not make any profits from the low quality good, on path 4 the stock of the high quality good does not depend on the stock of the low quality good.

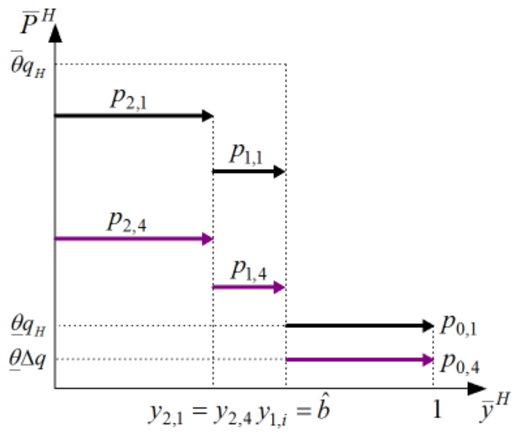

Figure 1.5: Path 1 vs Path 4

Figure 1.5 illustrates path 1 and path 4. We can see that the stock of the high quality good is the same on both paths. Moreover, since the low quality good is available for free on path 4, the information rent that the monopolist has to leave to a high type buyer is greater than the information rent on path 3. Hence, on path 4 the price of the high quality good is lower.

When none of the buyers hold the low quality good, the monopolist fully penetrates the market by charging $\underline{\theta}q_H$ for the high quality good. However, when some of the buyers hold the low quality good, the monopolist either fully penetrates the market with the high quality good at the price $\underline{\theta}\Delta q$ or sells the high quality good to some of the low type buyers at the price $\underline{\theta}q_H$ and reaches the steady state gradually. Moreover, we observe from Figure 1.3 and Figure 1.4 that as the stock of the low quality good increases, so does the real time that passes before the monopolist reaches the Coase Conjecture steady state. It follows that the depreciation rate supporting the Coase Conjecture equilibrium must be smaller when x^L is positive. Hence, when the durable goods monopolist produces a low quality good, the threshold depreciation rate $\overline{\mu}$ diminishes. Therefore,

Corollary 1 *The domain of the parameters consistent with the Coase Conjecture equilibrium is smaller when the monopolist can produce multiple goods that differ in quality.*

The monopolist fully penetrates the entire market with the high quality good in m_j periods on path j. Hence, the real time that elapses until the market is fully penetrated with the high quality good is $m_j z$. It is showed that m_j has a finite limit $\widehat{m}_j$ regardless of the state of the low quality good. Therefore,

Corollary 2 *In the Coase Conjecture equilibrium, the initial price of the high quality durable good converges to the lowest buyer valuation $\underline{\theta} q_H$ as the length of the time period between successive offers approaches zero.*

It is also showed that $\widehat{m}_1 = \widehat{m}_4 = \widehat{m}$ and $\widehat{m} < \widehat{m}_j$ for $j = 2, 3$ where $\widehat{m}$ is the corresponding limit of a durable goods monopolist selling a single version of the good and show that $\widehat{m}_j$ for $j = 2, 3$ increases with the state of the low quality good and decreases with the quality difference between the versions of the good. Therefore,

Corollary 3 *Suppose the time period between successive price changes is arbitrarily small. As the quality difference between two versions of the durable good increases, the real time that elapses until the market is fully penetrated with the high quality good converges to $\widehat{m}$.*

5.2 The Monopoly Equilibrium

Consider equilibria in which the monopolist credibly commits not to selling the high quality good to low type buyers. The monopoly steady states of such equilibria are $(\widehat{b}, 0)$ and $(\widehat{b}, 1 - \widehat{b})$.

The necessary conditions for the existence of a monopoly equilibrium are as follows. First, when the state before trade is $((1 - \mu)\widehat{b}, 0)$, the monopolist must prefer selling the high quality good to the high type buyers' replacement demand $\mu\widehat{b}$ at the price $\overline{\theta} q_H$ forever to selling the high quality good to buyers who do not hold the high quality good $(1-(1 - \mu)\widehat{b})$ at the price $P^H(1, 0)$ and continuing by selling the high quality good to all buyers' replacement demand μ at the price $P^H(1, 0)$ thereafter. If

$$\frac{\mu\widehat{b}\overline{\theta}q_H}{1-\delta} \geq (1-(1 - \mu)\widehat{b})P^H(1, 0) + \frac{\delta\mu P^H(1,0)}{1-\delta} \tag{5}$$

holds, the monopolist never cuts the price of the high quality good to $P^H(1,0)$ to serve the high quality good to all buyers. Since $P^H(1,0) \leq \underline{\theta} q_H$, (5) holds for $\mu \geq \frac{(1-\delta)(1-\widehat{b})\underline{\theta}}{\widehat{b}\Delta\theta - \delta(1-\widehat{b})\underline{\theta}} \equiv \underline{\mu}^{st}$.

Second, when the state before trade is $((1-\mu)\widehat{b}, 0)$, the monopolist must prefer selling the high quality good to the high type buyers' replacement demand $\mu\widehat{b}$ at the price $\overline{\theta} q_H$ forever to selling the high quality good to the high type buyers' replacement demand $\mu\widehat{b}$ at the price $P^H(\widehat{b}, 1-\widehat{b})$ forever and selling the low quality good to the low type buyers $1-\widehat{b}$ at the price $P^L(\widehat{b}, 1-\widehat{b})$ and continuing by selling the low quality good to their replacement demand $\mu(1-\widehat{b})$ at the price $P^L(\widehat{b}, 1-\widehat{b})$ thereafter. If

$$\frac{\mu\widehat{b}\overline{\theta} q_H}{1-\delta} \geq \frac{\mu\widehat{b}P^H(\widehat{b}, 1-\widehat{b})}{1-\delta} + (1-\widehat{b})P^L(\widehat{b}, 1-\widehat{b}) + \frac{\delta\mu(1-\widehat{b})P^L(\widehat{b}, 1-\widehat{b})}{1-\delta} \tag{6}$$

holds, the monopolist never cuts the price of the low quality good to $P^L(\widehat{b}, 1-\widehat{b})$ to serve the low quality good to the low type buyers. Since $P^H(\widehat{b}, 1-\widehat{b}) \leq \overline{\theta} q_H - \overline{\theta} q_L + P^L(\widehat{b}, 1-\widehat{b})$ and $P^L(\widehat{b}, 1-\widehat{b}) \leq \underline{\theta} q_L$, (6) holds if $\mu \geq \underline{\mu}^{st}$. Therefore, the monopolist does not deviate from the static monopoly steady state $(\widehat{b}, 0)$ for $\mu \geq \underline{\mu}^{st}$.

Third, when the state before trade is $((1-\mu)\widehat{b}, (1-\mu)(1-\widehat{b}))$, the monopolist must prefer selling the high quality good to the high type buyers' replacement demand $\mu\widehat{b}$ at the price $\overline{\theta} q_H - \overline{\theta} q_L + \underline{\theta} q_L$ and selling the low quality good to the low type buyers' replacement demand $\mu(1-\widehat{b})$ at the price $\underline{\theta} q_L$ forever to penetrating the entire market with the high quality good by charging $P^H(1, (1-\mu)(1-\widehat{b}))$ and continuing by selling the high quality good to all buyers' replacement demand μ at the price $P^H(1, x^L)$ thereafter. If

$$\frac{\mu\widehat{b}(\overline{\theta} q_H - \overline{\theta} q_L + \underline{\theta} q_L)}{1-\delta} + \frac{\mu(1-\widehat{b})\underline{\theta} q_L}{1-\delta} \geq (1-(1-\mu)\widehat{b})P^H(1, x^L) + \frac{\delta\mu P^H(1, x^L)}{1-\delta} \tag{7}$$

holds, the monopolist never cuts the price of the high quality good to $P^H(1, x^L)$ to serve the high quality good to all buyers. Since $P^H(1, x^L) \leq \underline{\theta}\Delta q$ for all $x^L > 0$, (7) holds if $\mu \geq \frac{(1-\delta)(1-\widehat{b})\underline{\theta}\Delta q}{\widehat{b}\Delta\theta\Delta q - \delta(1-\widehat{b})\underline{\theta}\Delta q + \underline{\theta} q_L} \equiv \underline{\mu}^{sg}$. Therefore, the monopolist does not deviate from the segmented monopoly steady state $(\widehat{b}, 1-\widehat{b})$ for $\mu \geq \underline{\mu}^{sg}$. Moreover, since $P^H(1,0) > P^H(1, x^L)$ for all $x^L > 0$, $\underline{\mu}^{sg} < \underline{\mu}^{st}$ must hold.

It is established that the necessary condition for the existence of a monopoly equilibrium, $\mu \geq \underline{\mu}^{sg}$, is also sufficient for the existence and the uniqueness of the equilibrium.

Theorem 2 *There exists at most one monopoly equilibrium iff $\mu \geq \underline{\mu}^{sg}$. The monopoly steady state(s) of such equilibrium is $\{(\widehat{b}, 1-\widehat{b})\}$ for $\underline{\mu}^{sg} \leq \mu < \underline{\mu}^{st}$ and $\{(\widehat{b}, 0), (\widehat{b}, 1-\widehat{b})\}$ for $\mu \geq \underline{\mu}^{st}$.*

In such an equilibrium, for $\mu \geq \underline{\mu}^{st}$ the monopolist initially charges $\overline{\theta} q_H$ for the high quality good and charges a price for the low quality good high enough that none of the buyers purchase it. Hence, from the initial state $(0,0)$ the monopolist brings the state to $(\widehat{b}, 0)$ by selling the high quality good to all high types. He then continues to charge the static monopoly prices to serve the replacement demand of the high type buyers for the high quality good. For $\underline{\mu}^{sg} \leq \mu < \underline{\mu}^{st}$ the monopolist initially charges $\overline{\theta} q_H - \overline{\theta} q_L + \underline{\theta} q_L$ for the high quality good and $\underline{\theta} q_L$ for the low quality good. Hence, from the initial state $(0,0)$ the monopolist brings the state to $(\widehat{b}, 1-\widehat{b})$ by selling the high quality good to all high types and selling the low quality good to all low types. He then continues to charge the segmented monopoly price to serve the replacement demands.

If the monopolist deviates from a monopoly steady state by selling more of the high quality good in an attempt to increase profits, the movement of the states in a monopoly equilibrium is as follows. The sequence of states $\{(\widetilde{x}^H_{k,j}, \widetilde{x}^L_{k,j})\}_{k=0}^{m_j+1}$ is constructed such that when that state is $(\widetilde{x}^H_{k,j}, \widetilde{x}^L_{k,j})$, the monopolist is indifferent between bringing the next period's state to $(\widetilde{x}^H_{k-1,j}, \widetilde{x}^L_{k-1,j})$ by charging $(\widetilde{p}^H_{k-1,j}, \widetilde{p}^L_{k-1,j})$ and staying at $(\widetilde{x}^H_{k,j}, \widetilde{x}^L_{k,j})$ by charging $(\widetilde{p}^H_{k,j}, \widetilde{p}^L_{k,j})$ forever. There exist three paths depending on the state of the low quality good. On all paths, the initial value of the state of the high quality good is $\widetilde{x}^H_{0,j} = (1-\mu)\widehat{b}$ and its end value is $\widetilde{x}^H_{m_j+1,j} \leq 1-\mu$.

On path 1, none of the buyers hold the low quality good. For all $x^H \leq \widetilde{x}^H_{m_1+1,1}$, path 1 reaches the static monopoly steady state. On path 2, some buyers hold the low quality good. For all $x^H \leq \widetilde{x}^H_{m_2+1,2}$, path 2 reaches the segmented monopoly steady state. Since the low type buyers anticipate that the price of the low quality good will be eventually equal to $\underline{\theta} q_L$, the monopolist cannot charge more than $\underline{\theta} q_L$

for the low quality good. Hence, $\widetilde{p}^L_{k,2} = \underline{\theta} q_L$ for all k. On path 3, there exists an excess supply of the low quality good. Hence, $\widetilde{p}^L_{k,3} = 0$ for all k. For all $x^H \leq \widetilde{x}^H_{m_3+1,3}$, path 3 reaches the segmented monopoly steady state.

If the good is sufficiently perishable, the monopolist will return to a monopoly steady state from any state of the high quality good above $(1-\mu)\widehat{b}$. Hence, $\widetilde{x}^H_{m_j+1,j} = 1-\mu$ holds. *Otherwise,* the Coase Conjecture steady state coexists with monopoly steady states. Because for sufficiently low μ, $\widetilde{x}^H_{m_j+1,j} < 1-\mu$ holds such that when the state is $(\widetilde{x}^H_{m_j+1,j}, \widetilde{x}^L_{m_j+1,j})$, the monopolist is indifferent between bringing the state to $(\widetilde{y}^H_{m_j,j}, \widetilde{y}^L_{m_j,j})$ and fully penetrating the market by selling the high quality good to all buyers and continuing by serving the replacement demand μ for the high quality good thereafter. It follows that the monopolist strictly prefers penetrating the market with the high quality good when the state of the high quality good is greater than $\widetilde{x}^H_{m_j+1,j}$.

When the state is moving towards a monopoly steady state, low type buyers purchase the high quality good at a price exceeding their valuation, in order to make capital gains by reselling it in the second-hand market at a later date. If none of the buyers hold the low quality good, the steady state of the market will be $(\widehat{b}, 0)$; otherwise it will be $(\widehat{b}, 1-\widehat{b})$.

The left graph of Figure 2.1 illustrates how states move to a monopoly steady state when the good is sufficiently perishable. The arrows indicate the direction of movement of the state at any state (y^H, y^L). When there exists no low quality good in the market, the states move towards the static monopoly steady state from any state $(y^H, 0)$ as follows. For $y^H \leq y$, the stock of the high quality good after trade in the next period will be $\widehat{b}$ and the monopolist will continue by selling the high quality good to the replacement demand of the high type buyers thereafter. For $y^H > y$, the stock of the high quality good after trade in the next period will be y. When there exists the low quality good in the market, the states move towards the segmented monopoly steady state from any state (y^H, y^L) as follows. If (y^H, y^L) is in region R_1 or in region R_2, the state after trade in the next period will be on the line that separates regions R_1 and R_2. Upon reaching this line, the monopolist will continue by charging monopoly prices and selling the high quality good to the replacement demand of the high type buyers and selling the low quality good to the replacement demand of low type buyers. If (y^H, y^L) is in region R_3, the state after trade will be on the line that separates the regions R_3 and R_2.

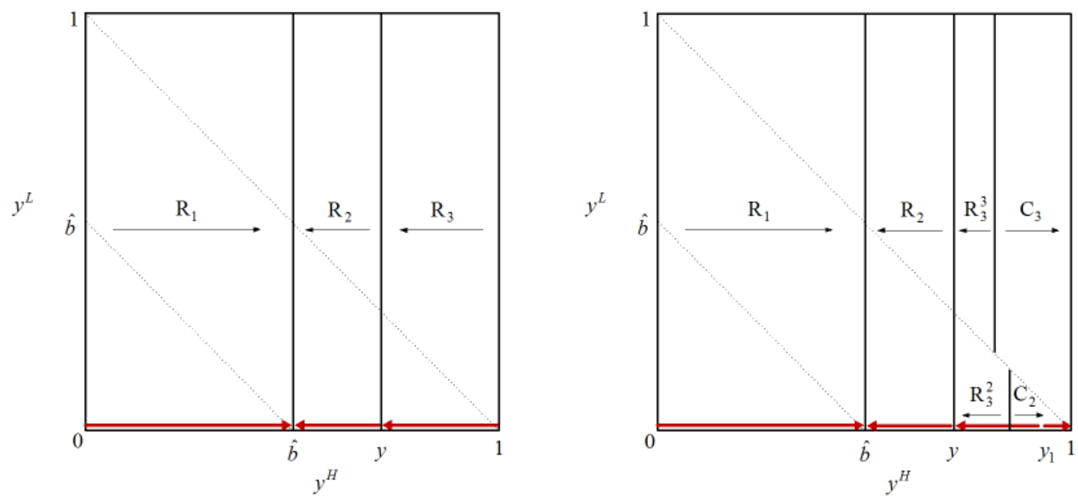

Figure 2.1: The Monopoly Equilibrium

The right graph of Figure 2.2 illustrates how states move to the a monopoly steady state when the good is not sufficiently perishable. The arrows indicate the direction of movement of the state at any state (y^H, y^L). When there exists no low quality good in the market, the states move towards the static monopoly steady state from any state $(y^H, 0)$ if $y^H \leq y_1$, otherwise the states move towards the Coase Conjecture steady state as follows. For $y^H \leq y$, the stock of the high quality good after trade in the next period will be $\widehat{b}$ and the monopolist will continue by selling the high quality good to replacement demand of the high type buyers thereafter. For $y < y^H \leq y_1$, the stock of the high quality good after trade in the next period will be y. For $y^H > y_1$, the stock of the high quality good after trade in the next period will be 1 and the monopolist will continue by selling the high quality good to replacement demand of all buyers thereafter. When there exists low quality good in the market, the states move towards the segmented monopoly steady state from any state (y^H, y^L) if y^H is sufficiently low, otherwise the states move towards the Coase Conjecture steady state as follows. If (y^H, y^L) is in region R_1 or in region R_2, the state after trade in the next period will be on the line that separates regions R_1 and R_2. Upon reaching this line, the monopolist will continue by charging monopoly prices and selling the high quality good to the replacement demand of the high type buyers and selling the low quality good to the replacement demand of low type buyers. If (y^H, y^L) is in region R_3^i, the state after trade will be on the line that separates the regions R_3^i and R_2. If (y^H, y^L) is in region C_i, the monopolist will charge low prices in an attempt to penetrate the entire market with the high quality good and continue by serving the high quality good to the replacement demand of all buyers thereafter.

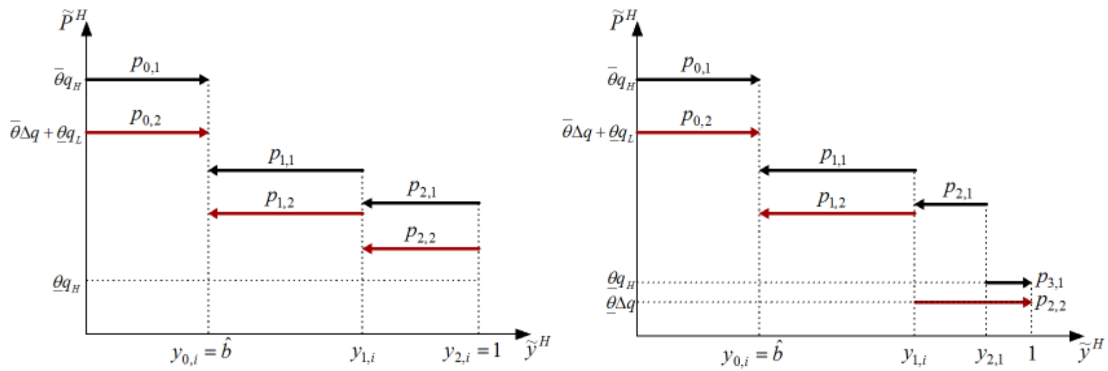

Figure 2.2: Path 1 vs Path 2

Figure 2.2 illustrates the movement of the high quality good towards a monopoly steady state and the corresponding prices. For expositional purposes only path 1 and path 2 are studied and when the monopolist moves towards the Coase Conjecture steady state, the monopolist is assumed to immediately penetrate the market with the high quality good by charging $\underline{\theta}\Delta q$ for the high quality good. For ease of exposition $\widetilde{y}_{k,j}^H$ is referred as $y_{k,j}$ and $\widetilde{p}_{k,j}^H$ is referred as $p_{k,j}$. On the left graph the state moves back to a monopoly steady state from any state, whereas on the right graph the state moves towards the Coase Conjecture steady state when the stock of the high quality good is high. The arrows indicate the direction of the movement. We can see from these graphs that production of a low quality good lowers the price of the high quality good in a monopoly equilibrium.

Let $\underline{\mu}^s(\delta)$ be the threshold depreciation rate derived in Deneckere and Liang (2008). When a monopolist produces a single version of a durable good, a monopoly equilibrium exists for all $\mu > \underline{\mu}^s(\delta)$. It is established that

Corollary 4 *The threshold depreciation rate supporting the static monopoly steady state $\underline{\mu}^{st}$ is a function of δ such that $\underline{\mu}^{st}(\delta) = \underline{\mu}^s(\delta)$ holds for all δ.*

Since the segmented monopoly steady state is supported for $\underline{\mu}^{sg} \leq \mu < \underline{\mu}^{st}$, the set of parameters consistent with the monopoly equilibrium expands when a monopolist produces a low quality good as well as a high quality good.

Now, consider the structure of the stationary path as the time period between successive offers of the monopolist diminishes. For $x^H > (1-\mu)\widehat{b}$, the state either immediately moves to the Coase Conjecture steady state or slowly goes back to a monopoly steady state. It is established that the rate at which a monopoly steady state is reached is independent of the state of the low quality good and that

Corollary 5 *As the length of the time period between successive price changes approaches zero, the state of the high quality good moves towards a monopoly steady state at the rate of $\dot{x}^H = \lambda x^H \left(1 - \frac{\underline{\theta}}{\overline{\overline{\theta}}} \left(\frac{y^H}{\widehat{b}}\right)^{\frac{\lambda+r}{\lambda}}\right)$ for $x^H > (1-\mu)\widehat{b}$.*

5.3 The Reputational Equilibrium

Consider equilibria in which the monopolist establishes a reputation by cutting the production of the high quality good. The stock of the high quality good at a reputational steady state falls short of the static monopoly output of the high quality good. The reputational steady states of such equilibria are $(\check{y}^H, 1-\check{y}^H)$ and $(\hat{y}^H, 0)$ where $\hat{y}^H$, $\check{y}^H \in (0, \widehat{b})$.

If μ is sufficiently low, from the initial state $(0,0)$, the monopolist will immediately bring the state to $(\check{y}^H, 1-\check{y}^H)$ by charging $\overline{\theta}q_H - \overline{\theta}q_L + \underline{\theta}q_L$ for the high quality good and $\underline{\theta}q_L$ for the low quality good and continue by selling to the replacement demand $\mu\check{y}^H$ for the high quality good at the price $\overline{\theta}q_H - \overline{\theta}q_L + \underline{\theta}q_L$ and to the replacement demand $\mu(1-\check{y}^H)$ for the low quality good at the price $\underline{\theta}q_L$ thereafter.

If μ falls above this threshold, from the initial state $(0,0)$, the monopolist will immediately bring the state to $(\hat{y}^H, 0)$ by charging $\overline{\theta}q_H$ for the high quality good and charging a price no less than $\overline{\theta}q_L$ for the low quality good and continue by selling to the replacement demand $\mu\hat{y}^H$ for the high quality good at the price $\overline{\theta}q_H$ thereafter.

If the monopolist penetrates the market by selling more of the high quality good, he loses his reputation for pricing high. Since buyers expect that the future prices will be lower, they are reluctant to pay a high price for the high quality good. Hence, the monopolist has to drastically lower the price of the high quality good, and the state slowly moves to the Coase Conjecture steady state $(1, 0)$. Therefore, upon deviation from a reputational steady state by increasing the stock of the high quality good, the game follows the Coase Conjecture path. When the steady state is $(\hat{y}^H, 0)$, if the monopolist penetrates the market by selling more of the low quality good, buyers expect that the monopolist will increase the sales of the low quality good and hence, they do not accept any price for the low quality good significantly greater than $\underline{\theta}q_L$. Hence, the monopolist immediately brings the state to $(\hat{y}^H, 1 - \hat{y}^H)$ by charging $\overline{\theta}q_H - \overline{\theta}q_L + \underline{\theta}q_L$ for the high quality good and charging $\underline{\theta}q_L$ for the low quality good and continues by selling to the replacement demands thereafter.

Theorem 3 *There exists a unique reputational equilibrium if and only if* $\underline{\mu}^{sg} < \mu \leq \overline{\mu}$. *The reputational steady state(s) of such equilibrium is* $\{(\check{y}^H, 1 - \check{y}^H)\}$ *for* $\underline{\mu}^{sg} < \mu \leq \mu'$ *and* $\{(\hat{y}^H, 0), (\hat{y}^H, 1 - \hat{y}^H)\}$ *for* $\mu' < \mu \leq \overline{\mu}$ *where* $\mu' \geq \underline{\mu}^{st}$.

The existence of the reputational equilibrium necessitates existence of the Coase Conjecture equilibrium and the monopoly equilibrium. The intuition behind this is as follows. If the Coase Conjecture equilibrium does not exist, neither does the reputational equilibrium. Because when the Coase Conjecture equilibrium does not exist, the steady state stock of the high quality good falls short of a certain level below which a path that the monopolist follows upon deviation from a reputational steady state cannot be constructed. Moreover, the monopoly equilibrium does not exist when the monopolist cannot resist penetrating the market further in an attempt to increase profits. Since, the monopolist makes less profits by limiting the production of the high quality good, when the monopoly equilibrium does not exist, neither does the reputational equilibrium.

The structure of the reputational equilibrium, when the time period between successive price changes is infinitesimal is studied and it is established that

Corollary 6 *Let* $\check{y}^H = \frac{\underline{\theta}}{\overline{\theta}}$ *and* $\hat{y}^H = \frac{(\lambda+r)\underline{\theta}}{\lambda\overline{\theta}+r\underline{\theta}}$. *As the length of the time period between successive price changes approaches zero, the reputational steady state converges to* $(\check{y}^H, 1-\check{y}^H)$ *for* $\mu \in (\underline{\mu}^{sg}, \underline{\mu}^{st}]$, *and converges to* $(\hat{y}^H, 0)$ *for* $\underline{\mu}^{st} < \mu \leq \overline{\mu}$.

6 Coexistence of Equilibria: Single Good vs Multiple Goods

In this section, how production of a low quality product affects the existence and the uniqueness of each type of equilibrium is discussed.

Let $\underline{\mu}^s(\delta)$ and $\overline{\mu}^s(\delta)$ be the threshold depreciation rates when the monopolist produces a single version of a durable good.[13] It is established that

Proposition 4 *The threshold depreciation rates are functions of* δ *such that* $\underline{\mu}^{sg}(0) < \overline{\mu}(0) < \underline{\mu}^s(0) = \overline{\mu}^s(0)$, *and* $\lim_{\delta\to 1}\underline{\mu}^{sg}(\delta) = \lim_{\delta\to 1}\underline{\mu}^{st}(\delta) = 0$ *and* $\lim_{\delta\to 1}\overline{\mu}(\delta) = \lim_{\delta\to 1}\overline{\mu}^s(\delta)$.

The range of (μ, δ) supporting each type of equilibrium is identified and how production of a low quality good affects the parameters consistent with each type of equilibrium is discussed with an example. Consistent with the example in Deneckere and Liang (2008), it is assumed that $\underline{\theta} = 0.6\,\overline{\theta}$, $\widehat{b} = 0.7$, and $q_H = 3\,q_L$.

First, the range of (μ, δ) where each type of equilibrium exists when the monopolist produces only a high quality good is illustrated in Figure 6.1. As discussed in detail by Deneckere and Liang (2008), for $\mu < \underline{\mu}^s(\delta)$ the Coase Conjecture equilibrium is the unique equilibrium, for $\underline{\mu}^s(\delta) \leq \mu \leq \overline{\mu}^s(\delta)$ all types of equilibrium coexist, and for $\mu > \overline{\mu}^s(\delta)$, the monopoly equilibrium is the unique equilibrium. The intuition behind this result is as follows. As depreciation factor μ increases, profits from replacement sales increase. Hence, rather than penetrating the market further, the monopolist prefers serving replacement demands.

[13] See Deneckere and Liang (2008) for details.

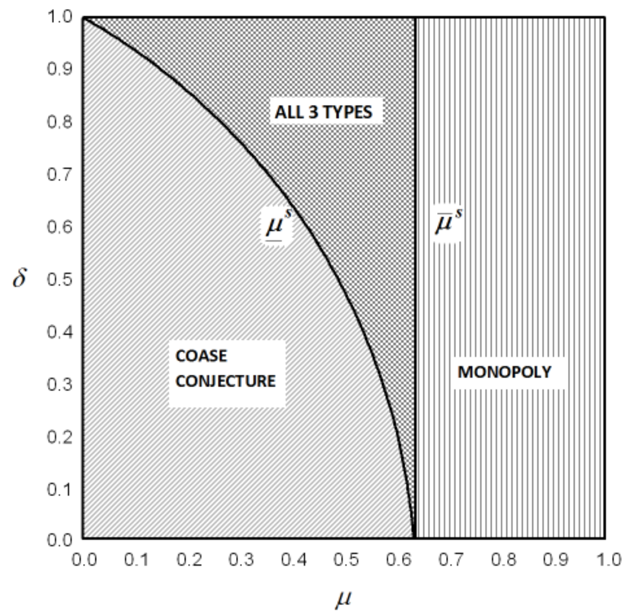

Figure 6.1: Support of Each Type of Equilibrium: Single Good

Now the range of (μ, δ) supporting each type of equilibrium when the monopolist produces a low quality good as well as the high quality good is illustrated in Figure 6.2. We can observe that with multiple goods differ in quality, the range of (μ, δ) consistent the Coase Conjecture equilibrium is smaller and the range of (μ, δ) consistent with the monopoly equilibrium is bigger. The economic intuition behind this result is as follows. For $\overline{\mu}(\delta) \leq \mu < \overline{\mu}^s(\delta)$, the Coase Conjecture equilibrium does not exist when the monopolist produces multiple goods, since the depreciation rate is high enough so that when the state is $(\widehat{b}, 1-\widehat{b})$, the monopolist prefers selling to replacement demands rather than fully penetrating the market with the high quality good. Moreover, for $\underline{\mu}^{sg}(\delta) < \mu \leq \underline{\mu}^s(\delta)$, a monopoly equilibrium exists when the monopolist produces multiple goods because for a given value of δ, $\mu \in (\underline{\mu}^{sg}(\delta), \underline{\mu}^s(\delta)]$ supports the segmented monopoly steady state but not the static monopoly steady state.

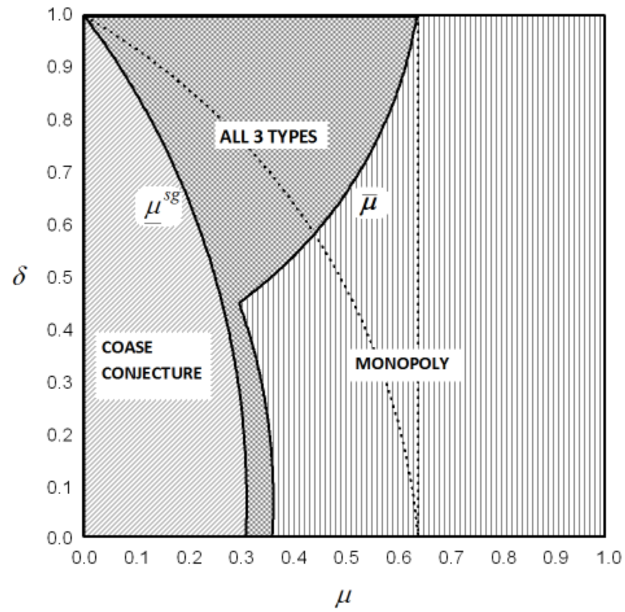

Figure 6.2: Support of Each Type of Equilibrium: Multiple Goods

Figure 6.2 also helps us identify the structure of equilibria when the monopolist can adjust the prices frequently. As the length of the time period diminishes, (μ, δ) converges to $(0, 1)$ for all $r > 0$ and $\lambda < \infty$. Since $\mu(\delta) = 1 - \delta^{\frac{\lambda}{r}}$, $(\mu(\delta), \delta)$ lies below $(\underline{\mu}(\delta), \delta)$ when $\frac{\lambda}{r}$ is small. Moreover, when the length of the time period between successive offers of the monopolist is arbitrarily small, the cutoff value λ_0 below which the Coase Conjecture equilibrium is the unique equilibrium is $\lambda_0 = \frac{r(1-\widehat{b})\underline{\theta}\Delta q}{(\widehat{b\theta}-\underline{\theta})\Delta q+\underline{\theta}q_L}$, whereas the corresponding threshold when the monopolist produces a single version of the good is $\lambda_0^s = \frac{r(1-\widehat{b})\underline{\theta}}{(\widehat{b\theta}-\underline{\theta})}$. Therefore,

Corollary 7 *(i) Let $\lambda_0 = \frac{r(1-\widehat{b})\underline{\theta}\Delta q}{(\widehat{b\theta}-\underline{\theta})\Delta q+\underline{\theta}q_L}$. As the length of the time interval between successive offers of the monopolist converges to zero, a Coase Conjecture equilibrium exists for all $\lambda < \infty$. When $\lambda < \lambda_0$, the Coase Conjecture equilibrium is the unique equilibrium. When $\lambda \geq \lambda_0$, all three types of equilibrium coexist.*

(ii) Let $\lambda_0^s = \frac{r(1-\widehat{b})\underline{\theta}}{(\widehat{b\theta}-\underline{\theta})}$. When the monopolist produces single version of the good, as the length of the time interval between successive offers of the monopolist converges

to zero, a Coase Conjecture equilibrium exists for all $\lambda < \infty$. When $\lambda < \lambda_0^s$, the Coase Conjecture equilibrium is the unique equilibrium. When $\lambda \geq \lambda_0^s$, all three types of equilibrium coexist.

Hence, since $\lambda_0 < \lambda_0^s$, it is concluded that for all $z \geq 0$, the monopoly equilibrium is more likely to occur, when the monopolist offers multiple goods differ in quality. For example, when $\frac{\underline{\theta}}{\overline{\theta}} = 0.6$, $\widehat{b} = 0.7$, $\delta = 0.60$, and $\frac{q_H}{q_L} = 3$, the threshold depreciation rates are $\underline{\mu}^{sg} = 0.15$, $\overline{\mu} = 0.41$, and $\underline{\mu}^s = 0.42$, $\overline{\mu}^s = 0.64$. Therefore, if the monopolist only produces the high quality good, the monopoly equilibrium exists when the turnover is less than 3 years, and is the unique equilibrium when the turnover is less than 1 year. However, if the monopolist produces the low quality good as well as the high quality good, the monopoly equilibrium exists when the turnover is less than 6 years, and is the unique equilibrium when the turnover is less than 2 years.

7 Literature Review

The time inconsistency problem a durable goods monopolist faces has been aggressively studied since Coase (1972) conjectured that the sequence of prices (or outputs) of a monopolist selling a perfectly durable good does not maximize his overall profitability. The goal of this section is to briefly survey the literature in an effort to understand the theory of durable goods monopolies and to place this study in the appropriate context.[14]

Early studies analyzing a monopolist selling a perfectly durable good establish that if buyers condition their strategies on payoff relevant part of the histories (that is, if buyers use Markov strategies), Coase's prediction holds. Bulow (1982) studies a durable goods monopolist in a two period model and shows that the optimum price charged by the monopolist is strictly less than the static monopoly price. Intuitively, unless the monopolist credibly precommit to a production plan, consumers anticipate that the monopolist will produce additional units to exploit residual demand which decreases the present value of the durable good. Therefore, since consumers

[14] See Waldman (2003) for a detailed literature survey on durable goods monopolies.

are reluctant to pay the static monopoly price in the first period, in response to the expectations of the consumers the monopolist cuts the price of the durable good. Stokey (1981) extends Bulow's (1982) setting into an infinite horizon setting and proves the existence of an equilibrium that is the limit of the unique equilibrium of the finite version of her model which satisfies the Coase Conjecture. Similarly, Gul, Sonnenschein, and Wilson (1986) show that even though a continuum of subgame perfect equilibria may exist in an infinite horizon model, the Coase Conjecture is verified for Markov strategies. Sobel (1991) extends these initial analyses by considering a market for a perfectly durable good in which demand expands over time. His study also verifies Coase's prediction for Markov strategies. Intuitively, if the monopolist charges the static monopoly price forever, as new consumers enter the market the number of low valuation consumers grows cutting the price then becomes inevitable.

On the contrary to the early studies, Ausubel and Deneckere (1989) show that if buyers condition their strategies on not only payoff relevant part of the histories but the past actions as well, there exist equilibria in which the monopolist creates a reputation and maintains some or all of his market power when the marginal cost of production is no less than the lowest valuation of the buyers. In addition to establishing a reputation, depreciation of a durable good can also help a durable goods monopolist avoid the time inconsistency problem. Bond and Samuelson (1984) show that in a discrete time, infinite horizon game when the good depreciates, replacement sales may deter the monopolist from cutting the price as long as the time period between successive offers of the monopolist is nonzero. However, in the limit, as the time period approaches zero, the competitive outcome is achieved and the Coase Conjecture holds. Karp (1996), on the other hand, by using a continuous time model with replacement sales constructs continuous time equilibria in which the monopolist can earn profits above the competitive level. However, Karp (1996) also shows the existence of an equilibrium that verifies the Coase Conjecture. Following Karp (1996), Deneckere and Liang (2008) characterize the effect of the depreciation rate on the market outcome of a durable goods monopoly when agents use Markov strategies. They conclude that below a certain level of durability, there exists a unique stationary equilibrium in which the monopolist charges the static monopoly price in each period which continues to exist even when the seller becomes highly impatient. Intuitively, when the product depreciates, replacement sales become more profitable than penetrating the market by cutting the price of the good.

A number of recent studies on durable goods monopolies revolves around the issue of introduction of new products with quality improvements. Levinthal and Purohit (1989), Fudenberg and Tirole (1998), and Lee and Lee (1998) study optimal sales strategy of a durable goods monopolist who may introduce an improved version of the good in a two period model. Waldman (1993, 1996), Choi (1994), Fishman and Rob (2000), and Kumar (2002) study the quality and pricing strategy of a durable goods monopolist. These studies develop the idea of planned obsolescence and established that Coase's insight holds. Intuitively, introduction of higher quality products lower the value of used units. Since consumers foresee that the units they have will become obsolete, they refuse to pay the static monopoly price of the good. Hence, the monopolist faces a problem of time inconsistency and the overall profitability is reduced. Kumar (2006) analyzes a discrete time, infinite horizon model in which the monopolist selling a perfectly durable good can vary the quality of the good and shows that when lowest buyer valuation is greater than the marginal cost of production every subgame perfect equilibrium verifies the Coase Conjecture. Anton and Biglaiser (2009) study an exogenous quality growth in an infinite horizon durable goods monopoly model. They show that, for any positive discount factor, the support Markov perfect equilibrium payoffs ranges from getting all the surplus to getting the single period flow value of each upgrade. Takeyama (2002) studies quality differentiation of a perfectly durable good and the possibility of upgrades in a two period model. Inderst (2008), on the other hand, studies optimal strategy of a durable goods monopolist who can offer perfectly durable goods in different qualities in an infinite horizon model and shows that when the monopolist becomes extremely flexible in adjusting the prices and qualities, he immediately loses his monopoly power and competitive outcome is achieved.

8 Conclusion

In this article, I study the effect of quality differentiation on the commitment problem of a durable goods monopolist. I extend the single good setting of Deneckere and Liang (2008) into a setting of a vertically differentiated market and consider a monopolist selling an imperfectly durable good available in two quality levels in an infinite horizon, discrete time game. I characterize the Markov perfect equilibria as a function of the common discount rate, the common depreciation rate of the goods, the length of the time period between successive price changes, and the quality levels of the goods. Similar to Deneckere and Liang (2008), I establish that there exist three types of Markov perfect equilibria: a Coase Conjecture equilibrium, a monopoly equilibrium, and a reputational equilibrium. For sufficiently low depreciation rates, the unique equilibrium is the Coase Conjecture equilibrium. The Coase Conjecture equilibrium has a unique steady state equal to the competitive quantity. For sufficiently high depreciation rates, the unique equilibrium is the monopoly equilibrium. This equilibrium has two monopoly steady states one of which is equal to the static monopoly quantity. The market at the other monopoly steady state is segmented into two: the monopolist serves the high quality good to the high type buyers and serves the low quality good to the low type buyers. For intermediate values of the depreciation rate, all three types of equilibria exist. In the reputational equilibrium, the monopolist creates a reputation of pricing high by cutting the production of the high quality good. Hence, the reputational steady state quantity of the high quality good falls short of the monopoly quantity of the high quality good. These results survive even when the agents become extremely patient. However, the set of parameters for which the Coase Conjecture equilibrium is unique vanishes. When the length of the time period between successive price changes is arbitrarily close to zero, the Coase Conjecture equilibrium always exists and the monopoly equilibrium exists only if the good is sufficiently perishable.

I prove that the set of parameters supporting the Coase Conjecture equilibrium is smaller and the set of the parameters supporting the monopoly equilibrium is larger when the monopolist can produce a lower quality good. Hence, my study establishes that quality differentiation may enhance market power of a durable goods monopolist and alleviate the commitment problem, and suggests that when the innate durability of a good is high, the monopolist will damage a portion of the goods and produce a lower quality good to credibly commit to high future prices for the higher quality good.

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