Feb. 11, 2009

Homework # 3 (Math 529)

Problem 1/ Show that

$$\int_0^1 \sin(mx)\sin(nx)\,dx = \begin{cases} 0 & \text{when } m \neq n \\ \frac{1}{2} & \text{when } m = n. \end{cases}$$

Problem 2/ Transform the PDE

$$u_t = u_{xx} \qquad 0 < x < 1, \quad 0 < t < \infty$$

$$\begin{cases} u(0,t) = 0 \\ u(1,t) = 1 \end{cases} \qquad 0 < t < \infty$$

$$u(x,0) = x^2 \qquad 0 \le x \le 1$$

to a PDE with zero boundary conditions. Then solve the homogeneous problem.

Problem 3/ Solve the 1^{st} order PDE

$$u_t + u_x = 0 \qquad -\infty < x < \infty, \qquad t > 0$$

$$u(x,0) = \cos x \qquad x \in \mathbb{R}$$

and graph the solution for a few different 't' values.

Problem 4/ Solve the first order PDE

$$x\,u_x + t\,u_t + 2u = 0 \qquad x \in \mathbb{R}, \quad \overset{\text{Note}}{\boxed{t > 1}}$$

$$u(x,1) = \sin x \qquad x \in \mathbb{R}$$

Problem 5/ Solve the following non-homogeneous heat equation IBVP by the eigenfunction expansion method:

$$u_t = u_{xx} + \sin(\pi x) \qquad 0 < x < 1, \quad t > 0$$

B.C. $\begin{cases} u(0,t) = 0 \\ u(1,t) = 0 \end{cases}$

I.C. $u(x,0) = 1$.