

TEST 1

Math 529
02/23/2009

Name: _____

by writing my name i swear by the honor code

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- You may use your notebooks and textbooks for the test.
- Justify your answers algebraically whenever possible to ensure full credit. When you do use your calculator, sketch all relevant graphs and explain all relevant mathematics.
- Write your name backwards on the top left corner of the last page for a bonus point.
- Circle or otherwise indicate your final answers.
- This test has 3 problems and is worth 100 points.
- Good luck!

1. Consider the Navier-Stokes equation given by

$$\rho(\mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u}) = \mu \Delta \mathbf{u} + \nabla p + f(x, y, z, t)$$

where $\mathbf{u} = \langle u(x, y, z, t), v(x, y, z, t), w(x, y, z, t) \rangle$.

a. Write down the components of the above PDE (10pts).

b. Suppose $\mathbf{u} = \langle u(y), 0, 0 \rangle$ and $\nabla p = \langle c, 0, 0 \rangle$ where c is a constant. Then what should $u(y)$ be in order for it to satisfy the Navier Stokes equation? (15pts).

2. Answer the following questions below.

a. Classify the equation $u_{xx} + 5u_x + 6u = 0$ defined over $-\infty < x, y < \infty$ (10pts).

b. Classify the equation $u_{xx} + \cos(x)u_{yy} = 0$ defined over $-\infty < x, y < \infty$ (10pts).

c. Let \mathbf{v} and ϕ represent a vector and a scalar, respectively. Show the following identities: (i) $\operatorname{div}(\operatorname{curl} \mathbf{v}) = 0$, (ii) $\operatorname{curl}(\operatorname{grad} \phi) = 0$. (15pts).

3. Consider the PDE

$$u_t + Qu_{xxxx} = 0$$

defined on $-\infty < x < \infty$, $Q > 0$, $t > 0$ and $u(x, t)$ bounded as $x \rightarrow \pm\infty$ and also $u(x, 0) = f(x)$.

a. Use Fourier Transform in x to find the solution in terms of the inverse transform involving $\hat{f}(s)$ (20pts).

b. Solve the same problem using separation of variables (20pts).

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