RESEARCH STATEMENT

Ashwin Vaidya

My research interest lies in applied mathematics, particularly in problems with applications to all aspects of fluid mechanics. I am drawn to problems with interesting physics and challenging mathematics. My past and current research have been focused on both fundamental problems in fluids and also those which are derived directly from applications. Some of the problems that I have worked on and my future plans are briefly discussed below.

1 Mathematical Fluid Mechanics

(**Collaborators**: G.P. Galdi, University of Pittsburgh , A. Silvestre, Institute Superior Technico, Portugal)

Existence for steady freefall of bodies of a Newtonian liquid in bounded and exterior domains is a well studied problem. However this problem remains unsolved in exterior domains when the liquid is non-Newtonian. Therefore we choose to fill this gap in the literature by choosing to work with the simplest model for a nonlinear viscoelastic liquid, namely a second order fluid model. This amounts to studying the following equations:

$$\begin{aligned} -\Delta u + \nabla p - \mathrm{We}\; u\cdot\nabla\Delta u &= f + \mathrm{We}\nabla\cdot[(\nabla u^T)A_1 + (1+\epsilon){A_1}^2] \\ \mathrm{div}\; u &= 0, \quad u = 0 \text{ on } \Sigma \\ \lim_{x\to\infty}(u+u_\infty) &= 0,\; u_\infty = \xi + \omega\times x \end{aligned} \tag{1}$$

where A_1 is the symmetric part of the velocity gradient, u_∞ represents the rigid body motion with ξ the constant translational motion and ω, the rotational motion, We refers to the Weissenberg number and ϵ is a material parameter depending upon the normal stress coefficients. Since the problem involves also the motion of the body, we must, in addition, specify equations for the body. These are given as linear and angular momentum equations with an additional constraint equation:

$$\begin{aligned} \int_\Sigma T(\omega,\pi)\cdot n &= mg \\ \int_\Sigma y\times T(\omega,\pi)\cdot n &= 0 \\ \omega\times g &= 0. \end{aligned} \tag{2}$$

Here, T is the Cauchy Stress tensor, g represents the gravity vector and m the mass. We have shown existence and uniqueness of solutions (u,p) for prescribed motion (ξ,ω) of the body, to this nonlinear, coupled system for small We and zero Re. The proof follows from the application of suitable fixed point arguments in complex function spaces [9, 10, 20, 16]. The more general case of existence of solutions (u,p,ξ,ω,g) to the full nonlinear coupled problem has also recently been shown by considering a fixed point theorem for multivalued functions. For the future, along with Dr. A. Silvestre, we are planning to look at (i) the issue of existence of solutions

(ξ, ω) for the motion of a rigid body in fluids with shear dependent normal stress effects and (ii) also plan on investigating the problem of existence of solutions, (u, p, ξ, ω, g), to the coupled problem in the presence of inertial effects.

2 Gravitational Settling of Bodies in Fluids

(**Collaborators**: G.P. Galdi, University of Pittsburgh, B.J. Chung, University of North Carolina)

It is a well established fact that homogeneous bodies of revolution around an axis (call it a) with fore-aft symmetry will orient themselves with respect to the direction of gravity ($\mathbf{g}$) depending upon their shape and upon the nature of the fluid in which they are immersed.

If, for instance, we are considering an ellipsoidal object falling in a Newtonian fluid such as water, then the body falls with a eventually becoming perpendicular to the direction of $\mathbf{g}$. However if the same body falls in a viscoelastic fluid where the inertial effects can be disregarded then a will eventually become parallel to $\mathbf{g}$. Furthermore, it has also been observed that elongated bodies falling in fluids with certain polymeric concentrations can taken on angles between the horizontal and vertical orientations.

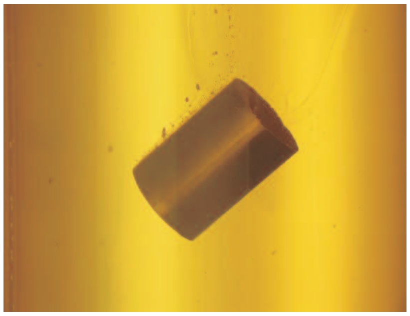

Fig.1: Particle orientation in a viscoelastic fluid.

These intermediate angles are referred to in the literature as *tilt angles*[5],[4]. With regards to this problem, we address several issues including the modeling, well-posedness of the equations, analysis of the terminal orientation phenomenon and even experimental verification of the phenomenon.

We have studied the steady free fall in Newtonian, viscoelastic and shear-thinning fluids modeled by the Navier-Stokes equations, second-order fluid equations and the power-law fluid equations respectively [9, 10, 19]. The principle objective of our research has been to explain the orientation phenomenon as a result of competing torques caused by different aspects of the fluid upon the sedimenting body [12] and to recognize the key elements in fluids that give rise to these unique angles. The terminal orientation can be seen to be the result of torques due to factors such as viscosity, inertia, elasticity and shear-thinning and the body tends to orient in the direction of the dominant torque. We are able to formulate the problem at first order in Re and We, i.e. for small Reynolds and Weisseneberg numbers respectively. We show that the net torque(indicated $\mathcal{M}_{tot}$) imposed upon the body by the fluid can be written by

$$\mathcal{M}_{tot} = \mathcal{M}_{viscous} + Re\, \mathcal{M}_{inertia} + We\, \mathcal{M}_{elasticity}. \tag{3}$$

The zero's of $\mathcal{M}_{tot}$ gives us the terminal steady angles. Our theoretical analysis, it turns out, is in perfect agreement with experimental observations for the cases when the particle turns either horizontal or perpendicular to the direction of gravity. We have recently resolved the issue of the *tilt-angle* by considering the freefall of symmetric bodies in a *generalized second order fluid*, where the normal stress can also vary with the shear rate.

3 Vortex Induced Oscillations

(**Collaborators**: R. Camassa, B.J. Chung, P. Howard, R. McLaughlin, University of North Carolina).

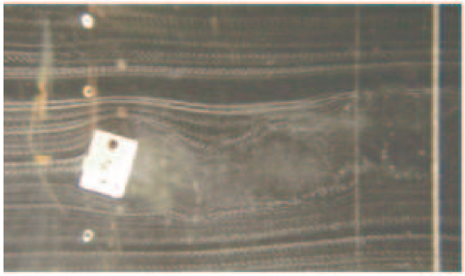

Fig.2: Hydrogen bubble visualization of an oscillating particle.

A more recent project that I have been involved in, concerns the unsteady interaction of symmetric rigid bodies at intermediate and high Reynolds numbers [3]. The phenomenon is being investigated experimentally and also numerically. The objective of our research has been to analyze the dynamics of bodies in a moving stream at varying Reynolds numbers when asymmetric vortex shedding effects behind the body starts to play an important role.

This problem can be thought of as a natural extension of the orientation problem described above to the unsteady regime. We study the orientational behavior of a hinged cylinder, of varying aspect ratio (ratio of length to diameter), suspended in a water tunnel in the presence of an incompressible flow with Reynolds number (Re), based on particle dimensions, ranging between 100-6000 and non-dimensional inertia (I^*) in the range 0.1-0.6. The cylinder displays four unique features, which include: **steady orientation, random oscillations, periodic oscillations and autorotation**. We illustrate these features displayed by the cylinder using a phase diagram which captures the observed phenomena as a function of Re and I^* and displays a unique pattern. A hydrogen bubble flow visualization technique was used to show vortex shedding structure in the cylinder's wake which seems to result in these oscillations.

For the future we hope to further refine these experiments and do a more quantitative study of the vortex shedding in the cylinder's wake by means of a constant temperature anemometer(CTA). Numerical simulation of this phenomenon is also being performed with Dr. B.J. Chung using the Chimera grid technique based on a finite volume method. The Chimera grid method is based on two overlapping grids, one for the object and the other for the surrounding fluid and can effectively handle a moving object in a fluid by means of an interpolation scheme. Finally, a theoretical model for this fluid structure interaction problem is lacking and needs to be developed.

4 Brachistochrones in Potential Flow Past Rigid Bodies

(**Collaborators**: R. Camassa, R.M. McLaughlin, M. Moore, University of North Carolina, Chapel Hill.)

It is well known that the unbounded potential and Stokes flow around a circular cylinder can be described in terms of a stream function. Consider the time taken by any given streamline to travel between points with x-coordinate $x_i = -|x_0|$ and $x_f = |x_0|$. It would seem logical to believe that as we move farther away from the obstacle, the travel time gets shorter. We can write the flight time in the following elliptic integral form

$$t = \int_{y_0}^{y_f} \frac{(x^2+y^2)^2}{2a^2xy}\, dy \tag{4}$$

for potential flow past a cylinder, where y_0 is the initial height and y_f is the final height. Remarkably enough, we find that in the case of potential flow, there is a critical finite height $y = y_c$ at which the streamline travels the fastest, i.e. the travel time is the least at y_c. We refer to this path of shortest time as the *brachistochrone*. This property is however unique to potential flow and does not exist in the case of Stokes flow. Figure 4(a) plots the travel time along different streamlines, indicating a minimum at a finite value close to the obstacle.

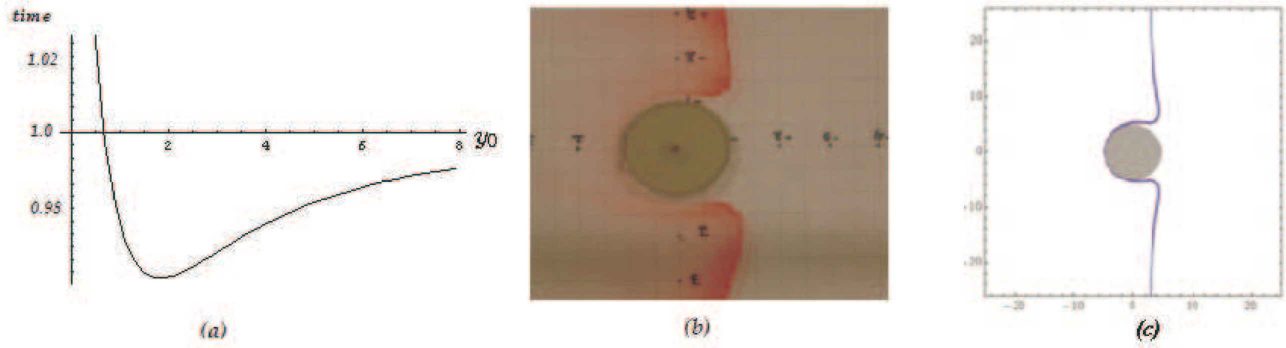

Fig.4: In figure (a) we denote the normalized time of flight as a function of stream value for potential flow past a cylinder. Figure (b) shows an image from an experiment in a Hele-Shaw cell where a line of dye is released upstream of the particle and moved due to a constant pressure gradient (from left to right). By the time the dye crosses the cylinder, there is a point near the cylinder where the dye is traveling the fastest which is indicated by the maximum. The final plot (c) shows a numerical simulation in two dimensions showing a very similar profile to that seen in experiments.

This property of potential flow is seen to persist in flow past bodies of any shape and can be explicitly computed for bodies such as cylinders, flat plates, ellipses and even in three dimensional spheres. We establish existence and non-existence of the minimum time for potential and Stokes flows, respectively and scaling laws have been derived by means of asymptotic methods. This phenomena has also been confirmed by means of numerical simulation of the problem. In our recent article on this subject[2], we also draw connections to the well known Darwin's theorem concerning the volume of fluid advected by a moving body as proposed by Sir Charles Darwin and later extended by Eames et al. and others [7, 8].

We are currently also trying to conduct an experimental study to visually identify the presence of the brachistochrone, in a small gap Hele-Shaw cell which recreates the potential flow past a body in two dimensions (see Figure 4(b)). As the gap thickness in the cell is increased the three dimensional effects become more prominent and flow transitions to a Stokes flow past the body. We are currently focused on (i) establishing the presence of a brachistochrone in the two dimensional potential flow case, (ii) understanding the transition from potential to Stokes flow and (iii) performing numerical simulations for flow past an obstacle in the presence of walls.

5 Modeling complex fluids and applications

(**Collaborators**: M. Massoudi, Department of Energy, National Energy Technology Lab, Pittsburgh.)

Non-Newtonian fluids differ from their Newtonian counterpart in that they may possess non-constant viscous and also elastic effects which can give rise to some remarkable phenomena such as rod climbing, tube siphon and several others. The modeling of viscoelastic fluids has been the topic of wide study for several decades now. However, several questions remain to be answered. For one, the flow properties of complex fluids in several engineering applications needs to be better understood. In one of our articles we have addressed the problem of natural convective flow of a second order viscoelastic fluid in a vertical channel which is heated from the sides and obtained numerical solutions to this coupled problem of fluid flow and heat transfer[17]. This study can be found to be very useful in understanding the efficient transport properties of coal slurries, for instance. A second issue of interest to us concerns the dependence of material parameters such as viscosity and normal stress coefficients upon additional complexities such as temperature, shear-rate, concentration of tracers and chemical reactions in order to model realistic fluids in biological and engineering applications. This question was to some extent tackled in our second paper [18] where we theoretically looked at the dependence of shear-rate upon the normal stress coefficients in *second grade fluid models.* We proposed an ad-hoc constitutive model called the *generalized second grade fluid model* given by the stress tensor

$$\mathbf{T} = -pI + 2\mu(\dot{\gamma}) \left(A_1 + \alpha_1 A_2 + \alpha_2 A_1^2\right).$$

Here p is the pressure field, I the identity matrix, μ the viscosity which can depend upon temperature, T and shear-rate $\dot{\gamma}$. A_1 and A_2 refer to the Rivlin-Ericksen tensors and α_1, α_2 are material parameters which are related to the normal stress coefficients. We studied the viscometric flow of such a fluid to better understand the effect of a more complex normal stress. Our work in this area has also proved beneficial in resolving issues discussed in Section 2, concerning the tilt-angle orientation of bodies in viscoelastic fluids. For the future we hope to examine more complex fluids and study them theoretically, numerically and even conduct experiments.

6 Network Analysis and Software Oriented Architecture

(**Collaborators**: Dr. Dakshinamurthy Kolluru, Prithvi Information Solutions Inc., Analytics Division, Hyderabad, India.)

Software Oriented Architecture (SOA) is a new buzzword in the Computer Science/IT community. It refers to the design and architecture of a business involving several processes each of which in turn is composed of several tasks. We are trying to model several questions of interest to the business community such as: (i) what are *optimal service compositions* in any given set of business processes, i.e. what sequence of tasks are most often repeated and can be optimally combined into a single service unit ? and (ii) how similar or dissimilar are two businesses which share the same tasks but which organize their processes slightly differently ? We are studying these problems using statistical, graph theoretic and linear programming techniques.

7 Future Work

In addition to the questions that have emerged from my current and previous work, I wish to investigate the following problems.

7.1 Unsteady shear flow past a body

The problem of steady Stokes shear flow past a fixed obstacle in an exterior domain is extremely interesting. Chwang and Wu [6] showed that in the case of two dimensional shear flow past a cylinder, there exists a *blocking region* in the fore and aft region of the cylinder. This suggests that whereas, far away from the cylinder the fluid flow crosses the obstacle, in a certain critical region, the motion of the fluid is confined to simply one side of it. Very recently, it was also shown (Camassa,McLaughlin and Zhao) that a similar blocking region exists when the flow is three dimensional and obstacle is a sphere. When the shear flow is not centered at the center of the sphere, the three dimensional flow structure also induces additional vortical patterns. In light of these interesting discoveries, it is then natural to ask what happens when the shear flow is unsteady, for instance, oscillatory in time. This problem has natural implications in rheology of complex fluids and fluid-solid mixtures. The governing equations for the problem, in the body frame, are

$$\begin{aligned} \rho\,\partial_t \mathbf{u}(\mathbf{x},t) &= \mu\,\Delta\mathbf{u}(\mathbf{x},t) - \nabla p \; in\ \Omega & (5)\\ \operatorname{div}\mathbf{u}(\mathbf{x},t) &= 0 & (6)\\ \mathbf{u}(\mathbf{x},t) &= 0\;\mathbf{x}\, on\ \ \partial\Omega & (7)\\ \mathbf{u}(\mathbf{x},t) &= \mathbf{V}(\mathbf{x},t)\mathrm{a}s\,\mathbf{x}\to\infty & (8) \end{aligned}$$

We conjecture that the problem can be resolved by an extension of the same singularity method employed by Chwang and Wu[6]. A detailed investigation is required to understand (1) the nature of temporal changes in the blocking region, (2) induced particle trajectories, (3) the contribution to unsteady effects in general other than simply the oscillatory shear flow case, (4) bifurcations and chaos in the system and (5) effect of shape of the obstacle upon the flow. We propose to tackle these issues theoretically, primarily using the singularity and asymptotic techniques, and also numerically.

7.2 Particle motion in a fluid

Another significant problem of interest relates to the modeling the equations of motions of a sedimenting body. The governing integro-differential equation, describing the unsteady translational motion of a spherical body in a Stokes fluid was first derived by Basset [1] and has also recently been modified to include inertial effects. The Basset equation has been re derived for the case of a spheroidal body by Lawrence and Weinbaum [14]. We wish to extend these earlier studies to derive equations for translational and rotational motion of a spheroidal body with inertial effects. This then allows us to address issues such as for instance (i) attainable steady orientations of a freefalling body and (ii) unsteady sedimentation behavior of falling bodies.

References

[1] Basset, A.B., Treatise on hydrodynamics, vol. 2, Deighton, Bell and Co., Cambridge, 1961.

[2] Camassa, R., McLaughlin, R., Moore, M. and Vaidya, A., to appear in Physics Letters A, 2008.

[3] Camassa, R., Chung, B.J., Howard, P., McLaughlin, R. and Vaidya, A., to appear in Mathematical Fluid Mechanics, Springer Verlag, 2008.

[4] Cho, K., Cho, Y.I., and Park, N.A. , Journal of Non-Newtonian Fluid Mechanics, 45, 105-145, 1995.

[5] Chiba, K., Song, K., and Horikawa, Rheologica Acta, 25, 380-388, 1986.

[6] A. Chwang and Y. Wu, Hydromechanics of low-Reynolds-number flow. Part 2. Singularity method for Stokes flows, J. Fluid Mech. 19761, vol. 67(4), 787-815.

[7] Darwin, C., Proceedings of the Cambridge Philosophical Society, 49, 342-354, 1953.

[8] Eames, I., Belcher, S.E. and Hunt, J.C.R., Journal of Fluid Mechanics, 275, 201-223,1994.

[9] Galdi G.P., and Vaidya A., Journal of Mathematical Fluid Mechanics, 3, 183-211, 2001.

[10] Galdi G.P., Vaidya A., Pokorny, M., Joseph, D.D., and Feng, J., Mathematical Models and Methods in Applied Sciences, 12, 1653-1690, 2002.

[11] Joseph, D.D., and Liu, Y.J., Journal of Rheology, **37**, 961-983, 1993.

[12] Joseph, D.D., and Feng, J., Journal Non-Newtonian Fluid Mechanics, **64**, 299-302, 1996.

[13] Joseph, D.D., in Proceedings of the Fifth World Congress of Chemical Engineering, Particle Technology Track, 6, 3-16, 1996.

[14] Lawrence, C.J. and Weinbaum S., Journal of Fluid Mechanics, 171, 209-218, 1986.

[15] Liu, Y.J., and Joseph, D.D., Journal Fluid Mechanics, **255** 565-595, 1993.

[16] Vaidya,A., Japan Journal of Industrial and Applied Math, 21, No.3, 299-321, 2004.

[17] Massoudi,M., Vaidya A. and Wulandana R., Nonlinear Analysis: Real World Applications, 9 (1), 80-93, 2008.

[18] Massoudi M. and Vaidya A., accepted for publication in Nonlinear Analysis: Real World Applications, 9(3), 1169-1183, 2008.

[19] Vaidya, A., Applied Math. Letters, 18, 1332-1338, 2005.

[20] Vaidya A., Nonlinear Analysis: Real World Applications,7(4), 748-768, 2006.