133 **Quiz 1.** Name: ______________________________

1. (5 points) Prove by induction that the sum of the first n odd numbers is n^2, that is prove:

$$1+3+5+\ldots+(2n-1)=n^2, \forall n \geq 1$$

Base case : $n=1$, $1+\ldots 2(1-1)$ is just $1 = 1^2$ ✓.

Induction step : Assume Statement for n, and prove for $n+1$

we know $1+3+\cdots+(2n-1) = n^2$

the previous odd # to $2n+1$

therefore $1+3+\cdots + (2(n+1)-1) = [1+3+\cdots+(2n-1)] + (2n+1)$

$= (n^2+2n+1) = (n+1)^2$

using induction hypothesis. Therefore, the statement is true for $n+1$

2. (5 points) Apply the binomial theorem to give a formula for $(a+b)^4$

$(a+4)^4 = \binom{4}{0}a^4 + \binom{4}{1}a^3b + \binom{4}{2}a^2b^2 + \binom{4}{3}ab^3 + \binom{4}{4}b^4$

$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

because $\binom{4}{0} = \frac{4!}{4!0!} = 1$

$\binom{4}{1} = \frac{4!}{3!1!} = \frac{4\cdot\cancel{3\cdot2\cdot1}}{\cancel{3\cdot2\cdot1}\cdot1} = 4$

$\binom{4}{2} = \frac{4!}{2!2!} = \frac{\cancel{4}\cdot3\cdot2\cdot1}{\cancel{2}\cdot\cancel{2}} = 6$

$\binom{4}{3} = \frac{4!}{1!3!} = 4$

$\binom{4}{4} = \frac{4!}{4!0!} = 1$.

3. (7 points) Prove that the fourth power of any integer n^4 is of the form $5k$ or $5k+1$.

division with remainder

First note that if $n = 5k+r$, then

$n^4 = (5k+r)^4 = (5k)^4 + 4(5k)^3r + 6(5k)^2r^2 + 4(5k)r^3 + r^4$

$= 5m + r^4$ for some m $[= (5k)^3 + 4(5k)^2r + 6(5k)r^2 + 4r^3]$

Case $r=0$

$n^4 = 5m+0^4$ of the form $5m$

Case $r=1$, $n^4 = 5m+1^4 = 5m+1$ of the form $5(?)+1$

Case $r=2$, $n^4 = 5m+2^4 = 5m+16 = 5(m+3)+1$

$r=3$

$n^4 = 5m+3^4$

$= 5m+81$

$= 5(m+16)+1$

$r=4$

$n^4 = 5m+4^4$

$= 5m+256$

$= 5(m+51)+1$

in each case answer is $5k+1$.