133 **Quiz 2.** Name: ________________________

1. (7 points) Use the euclidean algorithm to
   (a) Find gcd(98, 26).
   (b) Find integers x and y such that $98x + 26y = \gcd(98, 26)$.

$98 = (3)26 + 20$

$26 = (1)20 + 6.$

$20 = (3)6 + 2.$

$6 = 3 \cdot 2.$

$\Rightarrow \underline{\gcd = 2}.$

$2 = \underline{20} - 3(\underline{6})$

$= 20 - 3(26 - 20) = 4 \cdot \underline{20} - 3 \cdot \underline{26}$

$= 4(98 - 3(26)) - 3(26)$

$= 4 \cdot \underline{98} - 15(\underline{26})$

$=$

2. (4 points)Show that if $\gcd(a, b) = 1$ and c divides a, then $\gcd(b, c) = 1$.

let $a = cz$ $z \in \mathbb{Z}$

if $d | b$ and $d | c$ $\Rightarrow$ $d | cz \Rightarrow d | a$.

so common divisors of b and c are also the common divisors of a, b which is $\{\pm 1\}$ [since $\gcd(a,b) = 1$].

hence $\gcd(b,c) = 1$

3. (4 points) Prove that 8 divides $3^{2n} + 7$ for all n (positive integer).

$n = 1$, $8 | 3^2 + 7$ ✓ $8 | 16$

Assume $8 | 3^{2n} + 7$

then $3^{2(n+1)} + 7 = 3^{2n} \cdot 3^2 + 7$

$= 9 \cdot 3^{2n} + 7 = 9(3^{2n} + 7) + 7 - 63$

$= 9(3^{2n} + 7) - 56$

But $8 | 56$ and $8 | (3^{2n} + 7) \Rightarrow 8 | 3^{2(n+1)} + 7$ as desired