133 **Quiz 3.** Name: ______________________________

1. (7 points) Find all positive integer solutions to the equation $18x + 5y = 48$.

$18 = 3 \cdot 5 + 3$

$5 = 3 + 2$

$3 = 2 + 1$

$2 = 2 \cdot 1$

$1 = 3 - 2 = 3 - (5 - 3)$
$= 2 \cdot 3 - 5$
$= 2 \cdot (18 - 3 \cdot 5) - 5$
$= 2 \cdot 18 - 7 \cdot 5$

$48 = (2 \cdot 48)(18) - (7 \cdot 48)y.$

$x_0 = 48 \cdot 2 = 96$

$y_0 = -7(48)$

all solutions $x = 96 - 5t$
$y = -7(48) + 18t.$

need $96 \geq 5t$ a $t \leq \frac{96}{5}$, $t \leq 19$.

$18t - 7(48) \geq 0$
a $t \geq \frac{7 \cdot \cancel{48}\,8}{\cancel{18}\,3} = \frac{56}{3}$ $18.\ldots$ so $\boxed{t = 19}$ only choice

$x = 96 - 5(19) = 1$, $y = 18(19) - 7(48) = 6.$

2. (4 points) If p is a prime number and $p|a^5$ prove that $p^5|a^5$. Is this statement true if p is not a prime number?

If $p | a^5 = a \cdot a \cdot a \cdot a \cdot a$ then p divides one the factors, all a so $p|a$

$\Rightarrow a = dp$
$\Rightarrow a^5 = d^5 p^5$
so $p^5 | a^5.$

3. (4 points) Show that for any integer a, $\gcd(9a + 13, 7a + 10) = 1$.

$\gcd(9a+13, 7a+10) = \gcd(9a+13 - 7a+10, 7a+10)$
$= \gcd(2a+3, 7a+10)$
$= \gcd(2a+3, (7a+10) - 3(2a+3))$
$= \gcd(2a+3, 1) = 1.$