133 **Quiz 5.** Name: ____________________

1. (3 points) Given that $x = 6$ is a solution for $6x \equiv 15 \pmod{21}$, determine all solutions to $6x \equiv 15 \pmod{21}$.

$X_0 = 6$, the other solution are $6 + \frac{21}{d}$, $6 + 2\cdot\frac{21}{d} + \cdots$ $6 + (d-1)\frac{21}{d}$.

where $d = \gcd(21, 6)$
$= 3$

so $6, 6+7, 6+14$ ie $X \equiv 6, 13, 20 \mod 21$.

2. (8 points) Solve the system of congruences using the Chinese remainder theorem:

$$x \equiv 2 \pmod{3}$$
$$x \equiv 3 \pmod{5}$$
$$x \equiv 5 \pmod{7}$$

First solve

$(5\cdot 7)y_1 \equiv 1 \mod 3 \iff y_1 \equiv 2 \mod 3 \equiv -1 \mod 3.$

$(3\cdot 7)y_2 \equiv 1 \mod 5 \iff y_2 \equiv 1 \mod 5.$

$(5\cdot 3)y_3 \equiv 1 \mod 7 \iff y_3 \equiv 1 \mod 7.$

$3\cdot 7 = 21 \equiv 1 \mod 5.$
$5\cdot 3 = 15 \equiv 1 \mod 7$
$5\cdot 7 = 35 \equiv -1 \mod 3$

Solutions to given system

$X \equiv (35y_1)\cdot 2 + (21y_2)\cdot 3 + (15)y_3\cdot 5.$

$\equiv -70 + 21\cdot 3 + 75.$

$= 5 + 63 \equiv 68 \mod (3\cdot 5\cdot 7$
$= 105).$

3. (4 points) Use Fermat's theorem to show that $5^{32} - 3$ is divisible by 11 .

By Fermat, $5^{10} \equiv 1 \mod 11$

so $5^{32} = 5^{30}\cdot 5^2 = (5^{10})^3 5^2$
$\equiv 1\cdot 25 \equiv 3 \mod 11.$

so $11 \mid 5^{32} - 3$.