133 **Quiz 7.** Name: ______________________

1. (~~6~~ 4 points) How many zeroes does 152! end in?

Highest power of 5 is $\left[\frac{152}{5}\right] + \left[\frac{152}{25}\right] + \left[\frac{152}{125}\right] = 30 + 6 + 1$
$= 37.$

Highest power of 2 is $\left[\frac{152}{2}\right] + \left[\frac{152}{4}\right] + \cdots + \left[\frac{152}{28}\right]$

$= 76 + \cdots > 37$

so $152! = 2^{37} 5^{37} \underbrace{\quad}_{\leftarrow \text{not div by } 10} = 10^{37} x$ $\quad 10 \nmid x$ so # zeroes = 37.

2. (~~6~~ 7 points) Find and prove a formula for

$$F(n) = \sum_{d|n} d^2 \mu(d)$$

in terms of the prime factorization of n. Use this formula to evaluate $F(450)$ [Hint: Carefully show that F is multiplicative, then determine $F(p^k)$ ....]

$f(n) = n^2 \mu(n)$ is a multiplicative function of n because it is a product of mult. functions $\Rightarrow F(n) = \sum_{d|n} f(d)$ is also mult.

now $F(p^k) = \sum_{d|p^k} d^2 \mu(d) = 1^2\mu(1) + p^2\mu(p) + \underbrace{(p^2)^2 \mu(p^2)}_{=0} \cdots$

$k \geq 1$ $\quad \leadsto \{1, p, p^2, \ldots\}$ $\quad = \mu(1) + p^2 \mu(p)$

$= 1 - p^2.$

$\Rightarrow$ if $n = p_1^{k_1} \cdots p_r^{k_r}$, $\quad F(n) = (1-p_1^2)(1-p_2^2) \cdots (1-p_r^2)$

$k_i > 0$

p_i distinct

(~~6~~ 4 pts).

$n = 450 = 50 \times 9$
$= 2 \cdot 25 \cdot 9 = 2 \cdot 5^2 \cdot 3^2$

$F(450) = (1-2^2)(1-3^2)(1-5^2) = (-3)(-8)(-24) = -(24 \times 24) = -576.$

3. If $n = 2^k 3^j$ with k, j positive, show that $\phi(n)$ divides n.

$\phi(n) = (2^k - 2^{k-1})(3^j - 3^{j-1})$

$= 2^{k-1}(2-1)\, 3^{j-1} \underbrace{(3-1)}_{2}$

$= 2^{k-1} \cdot 1 \cdot 3^{j-1} \cdot 2 = 2^k 3^{j-1}$ which divides $2^k 3^j$

$2^k 3^j = 3(2^k 3^{j-1}).$

$\begin{array}{r} 24 \\ \times 24 \\ \hline 96 \\ 48 \\ \hline 576 \end{array}$