133 **Quiz 4.** Name: ______________________

1. (6) points) Find the remainder when 7^7 is divided by 11 (without using a calculator). Give details.

$7 \equiv -4 \mod 11$

$7^2 \equiv 16 \equiv 5 \mod 11$

$7^3 \equiv 35 \equiv 2 \mod 11$

$(7^3)^2 \equiv 4 \mod 11$

$\Rightarrow 7^6 \equiv 4 \mod 11$

$7^7 \equiv 28 \mod 11$

$\equiv 6 \mod 11$ $\Rightarrow$ remainder $= 6$.

2. (6 points) Show that for any odd integer a, $a^2 \equiv 1 \pmod 8$ by considering each of the eight cases $a \equiv r \pmod 8$ where $r = 0, 1, 2, \ldots, 7$ (some of which are not possible because a is odd).

$a \equiv r \mod 8 \Rightarrow a = 8q + r \Rightarrow$ if a is odd, r is odd.

so cases

$$\left.\begin{array}{l} a \equiv 1 \mod 8 \Rightarrow a^2 \equiv 1 \mod 8 \\ a \equiv 3 \mod 8 \Rightarrow a^2 \equiv 9 \equiv 1 \mod 8 \\ a \equiv 5 \quad '' \quad \Rightarrow a^2 \equiv 25 \equiv 1 \quad '' \\ a \equiv 7 \mod \Rightarrow a^2 \equiv 49 \equiv 1 \mod 8. \end{array}\right\}.$$

3. (3 points) Show that if $a \equiv b \pmod n$ and $m|n$ (m divides n), then $a \equiv b \pmod m$.

Assume $a \equiv b \mod n$ and $m | n$

$\Rightarrow a - b = nq$ and $n = ml$ for integers l, q $\Rightarrow a - b = mlq$

$\Rightarrow m | a - b \Rightarrow a \equiv b \mod m$.