Write *network* for an undirected graph whose edges e have positive real edge-lengths $\ell(e)$. In a n-vertex connected network, the *distance* $D(i,j)$ between vertices i and j is the length of the shortest route between them. Assuming generic edge-lengths, the shortest route is unique. For each ordered (source, destination) pair of vertices (i,j), send flow of volume $1/n$ along the shortest route from i to j. (The normalization $1/n$ is arbitrary but convenient for (1) below). For each directed edge e (i.e. an edge e and a specified direction across e) of the network, let $f(e)$ be the total flow across the edge in that direction. Note

$$n^{-1} \sum_{\text{directed } e} f(e)\ell(e) = n^{-2} \sum_i \sum_j D(i,j) := \bar{D} \tag{1}$$

where $\bar{D}$ is the average vertex-vertex distance.

One can formulate a project to study the distribution of such edge-flows $f(e)$ in different models of random n-vertex networks. Such models include both deterministic graphs to which random edge-lengths are assigned, and random graphs of both the classical Erdős - Rényi or random regular type [2] and the more recent *complex networks* types [1, 3, 4, 5] again with real edge-lengths attached.

We consider the complete n-vertex graph whose edge-lengths are independent exponentially distributed random variables. Simultaneously for each pair of vertices, put a constant flow between them along the shortest path. Each edge gets some random total flow. In the $n \to \infty$ limit we find explicitly the empirical distribution of these edge-flows, suitably normalized.

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