Motivated by Levys characterization of Brownian motion on the line, we propose an analogue of Brownian motion that has as its state space an arbitrary closed subset of the line that is unbounded above and below: such a process will be a martingale, will have the identity function as its quadratic variation process, and will be continuous in the sense that its sample paths dont skip over points. We show that there is a unique such process, which turns out to be automatically a reversible Feller-Dynkin Markov process. We find its generator, which is a natural generalization of the operator $f \to \frac{1}{2} f''$.

We then consider the special case where the state space is the self-similar set $\{q^k : k \in \mathbf{Z}\} \cup \{0\}$ for some $q > 1$. Using the scaling properties of the process, we represent the Laplace transforms of various hitting times as certain continued fractions that appear in Ramanujans lost notebook and evaluate these continued fractions in terms of basic hypergeometric functions (that is, q-analogues of classical hypergeometric functions). The process has 0 as a regular instantaneous point, and hence its sample paths can be decomposed into a Poisson process of excursions from 0 using the associated continuous local time. Using the reversibility of the process with respect to the natural measure on the state space, we find the entrance laws of the corresponding Ito excursion measure and the Laplace exponent of the inverse local time both again in terms of basic hypergeometric functions. By combining these ingredients, we obtain explicit formulae for the resolvent of the process. We also compute the moments of the process in closed form.