Risk Aversion and Asymmetry in Procurement auctions: Identification, Estimation and Application to Construction Procurements.*

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Abstract

This article studies a model of asymmetric risk averse bidding within the independent private value paradigm. The inherent asymmetry in cost and risk aversion imposes an original restriction on the observed bidding, an exact equality which leads to the primitives semiparametric identification and estimation. The unobserved arguments of this equality need to be simulated in order to estimate the bidders' Constant Relative Risk Aversion or Constant Absolute Risk Aversion parameters and their heterogenous cost distributions. In the Los Angeles City Hall construction contracts offered between 1994 and 2003, the model and methodology help reveal that financial asymmetries affect the firms' cost distribution, while experience influences their risk aversion behavior .

Fields: first-price auctions, independent private values, asymmetric risk aversion, semiparametric identification and estimation.

JEL classification: C14, C15, D44, L50, L74.

1 Introduction

The empirical auction literature often attributes asymmetric bidding to diverse cost functions (cf. Hendricks and Porter (1992), Porter and Zona (1993), Bajari (1997), Flambard and Perrigne (2000), Campo *et al.* (2003), Jofre-Bonet and Pesendorfer (2003)). This article studies a less investigated factor, risk aversion, and the problem of distinguishing its effect from other sources of asymmetry. In procurement auctions, financially constrained firms may face higher production costs and bid less competitively; however, more risk averse bidders may lower their mark-up to improve their winning odds.

In experiments where bidders share the same cost distribution, Cox *et al.* (1985) and, more recently, Goeree *et al.* (2002) recover the bidders' different attitudes toward risk. In timber auctions, Athey and Levin (2001) show how loggers spread the risk by splitting their bids across different species in a same lot. This evidence suggests that risk aversion and its diversity cannot be ignored in auction models, especially since cost asymmetries may have conflicting effects on a firm's bidding.

Such asymmetries create a unique environment for the resolution of identification and estimation issues. Within models of symmetric risk averse bidders, identification of a single risk aversion parameter is possible after imposing some parametric quantile restriction on the valuation distribution (see Campo *et al.* (2009)). With any added heterogeneity, one would expect that identification becomes more difficult. This preconception is wrong. This article shows that asymmetry helps recover the unique structure of the game, in all its diversity, if bidders share the same characteristics value along (at least) one dimension. The following sections also introduce a new estimation procedure, which relies on an exact equality derived from the model properties. This equation holds for pair of bids that cannot be observed in the data, but may be simulated. Then the exact equality becomes a regression equation where both the left- and right-hand side are estimated in an earlier step.

Section 1 defines the asymmetric risk averse model. Section 2 details the model main restrictions on the observations. These are the foundations of the identification result and the estimation procedure. Section 3 shows that the model is semiparametrically identified without imposing any additional restrictions on the model. Section 4 presents the new estimation procedure based on the approximation of an unobservable equality. Section 5 applies these results to the Los Angeles City Hall procurement contracts data.

2 The Model

I firms compete for a contract in a first-price sealed bid auction. They submit simultaneous bids, $\{b_i\}_{i=1}^I$, $I \geq 2$, in sealed envelopes. The low bidder wins the contract, and is paid the amount he bid.

Every firm i draws an independent cost c_i from the cumulative distribution $F_i(\cdot)$ which may differ from $F_j(\cdot)$, for $i \neq j = 1, \cdots, I$. As such, the cost distributions may depend on the firm's size (number of employees, assets holdings,...), distance from the project, and other physical and financial constraints. They are absolutely continuous on $[\underline{c}, \overline{c}] \subset I\!R^+$, with respective densities $f_i(\cdot)$, for $i = 1, \cdots, I$.

The firms may also be risk averse. The von Neuman Morgenstern (vNM) utility function, $U_i(\cdot)$, satisfies $U_i'(\cdot) > 0$ and $U_i''(\cdot) \leq 0$, $\forall i = 1, \cdots, I$. For a cost c_i and a bid b_i, bidder i enjoys a utility $U_i(b_i - c_i)$ if he wins, $U_i(0) = 0$ otherwise. Because firms may hold a variety of assets or have survived longer in the industry, they may exhibit different attitudes toward risk. Then, for at least one pair of bidders (i, j) $(i \neq j = 1, \cdots, I)$, we have $U_i(\cdot) \neq U_j(\cdot)$, where $U_i(\cdot)$ and $U_j(\cdot)$ belong to the same parametric family of functions but their measures of risk-aversion differ. For example, if agents i and j exhibit Constant Relative Risk Averse (CRRA) utility functions, $U_k(x) = x^{\theta_k}$, for $k = i, j$ and $i \neq j$, bidders are asymmetric firms if $\theta_i \neq \theta_j$.

While the costs are private information, the bidders' cost distributions $\{F_i(\cdot)\}_{i=1}^I$, utility functions $\{U_i(\cdot)\}_{i=1}^I$, and the number of participants I are common knowledge[1].

[1]The common knowledge assumption is more likely to hold either in small industries (where the

Firm i seeks to maximize the payoff from the auction

$$\begin{aligned} E_i(U_i) &= U_i(b_i - c_i)\Pr(b_i \le b_j, \forall j \neq i) \\ &= U_i(b_i - c_i) \prod_{j=1, j\neq i}^{I} \left[1 - F_j(s_j^{-1}(b_i))\right] \end{aligned}$$

by choosing the optimal bid $b_i = s_i(c_i)$, where $s_i(\cdot)$ is bidder i's equilibrium bidding strategy, for $i = 1, \cdots, I$. It is strictly increasing on $[\underline{c}, \overline{c}]$, and expresses the equilibrium bid as a function of bidder i's cost, the cost distribution functions $\{F_i(\cdot)\}_{i=1}^{I}$, the number of bidders I, and the bidders' utility functions $\{U_i(\cdot)\}_{i=1}^{I}$. The inverse bidding strategy $s_i^{-1}(\cdot)$ is defined on the support $[\underline{b}, \overline{b}]$.

After some rewriting, the Bayesian-Nash optimal bidding strategy $s_i(\cdot)$ satisfies the first-order condition:

$$1 = \frac{U_i(b_i - c_i)}{U_i'(b_i - c_i)} \sum_{j=1, j\neq i}^{I} \frac{f_j(s_j^{-1}(b_i))}{1 - F_j(s_j^{-1}(b_i))} \frac{1}{s_j'(s_j^{-1}(b_i))}. \tag{1}$$

with the initial conditions $s_i(\overline{c}) = \overline{c}$, and $s_i(\underline{c}) = s_j(\underline{c})$, for $i \neq j = 1, \cdots, I$, where $s_i(c) = b_i$, $i = 1, \cdots, I$ for $c \in [\underline{c}, \overline{c}]$[2].

The differential equation in $s_i(\cdot)$ does not have any analytical solution except under some limited parametric specifications. Guerre *et al.* (2000) propose to study the model identification and estimation procedure without solving explicitly for the equilibrium strategies. We choose to apply their "indirect approach" to derive the restrictions imposed by the model assumptions on the data and use these to facilitate the identification and estimation procedure.

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number of competitors is small) or in industries where firms organize themselves into associations, as in the construction industry. In both cases, we have described industries where firms know their competitors.

[2] We assume that the second-order conditions are automatically satisfied.

3 The Model Restrictions

If a theorist seeks to characterize the equilibrium existence and uniqueness conditions given the model assumptions, an applied economist follows the same reasoning backwards to find the existence and uniqueness of primitives to explain the observed equilibrium. The idea is that, because the model may impose unique predictions and properties on the observations, it may not fit every data set. This section discusses the restrictions that the asymmetric model structure imposes on the observations.

The asymmetric risk averse model is defined by $\left[\{F_i(\cdot), U_i(\cdot)\}_{i=1}^I\right]$. Whenever useful, we will use a slightly different notation where $F_i(\cdot) \equiv F(\cdot|x_i)$ and $U_i \equiv U(\cdot|w_i)$ for $x_i \in \mathcal{X}$ and $w_i \in \mathcal{W}$, bidder i's characteristics such that $\dim(\mathcal{W}) > \dim(\mathcal{X})$. Then the bid distribution may be denoted as $G_i(\cdot) \equiv G(\cdot|x_i, w_i)$, for $i = 1, \cdots, I$.

The first-order conditions in (1) define the observables $\{b_i\}_{i=1}^I$ as a function of the unobservables $b_i = s_i(c_i, \{U_i(\cdot), F_i(\cdot)\}_{i=1}^I)$. Under some parametric assumptions, $s_i(\cdots)$ may have a close form solution and help identify the primitives of our model. Guerre *et al.* (2000) prove that such restrictions are not needed to identify the cost distribution in a symmetric model with risk neutral bidders. In a model of asymmetric risk aversion, the same technique leads to an original identification result and suggests a new estimation procedure.

The F.O.C.s may be rewritten as a function of arguments which are either observed (the bids) or estimated (the bid distribution functions). Because $G_i(b) = F_i(s_i^{-1}(b, \cdots))$, equation (1) becomes

$$1 = \lambda_i(b_i - c_i) \sum_{j \neq i}^{I} \frac{g_j(b_i)}{1 - G_j(b_i)}, \tag{2}$$

where

$$\lambda_i(b_i - c_i) = \frac{U_i(b_i - c_i)}{U_i'(b_i - c_i)}.$$

Then bidder i's inverse bidding strategy, $\xi_i(\cdots)$, satisfies

$$c_i \;=\; \xi_i(b_i, \{U_i(\cdot), G_i(\cdot)\}_{i=1}^I, I) = b_i - \lambda_i^{-1}\left(1/Y_i(b_i)\right), \tag{3}$$

where $Y_i(b) = \sum_{j\neq i}^{I} \frac{g_j(b_i)}{1-G_j(b_i)}$, for $i = 1, \cdots, I$.

Equations (1) and (2) create a dual definition of the model and create a relationship between the firms' optimal bidding and their primitives. It follows that any assumptions made on $\left\{\{F_i(\cdot), U_i(\cdot)\}_{i=1}^{I}\right\}$ in (1) translate into conditions on the observations in equation (2). To express such constraints, we first need to lay out a few assumptions. First, three definitions:

Definition 1: Let $\mathcal{U}$ be the set of utility functions $U_i(\cdot)$ satisfying
(i) $U_i : [0, +\infty) \to I\!R^+$, $U_i(0) = 0$ and $U_i(1) = 1$.
(ii) $U_i(\cdot)$ is continuous on $[0, +\infty)$, and admits 2 continuous derivatives on $[0, +\infty)$ with $U_i'(\cdot) > 0$ and $U_i''(\cdot) \leq 0$ on $(0, +\infty)$.
(iii) $\lim_{x\to 0^+} \lambda_i'(x)$ is finite.

Note that condition (i) is a normalization of the agents' utility space. It implies that the bidders' utility in the case of a loss is equal to zero. Some may argue that agents may be differently affected by their gain or loss if they enjoy different initial wealth holdings (Gollier (2001)).[3] We do not consider this issue here. Popular risk-averse utility functions, such as CRRA and constant absolute risk averse (CARA) utility functions, satisfy definition 1.

Definition 2: Let $\mathcal{F}$ be the set of distributions $F_i(\cdot)$, $i = 1 \cdots, I$, such that
(i) $F_i(\cdot)$ is a c.d.f. defined over the support $[\underline{c}, \overline{c}]$, where $0 \leq \underline{c} < \overline{c} < \infty$.
(ii) $f_i(\cdot)$ is continuous on $[\underline{c}, \overline{c}]^I$.
(iii) $f_i(\cdot) > 0$ on $[\underline{c}, \overline{c}]$.

Within the Independent Private Value model, Athey (2001) shows that, under definitions

[3]Different wealth holdings may also lead to heterogenous bid distributions even if the cost distribution is homogenous. In a CRRA model, the first-order condition is a non-linear function in both the wealth and risk aversion parameters, which cannot be simplified. In a CARA model, the wealth holdings disappear from the first-order condition, and only affects the lower bound of the bid support. In such context, the identification result and estimation procedure developed in this paper cannot be applied.

1 and 2, the primitives satisfy the log-supermodularity and the single crossing of incremental returns properties. As such, an equilibrium exists and the agents' bid distributions satisfy:

Definition 3: Let $\mathcal{G}$ be the set of distributions $G_i(\cdot)$, for $i = 1, \cdots, I$, such that
(*i*) $G_i(\cdot)$ is a c.d.f. with support of the form $[\underline{b}, \bar{b}]$, where $0 \leq \underline{b} < \bar{b}$.
(*ii*) $G_i(\cdot)$ admits two continuous first derivative on $[\underline{b}, \bar{b}]$.
(*iii*) $g_i(\cdot) > 0$ on $[\underline{b}, \bar{b}]$.
(*iv*) $\lim_{b \to \bar{b}_+} d[1/Y_i(b_i)]/db = 0$.

These conditions result from definitions 1 and 2 and the fact that $b_i = s_i(c_i, \{U_i(\cdot), F_i(\cdot)\}_{i=1}^I)$ for $i = 1, \cdots, I$. Using definitions 1 through 3, Lemma 1 gathers the restrictions that any data set, candidate for the model estimation, should satisfy.

Lemma 1: Let $I \geq 2$ and $G(\cdots|\{x\}_{i=1}^I, \{w\}_{i=1}^I)$ be the distribution of $(b_1, \cdots, b_I)$. Then $G(\cdots|\{x\}_{i=1}^I, \{w\}_{i=1}^I)$ is rationalized by an Independent Private Value model with asymmetric risk averse agents $[\{U(\cdot|w_i), F(\cdot|x_i)\}_{i=1}^I] \in \mathcal{U}^I \times \mathcal{F}^I$ if and only if
(*i*) $G(b_1, \cdots, b_I|\{x\}_{i=1}^I, \{w\}_{i=1}^I) = \prod_{i=1}^I G(b_i|x_i, w_i) \equiv \prod_{i=1}^I G_i(b_i)$,
(*ii*) $G(\cdot|x_i, w_i) \equiv G_i(\cdot) \in \mathcal{G}$, for $i = 1, \cdots, I$,
(*iii*) $\forall (b_i^\alpha, b_j^\alpha)$ such that $x_i = x_j$ and $G(b_i^\alpha|x_i, w_i) = G(b_j^\alpha|x_j, w_j) = \alpha, \alpha \in [0, 1)$,
$\exists \; \{\lambda(\cdot|w_i) \equiv \lambda_i(\cdot)\}_{i=1}^I$ satisfying:

(a) $\lambda(\cdot|w_i) : I\!R_+ \to I\!R_+$ with one continuous derivative on $(0, +\infty]$, $\lambda(0|w_i) = 0$, $\lambda'(\cdot|w_i) \geq 1$, such that $\partial \xi(\cdots|x_i, w_i)/\partial b_i \geq 0$ for $\xi(b_i^\alpha, U_i, G, I|x_i, w_i) = b_i^\alpha - \lambda^{-1}[1/Y_i(b_i^\alpha)|\, w_i]$ and

(b) $b_i^\alpha - b_j^\alpha = \lambda^{-1}[1/Y_i(b_i^\alpha)|w_i] - \lambda^{-1}\left[1/Y_j(b_j^\alpha)|w_j\right]$.

These conditions are similar to those which apply to models with either risk neutral bidders and asymmetric valuation distributions models or symmetric risk averse bidders with independent private values (Campo *et al.* (2009)). Either model can explain individual bid distributions, $G_i(\cdot)$, $i = 1, \cdots, I$, which satisfy lemma 1 conditions (*i*) through (*iii*)(a). However, this model of asymmetric risk averse bidders imposes a distinctive restriction on the bid distribution, condition (iii)(b). If two bidders share the same cost characteristic x but exhibit different attitudes toward risk, they may draw the same cost but submit different bids. For example, suppose that two firms, 1 and 2, are characterized by $x_1 = x_2$ and $w_1 \neq w_2$. They happen to draw the same cost c, but bid b_1 and b_2, such that $b_1 \neq b_2$. Such bids satisfy $s_1(c, \cdots) = b_1$, $s_2(c, \cdots) = b_2$ and $F(c|x_1 = x_2) = G(b_1|x_1, w_1) = G(b_2|x_2, w_2)$, where $F(\cdot|x_1 = x_2)$ is the cost distribution, and $G(\cdot|x_i, w_i) \equiv G_i(\cdot)$ is bidder i's bid distribution, for $i = 1, 2$. Because their bidding strategies, $\{\xi_i(\cdots) = \xi(\cdots|x_i, w_i)\}_{i=1}^2$, may be defined as

$$\begin{aligned} \xi_1(b_1, \{U(\cdot|w_i), G(\cdot|x_i, w_i)\}_{i=1}^2, I = 2) &= c = b_1 - \lambda_1^{-1}\left(1/Y_1(b_1)\right), \text{and} \\ \xi_2(b_2, \{U(\cdot|w_i), G(\cdot|x_i, w_i)\}_{i=1}^2, I = 2) &= c = b_2 - \lambda_2^{-1}\left(1/Y_2(b_2)\right), \end{aligned}$$

for a cost c, one can subtract the first equation from the second and get

$$b_1 - b_2 = \lambda_1^{-1}\left[1/Y_1(b_1)\right] - \lambda_2^{-1}\left[1/Y_2(b_2)\right],$$

i.e. condition $(iii)b$.

The auction theory defines the Bayesian Nash equilibrium in $(b_1 = s_1(\cdot), \cdots, b_I = s_I(\cdot))$, and imposes conditions on the model primitives to derive the optimal bidding strategy properties and other comparative statics results. Lemma 1 gives an alternative definition of the equilibrium, which can now be described as the vector of increasing functions $(\xi_1(\cdots), \cdots, \xi_I(\cdots))$, which satisfy

$$\left\{ \begin{aligned} \xi_i(b_i^\alpha, \{U_i, G_i\}_{i=1}^I, I) &= b_i^\alpha - \lambda_i^{-1}\left(1/Y_i(b_i^\alpha)\right) \\ b_i^\alpha - b_j^\alpha &= \lambda_i^{-1}\left[1/Y_i(b_i^\alpha)\right] - \lambda_j^{-1}\left[1/Y_j(b_j^\alpha)\right] \end{aligned} \right. \tag{4}$$

for $i \neq j = 1, \cdots, I$ such that $x_i = x_j$ and $\alpha \in [0, 1)$. This system defines the equilibrium in terms of the inverse bidding strategies $\xi_i(\cdots)$, $i = 1, \cdots, I$. It highlights one crucial property of the model: bidders i and j share the same cost distribution whenever $x_i = x_j$. In a model of symmetric or even asymmetric private value distributions, the first equation in system (4) defines the equilibrium in terms of the inverse bidding strategies. The second equation distinguishes the asymmetric risk averse model from other competing frameworks. It restricts the set of distributions that may be explained by the model primitives. The following example illustrates my point. Suppose that two types of bidders, 1 and 2, exhibit CRRA utility functions, $U_i(x) = x^{\theta_i}$, where $\theta_i \in (0,1]$ for $i = 1, 2$ and $\theta_1 \neq \theta_2$. There are n_i bidders of type i such that $I = n_1 + n_2 \geq 2$. Their observed distribution functions are: $G_1(b) = b$, for $b \in [0,1]$ and $G_2(b) = (5/8)b$, for $b \in [0, 4/5]$, $G_2(b) = -3/2 + 5/2b$ for $b \in [4/5, 1]$. Then, for $b_1 = 2/3$ and $b_2 = 13/15$, we have $G_1(b_1) = G_2(b_2) = 2/3$. We hereby show that such distributions cannot be explained by a model of asymmetric risk averse bidders as defined in section 2.

Given $G_1(\cdot)$ and $G_2(\cdot)$, condition (iii)(b) becomes

$$\begin{aligned} \frac{2}{3} - \frac{\theta_1}{3(n_1-1)+\frac{15}{7}n_2} &= \frac{13}{15} - \frac{2\theta_2}{15(n_2+n_1-1)}, \\ \frac{3}{15} + \frac{7\theta_1}{21(n_1-1)+15n_2} &= \frac{2\theta_2}{15(n_1+n_2-1)}, \end{aligned}$$

where, when solving for θ_2, one gets

$$\frac{[3(n_1+n_2-1)][21(n_1-1)+15n_2] + 7 \times 15(n_1+n_2-1)\theta_1}{2[21(n_1-1)+15n_2]}.$$

Since n_1 and n_2 satisfy $n_1 + n_2 \geq 2$, θ_2 is greater than one, which contradicts the assumption $\theta_2 \in (0,1]$. Condition (iii)(b)is violated for at least one pair of bids (b_1, b_2). This example proves that lemma 1 defines a new and relevant restriction on the observed bid distributions in (iii)b. Athey and Haile (2007) give yet another example of how the condition may exclude some bid distribution functions from explaining the observations. They show that, if $G_1(b) = b$ and $G_2(b) = b^{\gamma}$ ($\gamma \in I\!R$), condition (iii)b holds if and only if $\gamma = 1$: bidders are homogenous. Along the same lines, Parreiras (2007) shows that the same condition is violated whenever the bidders' risk aversion parameters are too spread out when one assumes a common cost support.

The remaining sections are built around condition (iii). Its distinctive role in defining the asymmetric risk averse model explains its key role in defining the model identification and estimation procedure. We may refer to it as the "quantile" equality in the remainder of the paper.

4 Identification

Identification is an interesting issue in auction models, in part because, unless one agrees to make restrictive parametric assumptions on the model primitives (utility functions and cost distribution functions), the bidding strategy, $b = s_i(\cdot)$, for $i = 1, \cdots, I$, does not have an explicit functional form. Identification may still be achieved by analyzing the model restrictions on the data. In this model, the quantile equality leads to the desired result for common CRRA and CARA utility functions, without the need to impose any other assumption (in particular on the cost distribution). Hereafter, we show that there do not exist two structures $[\{U_i(\cdot), F_i(\cdot)\}_{i=1}^I]$ and $[\{\tilde{U}_i(\cdot), \tilde{F}_i(\cdot)\}_{i=1}^I]$- where the utility functions are either all CARA or all CRRA- which may explain the observed bids (or bid distribution).

Identification proceeds by steps. First, we show that the bidders' utility functions parameters are unique, then, given the newly identified parameters, one can prove the existence of unique cost distributions $\{F_i(\cdot)\}_{i=1}^I$ using results derived in previous articles (for example, in Campo *et al.* (2003)).

Guerre *et al.* (2000), Li *et al.* (2002) and Athey and Haile (2002) show that identification is possible under different cost distribution assumptions (Independent and Affiliated Private Value, (I.P.V. and A.P.V)) when bidders are risk neutral bidders, same utility function), but remain unsolved under the common value model (see Laffont and Vuong (2001)). In an I.P.V. model, Campo *et al.* (2009) prove that one may recover a unique structure $(U(\cdot), F(\cdot))$ for symmetric risk averse bidders under some mild assumptions on the valuation distribution quantiles. In this paper, we show that identification is actually made easier by exploiting the agents' asymmetry. The common perception is that identification gets more complex after adding more parameters or dimensions to a model specification. Here, it is the opposite. The bidders' diverse utility functions are more easily recovered because they change against a common background, the agents' cost distribution (or whenever bidders share the same cost characteristics).

Identification is possible whether bidders exhibit a CRRA or CARA utility function. The utility functions are respectively defined by $U_i(x) = x^{\theta_i}$, where $\theta_i \in (0,1]$, and $U_i(x) = a_{1i}\left[1 - \exp(-\beta_i x)\right]$, where $a_{1i} > 0$ and $\beta_i \geq 0$, for $i = 1, \cdots, I$ (Pratt(1964)). Both specifications share the same appealing feature. One parameter gives the measure of the bidders' risk aversion: $U_i''(x)x/U_i'(x) = 1 - \theta_i$ in the CRRA model and $-U_i''(x)/U_i'(x) = \beta_i$ in the CARA model. The bidders' inverse bidding strategies and condition $(iii)b$ also simplify under the two specifications. In the CRRA model, bidder i's inverse bidding strategy is

$$\xi_i(b_i) = b_i - \frac{\theta_i}{Y_i(b_i)}, \tag{5}$$

and condition $(iii)b$,

$$b_1^\alpha - b_2^\alpha = \begin{bmatrix} \frac{1}{Y_1(b_1^\alpha} & \frac{-1}{Y_2(b_2^\alpha} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}.$$

In the CARA model, the same equations become

$$\xi_i(b_i) = b_i - \frac{1}{\beta_i} \log\left[\frac{\beta_i + Y_i(b)}{Y_i(b)}\right] \tag{6}$$

and

$$b_i^\alpha - b_j^\alpha = \frac{1}{\beta_i} \log\left[\frac{\beta_i + Y_i(b_i^\alpha)}{Y_i(b_i^\alpha)}\right] - \frac{1}{\beta_j} \log\left[\frac{\beta_j + Y_j(b_j^\alpha)}{Y_j(b_j^\alpha)}\right], \tag{7}$$

respectively.

Lemma 2: The independent asymmetric cost distribution function model with either constant absolute or constant relative asymmetric risk averse bidders is identified.

The identification result in both the CRRA and CARA model relies on condition $(iii)b$, which states that for a common cost draw c and a same characteristic x,

$$b_i^\alpha - b_j^\alpha = \lambda_i^{-1}(1/Y_i(b_i^\alpha); \gamma_{-i}) - \lambda_j(1/Y_j(b_j^\alpha; \gamma_{-j})), \tag{8}$$

where (γ_i, γ_{-i}) is the bidders' vector of utility parameters. Suppose we have a two-bidder auction (the same reasoning may apply to a I-bidder auction), the equation depends then on two unknowns (γ_i, γ_{-i}) since everything else is either observed or estimated.

Identification is not yet possible based on equation (8) alone. Suppose that these two bidders draw again a same cost $\tilde{c} \neq c$ then

$$b_i^{\tilde{\alpha}} - b_j^{\tilde{\alpha}} = \lambda_i^{-1}(1/Y_i(b_i^{\tilde{\alpha}};\gamma_{-i}) - \lambda_j(1/Y(b_j^{\tilde{\alpha}};\gamma_{-j})), \tag{9}$$

where $F(\tilde{c}|x_i = x_{-i} = x) = \tilde{\alpha}$. Then equations (8) and (9) form a system of two equations in two unknowns. Under our specifications, a unique pair of parameters satisfies both equations because the ratio of parameters θ_i/θ_j (in the CRRA), β_i/β_j (in the CARA), where $i \neq j$ but $x_i = x_j$, stay unchanged across auctions. A rigorous proof may be found in the appendix.

Thus it is possible to find a unique structure $[\{U_i(\cdot), F_i(\cdot)\}_{i=1}^I]$ because the bidders' taste for risk depends on a characteristic w, which does not affect their cost distribution $F(\cdot|x)$. Condition (iii), which formalizes this idea, defines also a new strategy of estimation as explained in the next section.

5 Estimation Procedure

The purpose here is to semiparametrically estimate the model primitives, $(\{F_i(\cdot)\}_{i=1}^I, \{U(\cdot;\gamma_i)\}_{i=1}^I)$, where γ_i equals either β_i or θ_i depending on the chosen specification (either CARA or CRRA). As mentioned earlier, the estimation steps rely on the identifying restriction $(iii)b$ in lemma 1.

Assume firm i's cost distribution, $F_i(\cdot) \equiv F(\cdot|z_\ell, x_i)$ is a function of contract ℓ and firm i's characteristics, denoted $z_\ell \in \mathcal{Z}$ and $x_i \in \mathcal{X}$ respectively, for $\ell = 1, \cdots, L$ and $i = 1, \cdots, I$. Firm i's coefficient of risk aversion $\gamma_i \equiv \gamma(w_i)$ depends on its characteristics $w_i \in \mathcal{W}$. The firms' cumulative bid distributions are denoted by $G_i(\cdot) \equiv G(\cdot|x_i, w_i, z_\ell)$, for $i = 1, \cdots, I$.

The estimation steps are:

Step 1. Estimate $G(\cdot|x_i, w_i, z_\ell)$ and the densities $g(\cdot|x_i, w_i, z_\ell)$. The estimates are denoted by $\hat{G}_i(\cdot)$ and $\hat{g}_i(\cdot)$ respectively, for $i = 1, \cdots, I$. Estimate also $Y_i(\cdot)$ as $\widehat{Y}(\cdot|x_i, w_i, z_\ell) = \sum_{j \neq i} \hat{g}_j(\cdot|x_i, w_i, z_\ell)/(1 - \widehat{G}_j(\cdot|x_i, w_i, z_\ell))$, for all $i \neq j = 1, \cdots, I$ such that $x_i = x_j$.

Step 2. Recover the pair of bids $(\hat{b}^{\alpha}_{i\ell}, \hat{b}^{\alpha}_{j\ell})$ such that $\widehat{G}_i(\hat{b}^{\alpha}_{i\ell}) = \widehat{G}_j(\hat{b}^{\alpha}_{j\ell}) = \alpha$, and $x_i = x_j$, for $i \neq j = 1, \cdots, I$, where $\hat{b}^{\alpha}_{i\ell} \equiv \hat{b}_i(\alpha, z_\ell)$.

Step 3. Replace estimates from step 1 and 2 into condition $(iii)b$. Since the left- and right-hand side variables are estimated, the condition cannot hold exactly unless one adds a residual term such that

$$\hat{b}^{\alpha}_{1\ell} - \hat{b}^{\alpha}_{2\ell} = \lambda_1^{-1}\left[1/\widehat{Y}_1(\hat{b}^{\alpha}_{1\ell})\right] - \lambda_2^{-1}\left[1/\widehat{Y}_2(\hat{b}^{\alpha}_{2\ell})\right] + \epsilon_{ij\ell}, \tag{10}$$

where the expected value of ϵ conditional on the estimates is zero. One may apply the GMM to the above regression to estimate the parameters of risk aversion. Instrumental variables may be added to improve the estimates efficiency.[4]

Step 4. Firm i's cost distribution may be either parametrically or nonparametrically estimated (the dimensionality curse may limit the analysis to parametric estimations), by computing the so called firm's "pseudo" costs. They are defined in equations (5) and (6), for contracts $\ell = 1, \cdots, L$, and bidders $i = 1, \cdots, I$. The estimated cost distribution $\widehat{F}(\cdot|x_i, z_\ell)$ is the empirical distribution of the pseudo costs $\hat{c}_i$. Guerre *et al.* (2000) show that $\widehat{F}(\cdot|x_i, z_\ell)$ is consistent for all $i = 1, \cdots, I$, even for a nonparametric specification.

5.1 The risk aversion estimation

The bid distribution:

As in previous articles (Laffont et al. (1995), Hong and Shum (2002)), we assume that bids follow a log-normal distribution $\mathcal{N}(\mu_i, \sigma_i)$, $\forall i = 1, \cdots, I$. We denote the conditional cumulative distribution of $B_i = \log(b)$ by $H_i(\cdot|\cdot)$, and its respective density by $h_i(\cdot|\cdot)$. The distribution mean $\mu_i(z) = d_0 + d_1 z_\ell + d_2 x_i$ is linear in the contract ℓ and bidder i's characteristics for $\ell = 1, \cdots, L$ and $i = 1, \cdots, I$. We estimate (d_0, d_1, d_2) using the

[4] In situations where one may use a different set of regressors to estimate the left- and right-hand side variables in (10), the LHS residuals may be correlated with the RHS regressors. Instrumental variables should then be used to solve for any endogeneity issue.

maximum likelihood method.[5] From the bid distribution estimates, we construct Step 2's identifying condition.

The identifying condition:

For illustrative purposes, we consider the CRRA model where bidders are heterogenous along two dimensions: their risk aversion parameter θ, and a one dimensional variable X_1 which affect the bidders' cost distribution but not their attitude toward risk. The same logic applies to the CARA model where one needs to apply nonlinear estimation procedures.

Within the CRRA specification, the identification condition states that

$$b_i^\alpha(z_\ell) - \frac{\theta_i}{Y_i(b_i^\alpha(z_\ell))} = b_j^\alpha(z_\ell) - \frac{\theta_j}{Y_j(b_j^\alpha(z_\ell))},$$

holding for all the pairs of bids $(b_i^\alpha(z), b_j^\alpha(z))$ such that, for $i \neq j$, $G_i(b_i^\alpha(z_\ell)|Z = z_\ell, X_1 = x) = G_j(b_j^\alpha(z_\ell)|Z = z_\ell, X_1 = x) = \alpha$, where bidder i and j differ in their risk aversion parameter but share the same characteristic $X_1 = x$. Given the bids log-normal distribution $H(\cdot|\cdot)$, the equality becomes

$$\exp(B_i^\alpha) - \exp(B_j^\alpha) = \frac{\theta_i}{Y_{Hi}(B_i)} - \frac{\theta_j}{Y_{Hj}(B_j)}, \tag{11}$$

where $Y_{Hi}(B_i^\alpha)$ stands for the L-vector whose ℓ^{th} element is

$$\sum_{j\neq i} \frac{h_j(B_{i\ell}^\alpha|Z = z_\ell)}{1 - H_j(B_{i\ell}^\alpha|Z = z_\ell)} \frac{1}{\exp(B_{i\ell}^\alpha)},$$

and $(B_i^\alpha(z), B_j^\alpha(z))$ satisfy $H_i(B_i^\alpha|z_\ell) = H_j(B_j^\alpha|z_\ell)$, where i and j share the same $X_1 = x$.

Unfortunately, we do not observe pairs of bids $(B_i^\alpha(z_\ell), B_j^\alpha(z_\ell))$ satisfying this equality for any given auction $\ell = 1, \cdots, L$. To circumvent this problem, we estimate $(\hat{B}_i^\alpha(z_\ell), \hat{B}_j^\alpha(z_\ell))$,

[5] One may define truncation points to the bid distribution such that $H(\cdot|\cdot)$ is defined over $[\underline{B}, \overline{B}]$. The distribution lower and upper bounds may be estimated using Korostelev and Tsybakov (1993)'s trapezoid method as used in Campo *et al.* (2009). Because in the application the bounds do not improve the estimation results, I decide not to define any.

which satisfy $\hat{H}_i(\hat{B}_i^\alpha(z_\ell)|Z = z_\ell) = \hat{H}_j(\hat{B}_j^\alpha(z_\ell)|Z = z_\ell) = \alpha$, for any $\ell = 1, \cdots, L$, at $X_1 = x$ and any given $\alpha \in [0, 1)$, using the estimates from Step 1. Equation (11) becomes

$$\exp(\hat{B}_i^\alpha(z_\ell)) - \exp(\hat{B}_j^\alpha(z_\ell)) = \frac{\theta_i}{\widehat{Y}_{Hi}(\hat{B}_i^\alpha(z_\ell))} - \frac{\theta_j}{\widehat{Y}_{Hj}(\hat{B}_j^\alpha(z_\ell))} + \epsilon_{i\ell}. \tag{12}$$

where the coefficients (θ_i, θ_j), $i \neq j = 1, \cdots, I$, are the only remaining unknowns. An important feature is to be stressed here. The reader should remember that all the variables in equation (12) are estimated, and not observed, which justifies the introduction of a residual term ϵ, orthogonal to the right-hand side variables by construction.

The introduction of ϵ makes the parameters of interest appear naturally as the solution of a regression in θ_i and θ_j. Since the equation holds for every pair of bidders, $i, j = 1, \cdots, I$, $i \neq j$ such that $x_i = x_j$ and any value $\alpha \in [0, 1)$, the estimator efficiency increases by pooling all bidders' (1 through I) identification conditions into one equation,

$$\begin{bmatrix} \hat{b}_1^\alpha - \hat{b}_2^\alpha \\ \vdots \\ \hat{b}_1^\alpha - \hat{b}_I^\alpha \\ \hat{b}_2^\alpha - \hat{b}_3^\alpha \\ \vdots \\ \hat{b}_{I-1}^\alpha - \hat{b}_I^\alpha \end{bmatrix} = \begin{bmatrix} \frac{1}{\hat{Y}_{H1}(\hat{B}_1^\alpha)} & \frac{-1}{\hat{Y}_{H2}(\hat{B}_2^\alpha)} & 0 & \cdots & \cdots \\ \vdots & 0 & \frac{-1}{\hat{Y}_{Hi}(\hat{B}_i^\alpha)} & 0 & 0 \\ \frac{1}{\hat{Y}_{H1}(\hat{B}_1^\alpha)} & 0 & \cdots & \cdots & \frac{-1}{\hat{Y}_{HI}(\hat{B}_{HI}^\alpha)} \\ 0 & \frac{1}{\hat{Y}_{H2}(\hat{B}_2^\alpha)} & \frac{-1}{\hat{Y}_{Hi}(\hat{B}_i^\alpha)} & 0 & 0 \\ 0 & \cdots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & \frac{1}{\hat{Y}_{H(I-1)}(\hat{B}_{I-1}^\alpha)} & \frac{1}{\hat{Y}_{HI}(\hat{B}_I^\alpha)} \end{bmatrix} \begin{bmatrix} \theta 1 \\ \vdots \\ \theta_i \\ \vdots \\ \theta_I \end{bmatrix} + \epsilon, \tag{13}$$

where $\hat{b}_i^\alpha - \hat{b}_j^\alpha = \exp(\hat{B}_i^\alpha) - \exp(\hat{B}_j^\alpha)$ and $1/\widehat{Y}_{Hi}(B_i^\alpha)$ are $L_{i,j} \times 1$ vectors which typical elements are $b_i^\alpha(z_\ell) - b_j^\alpha(z_\ell)$ and $1/\widehat{Y}_{Hi}(B_i^\alpha(z_\ell))$ respectively. An observation is a triplet (i, j, α) for auctions $\ell = 1, \cdots, L_{i,j}$, where $L_{i,j}$ is the number of auctions such that $x_i = x_j$ for $i \neq j = 1, \cdots, I$.[6] The error term ϵ is a $(L_{1,2} + \cdots + L_{I,I-1}) \times 1$ vector for a given α value. Notice that the optimal regression would consider every value of $\alpha \in [0, 1)$. For simplicity, the estimation is conducted for three different sets of quantiles, $\alpha = 0.1, \cdots, 0.9$, $\alpha = 0.1, 0.5, 0.9$ and $\alpha = 0.25, 0.5, 0.75$, such that $G_i(b_i(\alpha, z)) = G_i(b_j(\alpha, z)) = \alpha\imath$. ($\imath$ is the $L \times 1$ unit vector). Estimating over three different options of quantiles is an informal

[6] The size of vectors $\hat{b}_i^\alpha - \hat{b}_j^\alpha$ and $1/\widehat{Y}_{Hi}(B_i^\alpha)$ varies with the number of auctions where agent i and j compete.

test of robustness.

To improve the estimates efficiency, one may want to introduce instruments. In the application, projecting the left- and right-hand side variables onto Z and Z^2 resulted in more efficient estimates.[7]

5.2 Consistency and Asymptotic distribution

Using a different notation, I rewrite equation (10) as

$$\widehat{DB} = \Lambda(\widehat{Y}, \gamma) + \epsilon, \tag{14}$$

where $\Lambda(\cdot, \gamma)$ may be linear in γ as in (13) in the CRRA model or describe the non-linear equality that holds in CARA models. One typical element of the left-hand side variable $\widehat{DB}$ is $\widehat{DB}_{ij\ell} = DB_{ij\ell} - e_{DB_{i\ell}}$, where $DB_{ij\ell} = \exp B_i^\alpha(z_\ell) - \exp B_j^\alpha(z_\ell)$ and $e_{DB_{i\ell}}$ is the residual. Similarly, one typical element of the right-hand side variable is $\Lambda(\widehat{Y}_{i\ell}, \gamma) = \Lambda(Y_{i\ell}, \gamma) - \Big(\partial\Lambda(\widehat{Y}_{i\ell}, \gamma)/\partial Y\Big)\, e_{Y_{i\ell}}$, where $e_{Y_{i\ell}}$ is the residual resulting from the estimation of $Y_{i\ell}$. In equation (14), ϵ appears then as the residual $(\partial\Lambda(Y_{i\ell}, \gamma)/\partial Y)\, e_{Y_{i\ell}} - e_{DB}$, which converges to zero as $N \to +\infty$. This property helps us derive the consistency and asymptotic normality of $\hat{\gamma}$ as follows:

Lemma 3: Suppose $Y = \Lambda(X, \gamma_0)$. Variables Y and X are unobserved, but $Y \equiv Y(\beta_0)$ and $X \equiv X(\beta_0)$. The respective estimates of X and Y, $\hat{X} \equiv X(\hat{\beta}) = X - e_X =$ and $\hat{Y} \equiv Y(\hat{\beta}) = Y - e_Y$, satisfy $E(e_X|\hat{X}) = 0$ and $E(e_Y|\hat{Y}) = 0$ where e_x and e_Y are the residuals of the estimation procedure, and $\hat{\beta}$ is an estimator of β_0 such that $\sqrt{L}(\hat{\beta} - \beta_0) \to \mathcal{N}(0, \Sigma_\beta)$, where L is the number of observations. Define ϵ as the combined residual $\partial\Lambda(X, \gamma_0)\partial X e_X - e_Y$.

For the appropriate choice of instruments $W \in I\!R^K$, $K \geq dim(X)$, such that $E(W'\epsilon) = 0$ and $E(W'\hat{X}) \neq 0$, the estimator $\hat{\gamma}$ solution of

$$\min_\theta \Psi(\gamma, \hat{\beta}, z_\ell) = P'_W \epsilon,$$

[7]Simulations were very sensitive to any other transformations, even more under the CARA nonlinear specification. It explains why optimal instruments could not be successfully applied here.

where P_W is the orthogonal projection matrix onto the space spanned by the columns of W, satisfies the following properties

(i) $\hat{\gamma}$ is a consistent estimator of γ_0, and
(ii) $\sqrt{L}(\hat{\gamma}-\gamma_0)$ is asymptotically normal $\mathcal{N}(0,\Sigma_\gamma)$ where

$$\Sigma_\gamma = K_1^{-1}K_2\Sigma_\gamma K_2 K_1^{-1},$$

with

$$K_1 \equiv E\left[\frac{\partial^2\Psi(\gamma_{,0}\,\beta_0,z)}{\partial\gamma\partial\gamma'}\right], \quad \text{and} \quad K_2 \equiv E\left[\frac{\partial^2\Psi(\gamma_0,\beta_0,z)}{\partial\gamma\partial\beta'}\right],$$

whenever

$$\text{plim}_{L\to\infty}\frac{1}{L}\sum_{\ell=1}^{L}\frac{\partial^3\Psi(\gamma_0,\beta_0,z_\ell)}{\partial\gamma\partial\gamma'\partial\beta} = o(1).$$

The asymptotic properties rely on the observation that the randomness in (14) lies in the estimates of $\hat{\beta}$. It implies that the estimator properties, in particular its rate of convergence, depends on the first initial estimator of β. The proof is provided in the appendix B.

Lemma 3 can be applied to our system of equations (13), where
$\widehat{DB}_{ij\ell}$ can be rewritten as $\exp B^\alpha(z_\ell,x_i,w_i,\hat{\beta}) - \exp \hat{B}^\alpha(z_\ell,x_j,w_j,\hat{\beta})$, and
$\Lambda(\widehat{Y}_{i\ell},\widehat{Y}_{j\ell},\gamma) = \lambda_i^{-1}(\widehat{Y}_{iH}(B^\alpha(z_\ell,x_i,w_i,\hat{\beta}),\hat{\beta},w_i),\gamma) - \lambda_j^{-1}(\widehat{Y}_{jH}(B^\alpha(z_\ell,x_j,w_j,\hat{\beta}),\hat{\beta},w_j),\gamma)$, where $\hat{\beta}$ stands for vector of estimates of β, which define the bid distributions $\hat{G}_i(b|z_\ell) = \hat{G}(b|z_\ell,x_i,w_i;\hat{\beta})$, for $x_i = x_j$ and $i \neq j = 1,\cdots,I$. In a sample of size N, $\sqrt{N}(\hat{\beta}-\beta)$ follows asymptotically $\mathcal{N}(0,I(\beta)^{-1})$, $I(\cdot)$ is the information matrix derived from the maximum likelihood estimator. Then $\hat{\theta}$ (parameter of interest estimator) has a $\sqrt{N}$ rate of convergence.

Gourieroux, *et al.* (1993) and Gallant and Tauchen (1996) develop a similar estimation method: the "indirect inference" approach. It applies when the parameter of interest is the solution to an untractable equation. The equation is first simulated by defining a consistent approximation to the true equation. Their simulated observations come from a simplified model of the observations, as in the first step of our estimation procedure, where we estimate the equation left- and right-side variables using the models $X(\beta)$ and $Y(\beta)$ to "approximate" the true model. Their estimator is consistent and asymptotically normal under mild regularity conditions for an exactly identified model.[8]

6 The Data

We apply the asymmetric risk averse model to the study of the construction procurement contracts offered by the Los Angeles City Hall between February 1993 and February 2003. The projects range from sewage and street repairs to library branches construction. We first collected 4,757 bids over 951 contracts and 700 firms from the department of Public Works web page. The data set details the date of the auction, the number of bidders per auction, the city's appraisal value of the contract and the firms' identity. Since there is evidence that building projects may satisfy the common value assumption (see Hong and Shum (2002)), we drop these from the sample. Three thousand five hundred and thirty eight bids remain, across six hundred and sixty five auctions and four hundred and seventy firms. The plot of the observations is displayed in Figure 1, where numbers are to be read as millions of dollars.

Then, from the California Construction State License Board web site, we gathered the firm's foundation years and professional licenses (General Engineering, type A, General Building, type B, or specialty licenses, type C and above).Our bidders' age ranges from 0 to 92 years, with 14 and half years of experience on average. Among them, 25.11% of them own an A only license, 8.30% own a B only license, 18.94% hold both.

The following summary statistics describe the contracts and firm sample in more detail:

[8]Dridi (2000) proves that their estimation approach may be extended to the case of over-identifying restrictions as well.

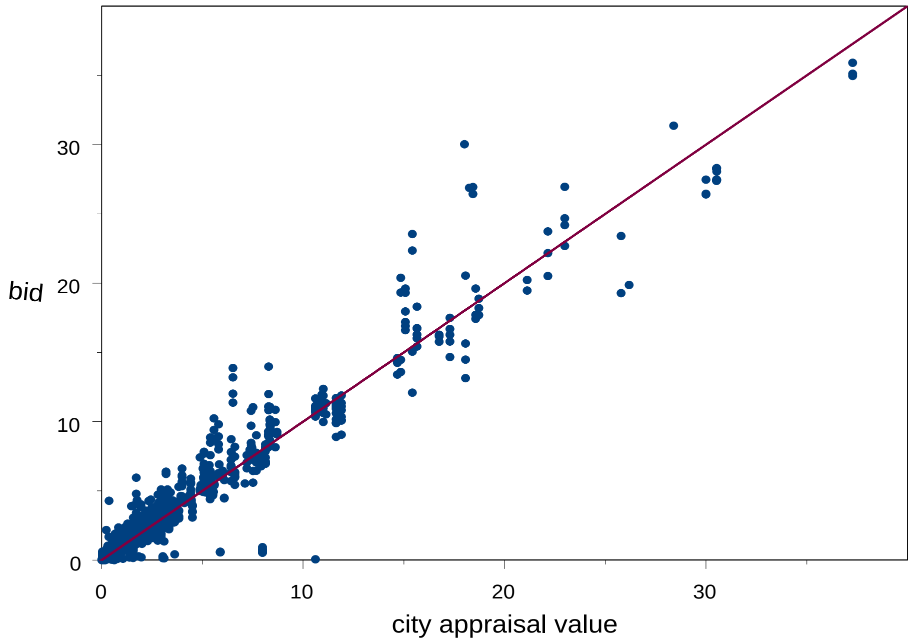

Figure 1: Bids and city's appraisal value for the project

Table 1: Auctions Summary Statistics				
variable	**mean**	**stdv**	**min**	**max**
bids	1.91	13.90	1.39e-4	280.58
winning bids	2.11	15.55	1.39e-4	217.12
city's estimate	2.25	16.46	1.30e-2	230.35
# of bidders	5.26	2.60	2	17

Figures are expressed in millions of dollars, and deflated according to the construction cost index, base(1997)=100, provided by the U.S. Census Bureau.

Table 2: Firms' Summary Statistics					
variable	**# obs**	**mean**	**stdv**	**min**	**max**
experience	1379	14.50	13.89	0	92
paydex score	295	68.86	11.65	13	94
financial stress class	458	1.70	1.05	1	5

The consulting firm D&B provided some information on the bidders' financial health. The paydex (pdx) measures the average number of days a firm takes to pay back its creditors.

For a $pdx = 80$, a firm pays its debts upon receipt (on average). If $pdx = 80 - x$, the firm pays x days after receipt. The financial stress class is a number between 1 (lower risk) and 5 (greater risk), where a firm in class 1 has a .6% chance of discontinuing operations with loss to creditors in the coming year. The same percentage raises to 15.71 for firms in class 5[9]. A few regressions reveal that the firms' experience, paydex and financial score may capture the firms' asymmetries and explain their bid distribution. These results suggest that firms may draw from different cost distributions due to their financial situation, but share the same attitude toward risk if they have the same number of years of experience.[10] Risk aversion is assumed to be a function of a bidder's experience (or number of years in operation) $\gamma_i = \gamma_1 + \gamma_2 exper_i$, for $i = 1, \cdots, I$. This assumption is supported by the following observation: older firms in our sample acquire specialty permits to complement their general licenses acquired earlier in their business history. As such, they diversify their activity and may be less dependent on the L.A. public contracts for their survival. The bid distributions are log-normal, $\log\mathcal{N}(\mu_g, \sigma_g)$, where $\mu_g = \beta_0 + \beta_1 city + \beta_2 nb + \beta_3 expr + \beta_4 pdx$ in a first specification of the model (model I) and $\mu_g = \beta_0 + \beta_1 log(city) + \beta_2 nb + \beta_3 expr + \beta_4 fs$ in an alternative specification (model II)[11].

Because the original sample size is small, we simulate 1000 bootstrap samples to conduct the estimation and repeat steps 1 through 4 of the estimation procedure. The quantile regression, equation (12), was estimated using different set of quantiles to check for the robustness of the results and using the instruments $(log(\text{city}), log(\text{city})^2)$ to reduce the variance of our estimates. The estimates reported in Table 4 and 5 were obtained for the set of 9 quantiles $(0.1, 0.2, \cdots, 0.9)$[12].

[9] Dun & Bradstreet updates the percentage of failure or discontinued operations every year. It is based on the national average, and corresponds here to the 2005 figures.

[10] In Zheng(2001) or Che and Gale(1998), financially constrained bidder play a common value game and a double signal game in private value and budget respectively. In my model, firms do not face a strict budget constraint, but need to borrow to cover their cost.

[11] Different specifications were tried for the bid distributions to try to capture one bidder's competition in a given auction. In particular, I used the competitors' minimum/maximum/average experience level in each auction. None of these variables seemed to capture consistently the bids' distribution across different quantile sets. They also lead to more outliers.

[12] This quantile set lead to a lesser number of outlier estimates. Other estimation results are available upon request.

The CRRA specification results:

In the CRRA case, the mean estimates of the risk aversion parameters (γ_0, γ_1) in $\theta_i = \gamma_0 + \gamma_1 expr_i$, for $i = 1, \cdots, I$ and their 95% confidence intervals are:

Table 3: Bootstrap Estimates		
Estimates	**Model I**	**Model II**
γ_0	-2.47e-1	1.45e-1
IC(95%)	$[-5.81e-1, 8.82e-2]$	$[-1.61e-2, 3.06e-1]$
γ_1	1.51e-2	1.71e-2
IC(95%)	$[1.26e-2, 1.75e-2]$	$[1.36e-2, 2.07e-2]]$

Under both specifications, a firm becomes less risk averse as it grows older. The same result was found in Campo (2005), where it was argued that firms shared the same cost distribution but different risk aversion parameters. Asymmetry in risk aversion remains even after controlling for a firm's financial health. As γ_1 is significantly different from zero, one may conclude that experience remains a decisive factor in explaining a firm's attitude toward risk and thus its bidding. Asymmetry in risk aversion cannot be ignored in the recovery of the firms' true production cost.

The CARA specification results:

Estimating a CARA model turns out to be more difficult because the equation (7) is highly non linear in the parameters β_i. As in the CRRA specification, we estimate a function of the CARA parameters linear in the bidder's number of years of experience.

The estimates are

Table 4: Bootstrap Estimates		
Estimates	**Model I**	**Model II**
γ_0	19.67	9.13
IC(95%)	$[7.29, 67.72]$	$[5.21, 19.28]$
γ_1	-1.10	-0.64
IC(95%)	$[-2.88, -0.51]$	$[-1.29, -0.38]$

In a CARA specification, a firm becomes more risk averse with experience, which goes against the economic intuition laid out in the previous section and confirmed in an earlier paper (see Campo (2005)). The estimates suggest that the CARA specification can not explain the firms' behavior. In the following sections, we use the CRRA estimate to compute the firms' production cost and evaluate their rent.

<u>The Cost distribution:</u>

We recover the production pseudo-costs $\{\hat{c}_i\}_{i=1}^{I}$ after replacing the estimates $\hat{G}_i(\cdot|\cdot)$, $\widehat{Y}_i(\cdot)$ and $\hat{\theta}_i$ into equation (5). We get

$$\hat{c}_i = b_i - \frac{\hat{\theta}_i}{\widehat{Y}_i(b_i)},$$

for all $i = 1, \cdots, I$. If any cost asymmetry exists due to a firm's financial health, it should be confirmed by the estimated pseudo-cost distribution. Suppose a firm's cost follows the log-normal distribution, $\log \mathcal{N}(\mu_c, \sigma_c)$, where either $\mu_c = \alpha_0 + \alpha_1 log(city) + \alpha_p pdx$ or $\mu_c = \alpha_0 + \alpha_1 city + \alpha_f fs$, we would want to find that $\alpha_p \neq 0$ and $\alpha_f \neq 0$. Table 5 confirms these results and reveals a few trends.

Table 5: Asymmetric cost distributions

variable	**Model I**	**Model II**
cst	7.40e-2	1.29e-2
	(4.68e-2)	(1.19e-2)
log(city)	9.72e-2	9.89e-1
	(4.44e-3)	(3.65e-3)
px	-1.32 e-3	NA
	(6.66e-4)	(NA)
fs	NA	-2.18e-2
	(NA)	(6.32e-3)
R^2	0.95	0.98

Standard errors appear here in parentheses.

The paydex specification suggests that as the firms' paydex increases their cost decreases. Such result is justified if one considers that a low paydex firm will have to borrow money to cover the initial cost of construction until payment. The City Hall's official payment policy stipulates that firms will be paid for their services on a monthly basis and only after their engineers have verified the construction quality. It may mean a long delay between the start of the project (and the purchase of raw materials) and the actual final payment. The same trend is observed in the financial stress score specification. As a firm's bankruptcy risk increases, its cost increases. It is likely that the "at-risk" firm does not face the same material prices and/or borrowing rates that healthier firms may get. These results suggest that a cost may be split between the actual production cost and the interests on the debt. In such case, the government should be able to observe the firms' financial status and design rules to encourage competition between financially unconstrained firms. The common practice is to collect such financial health information to check on a firm's reliability but not on its efficiency.

Efficiency itself is measured by the informational rents. They are defined as the difference between the winning bid and the estimated winning cost, $b_w - \hat{c}_w$. The rent percentage, $(b_w - \hat{c}_w)/b_w * 100$, is another measure of efficiency. Furthermore, since the theory predicts that, in the second-price sealed bid auction, firms bid their exact cost, $b_i = c_i$, the rent becomes an instrument of comparison between the first and second-price auction formats. In Table 6, we detail the procurement performance across specifications by looking at the statistics on the pseudo-cost and rents (figures are reported in million of dollars). These are:

Table 6: Cost and informational rent				
in the paydex model				
	average	std	min	max
cost:	2.57	15.07	1e-4	221.50
rent:	2.30e-1	9.36e-1	1e-4	16.28
percentage rent:	10.95	13.56	1.87e-1	93.13
in the financial stress model				
	average	std	min	max
cost:	2.99	16.49	1.40e-4	212.12
rent:	2.68e-1	1.15	0	17.92
percentage rent:	11.58	12.45	0	90.64

These numbers confirm the general perception that the construction sector is a highly competitive market: on average, firms capture roughly 11% of the game rent.

<u>The financial constraint:</u>

Since firms exhibit different cost distributions due to their paydex value, the estimated costs should reveal how their financial health status may affect their bidding. Firms bidding for projects of the Public Works department (DPW) may be split between the financially constrained firms, with a paydex score below 80 (group 1), and those which are unconstrained with a paydex score of 80 or higher (group 2). The firms in group 1 draw higher costs than their competitors in group 2, since they are bearing the additional financing cost associated with a lower budget. This finding suggests that the DPW may want to stimulate competition by changing its contract terms (the payment and financing section) to make financially constrained firms more competitive against their unconstrained competitors after entry. Suppose firm $1i$ in group 1 draws a cost $\tilde{c}$ from the cumulative distribution $\tilde{F}(\cdot)$, while firm $2i$ draws its cost c_i from the distribution $F(\cdot)$, such that

$$\tilde{c} = \begin{cases} (1+r)c_i - rm_i & \text{if } c_i \geq m_i \\ c_i & \text{otherwise} \end{cases}$$

where $\tilde{c}$ is the budget-constrained cost, c_i the unconstrained cost, r is the interest rate on the debt and m_i, firm i's budget. Because the number of observations per firm is small, we impose the condition $m_i = m$ for all $i = 1, \cdots, I$[13]. To elicit r and m average values (average since now they are representative of an average budget constrained bidder), we apply the generalized least square regression

$$\tilde{c}_i = \alpha_1 + \beta_1 \log(z) + u_1$$
$$c_i = \alpha_2 + \beta_2 \log(z) + u_2$$

on the two groups sub-samples, where z stands for the city appraisal value, and (u_1, u_2) are independent error terms, normally distributed with mean zero. The GLS yields $(\hat{\alpha}_1, \hat{\beta}_1) = (0.470, 0.443)$ and $(\hat{\alpha}_2, \hat{\beta}_2) = (0.467, 0.427)$ with respective standard errors $(0.023, 0.015)$ and $(0.066, 0.031)$. Estimates of the interest rate and budget follow

$$\begin{aligned} (1+\hat{r}) &= \hat{\beta}_1/\hat{\beta}_2 = 1.199 \\ \hat{m} &= -(\hat{\alpha}_1 - (1+\hat{r})\hat{\alpha}_2)/\hat{r} = \$0.824 \text{ million.} \end{aligned}$$

From these figures, one gathers that, on average, a construction firm is financed at a 19.9% interest rate and has a budget of \$824,000 to allocate to each project. If the rate seems rather high, the second figure reveals that the interest rate does not apply to a majority of the projects in the sample, since approximately 69% of them are worth less than \$0.9 million to the city hall (according to the engineers' appraisal value). The estimated averages may also be sensitive to extreme cases where whenever large amounts need to be financed, interest rates can be smaller, while for smaller amounts, interest rates may climb. One should also consider the possibility that the so-called "interest rate" may be more representative of a "financing rate" where suppliers' and banks' terms may all well be confused. The two sources cannot be separately identified given the available data.

If we were to change the terms of the auction and pay the winning firm before completion of the project, such that the firms would not face any financial constraint, how would the new payment rule affect the City Hall's payments? To analyze the effect of such policy, we simulate auctions where only unconstrained firms participate. These firms have a paydex greater or equal to 80 (a paydex of 80 applies to firms who pay their creditors

[13] One might also expect that each firm gets a different interest rate $r_i \neq r_j$, for $i \neq j = 1, \cdots, I$. For lack of more individual information, $r_i = r \; \forall i$.

on time, anything greater than 80 to firms who pay their creditors ahead of time). We should expect bids to decrease since, according to our estimates, unconstrained bidders bear lower costs than their more unfortunate competitors. Notice that the number of bids and all other auction characteristics remain unchanged to avoid any confusion on what may cause the increase or decline in payments in the policy simulation. For this purpose, we force constrained firms to draw their cost from their competitors' unconstrained cost distribution. The results confirm the intuition that competition is more intense between budget-unconstrained firms simply because they draw more advantageous costs. They also suggest that the difference in payments is substantial as the winning costs decrease by 23.67% and the corresponding bids (City Hall expenses) by 29.54% on average.[14]

7 Conclusion

This article introduces a new semiparametric identification result and estimation procedure for auction models with asymmetric risk averse bidders. Identification is achieved using one of the model main restriction: whenever two bidders draw their cost from the same distribution, but differ in their risk aversion (either CRRA or CARA), their inverse bidding functions lead to the same cost for a given quantile of their asymmetric bid distributions. The estimation procedure relies on the same exact equality to recover the model primitives (the bidders' risk aversion parameters and their cost distributions). The equality which holds for an unobserved common cost may be simulated using the bid distributions quantiles. The simulation creates a regression where both the right- and left-hand side variables are estimated and the error term gathers the residuals. Whenever the estimates of variables are consistent, the parameters of interest share the same property.

This methodology helps explain the construction firms bidding behavior for the Los Angeles Department of Public Works contracts. Firms become less risk averse with age (a proxy for experience, permit numbers, capital accumulation) and bid less aggressively. They are less cost efficient if they suffer from bad credit and, as a result, submit higher

[14] In a model with endogenous entry, one would be able to consider the effect of such policy on the bidders' participation as well as their bid distribution.

quotes. Borrowing rates as high as 20% penalize budget-constrained firms in the studied procurements (firms need financing in 30% of the projects in our sample). Even though, measures of experience and financial health are readily available, they are not used by the awarding bureau to promote competition between firms. It is likely that the city of Los Angeles is not allowed to discriminate based on such criteria. The city may still elicit better quotes, if it were to consider paying the winning bidder before the project starting date. Note that, in a model of endogenous entry, a firm's experience and/or financial strength may also affect its entry decision, and thus the competition structure. Our conclusions may then be different. Further research may be able to measure the asymmetries impact on a firm's entry decision, but this topic is out of scope here.

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APPENDIX

Proof of Definition 3:

Conditions (i) through (iii) are straightforward consequences of Definition 2 (i) and (ii). Condition (iv) is proven as follows.

Note that by definition,

$$\lambda_{\omega_i,i}(b_i - c_i) = \frac{U_{\omega_i,i}(b_i - c_i)}{U'_{\omega_i,i}(b_i - c_i)},$$

Along with condition (iii) in Definition 1, it implies that $\lambda(\cdot)$ admits one continuous derivative over its support $[0, +\infty)$.

Condition (iv) then follows from equation (2)

$$\sum_{j \neq i} \frac{g_j(b_i)}{1 - G_j(b_i)} = \lambda_i(b_i - c_i),$$

for $i = 1, \cdots, I$, where $g_j(\cdot)$ is once continuously differentiable since $\lambda_i(\cdot)$ is once continuously differentiable, and $G_j(\cdot)$ admits then two continuous derivatives over $(\underline{b}, \overline{b}]$, for $j = 1, \cdots, I$. Note that this support is not closed as $G_i(\cdot)$ appears in the denominator of the expression here above.

Proof of Lemma 1 condition 3:

Necessary conditions:

Let's prove that, given the distribution and utility functions $\{F_i(\cdot)\}_{i-1}^{I}$, there exists a function $\tilde{\lambda}_i(\cdot)$ such that the inverse bidding functions $\tilde{\xi}_i(\cdots)$ is increasing in b_i, for all $i = \cdots, I$.

Consider bidder i's F.O.C. where I define $\tilde{\lambda}_i(x)$ as $\frac{U_i(x)}{U'_i(x)}$:

$$1 = \tilde{\lambda}_i(b_i - c_i) \sum_{j=1, j \neq i}^{I} \frac{f_j(s_j^{-1}(b_i))}{1 - F_j(s_j^{-1}(b_i))} \frac{1}{s'_j(s_j^{-1}(b_i))}. \tag{B.1}$$

The function $\tilde{\lambda}_i(x) = \frac{U_i(x)}{U'_i(x)}$ is continuous over $[0, +\infty[$, $\tilde{\lambda}_i(0) = 0$ and admits a derivative $\tilde{\lambda}'_i \geq 1$ by definition of $U_i(\cdot)$, for all $i = 1, \cdots, I$.

Isolating c_i in equation $(B.1)$, I get

$$c_i = b_i - \tilde{\lambda}_i(1/Y_i(b_i)) \equiv= \tilde{\xi}_i(b_i, \{G_j(\cdot)\}_{j=1}^{I}),$$

where $Y_i(b_i) = \sum_{j=1,j\neq i}^{I} \frac{f_j(s_j^{-1}(b_i))}{1-F_j(s_j^{-1}(b_i))} \frac{1}{s_j'(s_j^{-1}(b_i))}$. Given the aforementioned properties of $\tilde{\lambda}_i(\cdot)$, $\tilde{\xi}_i(b_i)$ is increasing in b_i, for $i = 1, \cdots, I$. It implies that $\tilde{\lambda}_i(\cdots) = \lambda_i(\cdots)$ and $\tilde{\xi}(\cdots) = \xi(\cdots) =$.

Consider now two bidders i and j, who share the same cost function, and happen to draw the same cost c_0. They bid b_{i0} and b_{j0} respectively. The common cost c_0 satisfies $c_0 = b_{i0} - \lambda_i(1/Y_i(b_{i0})) = b_j - \lambda_j(1/Y_j(b_{j0}))$, or $b_{i0} - b_{j0} = \lambda_i(1/Y_i(b_{i0})) - \lambda_j(1/Y_j(b_{j0}))$. Since $b_{i0} = s_i(c_0, \cdots)$ and $b_{j0} = s_j(c_0, \cdots)$, we also have

$$F(c_0) = \Pr(c \leq c_0) = \Pr(s_i(c, \cdots) \leq s_i(c_0, \cdots)) = \Pr(b_i \leq b_{i0}) = G_i(b_{i0})$$

and

$$F(c_0) = \Pr(c \leq c_0) = \Pr(s_j(c, \cdots) \leq s_j(c_0, \cdots)) = \Pr(b_j \leq b_{j0}) = G_j(b_{j0})$$

or, in other words, $G_i(b_{i0}) = G_j(b_{j0})) = \alpha$ where $\alpha \in [0, 1[$. The firms' bids (b_{i0}, b_{j0}) are the quantile bids $(b_i^{\alpha}, b_j^{\alpha})$.

Sufficient conditions:
Suppose we observe that two bids (b_{i0}, b_{j0}) satisfy

$$b_{i0} - b_{j0} = \lambda_i^{-1}\left[1/Y_i(b_{i0})\right] - \lambda_j^{-1}\left[1/Y_j(b_{j0})\right].$$

whenever $G(b_{i0}|x_i, w_i) = G(b_{j0}|x_j, w_j) = \alpha$ and $w_i = w_j$, then

$$\begin{aligned} G(b_{i0}|x_i, w_i) &= \Pr(b_i \leq b_{i0}|x_i, w_i) = \Pr(s_i^{-1}(b, \cdots) \leq s_i^{-1}(b_{i0}, \cdots)|x_i) \\ &= \Pr(c \leq c_1|x_i) = F(c_1|x_i) = \alpha \end{aligned}$$

and

$$\begin{aligned} G(b_{j0}|x_j, w_j) &= \Pr(b_j \leq b_{j0}|x_j, w_j) = \Pr(s_i^{-1}(b, \cdots) \leq s_i^{-1}(b_{i0}, \cdots)|x_j) \\ &= \Pr(c \leq c_2|x_j) = F(c_2|x_j) = \alpha. \end{aligned}$$

Since $G(b_{i0}|x_i, w_i) = G(b_{j0}|x_j, w_j)$ and $x_i = x_j$, $c_1 = c_2 = c_0$ is the alpha-quantile of the cost distribution $F(\cdot|x_i) \equiv F(\dot{|}x_j)$, where the cost may be defined by the inverse bidding function $\xi_i(b_{i0}, \cdots |x_i, w_i)$ as defined in condition $(iii)a$.
Condition $(iii)a$ also defines the function $\lambda(\cdot|w_i)$ which integral yields

$$\int_0^y \lambda(u|w_i)du = \log\left[U(y|w_i)\right],$$

bidder i's logarithmic transformation of his utility function. Thus, given $G_i(\cdot)$ and the observed characteristics (x_i, w_i), I have recovered the cost distribution function $F(\cdot|x_i)$ and the utility function $U(\cdot|w_i)$ for $i = 1, \cdots, I$.

Proof of Lemma 2:

CRRA utility functions:

Suppose bidder i exhibit the constant relative risk averse utility function

$$U_i(x) = x^{(1-\theta_i)}.$$

Consider two agents, arbitrarily called agent 1 and agent 2. For $\alpha = 0$, the bidders' inverse bidding strategies yield the identifying condition

$$\underline{b} - \underline{b} = \begin{bmatrix} \frac{1}{g_1(\underline{b})} & \frac{-1}{g_2(\underline{b})} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

so that

$$\theta_1 = \frac{g_2(\underline{b})}{g_1(\underline{b})}\theta_2 \tag{B.2}$$

Assume instead that α belongs to $(0, 1)$. Any pair (b_1^α, b_2^α), which satisfies $G_1(b_1^\alpha) = G_2(b_2^\alpha) = \alpha$, also satisfies

$$b_1^\alpha - b_2^\alpha = \begin{bmatrix} \frac{1}{Y_1(b_1^\alpha} & \frac{-1}{Y_2(b_2^\alpha} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{B.3}$$

where I can substitute θ_1 for its expression in $(B.2)$, so that equation $(B.3)$ becomes

$$b_1^\alpha - b_2^\alpha = \left[\ \tfrac{g_2(\underline{b})}{g_1(\underline{b})} \tfrac{1}{Y_1(b_1^\alpha)} - \tfrac{1}{Y_2(b_2^\alpha)} \ \right] \theta_2 \tag{B.4}$$

Since $b_1^\alpha - b_2^\alpha \neq 0$, neither the first nor the second term in equation $(B.4)$ right-hand side is equal to zero, thus I can solve for θ_2. Parameter θ_2 is uniquely defined as

$$\theta_2 = (b_1^\alpha - b_2^\alpha) \,/\, \left[\ \tfrac{g_2(\underline{b})}{g_1(\underline{b})} \tfrac{1}{Y_1(b_1^\alpha)} - \tfrac{1}{Y_2(b_2^\alpha)} \ \right]$$

Then parameters θ_j, $j = 3, \cdots, I$, are uniquely recovered from the equation

$$b_2^\alpha - b_j^\alpha = \frac{\theta_2}{Y_2(b_2^\alpha)} - \frac{\theta_j}{Y_j(b_j^\alpha)}$$

, where $G_2(b_2^\alpha) = G_j(b_j^\alpha)$, θ_2 is known, and α belongs to the interval $[0, 1)$.
The model of CRRA-asymmetric risk averse bidders is identified.

CARA utility functions:
Suppose bidder i exhibit the constant absolute risk averse utility function

$$U_i(x) = a_{1i} \left[1 - \exp(-\beta_i x)\right].$$

Bidder i's bidding strategy is

$$b_i = c + \frac{1}{\beta_i} \log \left[\frac{\beta_i + Y_i(b)}{Y_i(b)} \right],$$

for a given cost c. The "a_1" coefficient, which does not appear here, helps normalize the utility function, which should satisfy $U(0) = 0$ and $U(1) = 1$. Then a unique β_i yields a unique vector a_1.
I show that given the observations b_i and the estimated $\hat{Y}_i(b_i)$, $i = 1, \cdots, I$, there exists a unique vector $(\beta_1, \cdots, \beta_I)$ satisfying the bidders' first order conditions.

Consider 2 bidders i and j, and the quantile $\alpha \in [0, 1)$, such that

$$\begin{aligned} b_i^\alpha - \frac{1}{\beta_i} \log \left[\frac{\beta_i + Y_i(b_i^\alpha)}{Y_i(b_i^\alpha)} \right] &= b_j^\alpha - \frac{1}{\beta_j} \log \left[\frac{\beta_j + Y_j(b_j^\alpha)}{Y_j(b_j^\alpha)} \right] \\ b_i^\alpha - b_j^\alpha &= \frac{1}{\beta_i} \log \left[\frac{\beta_i + Y_i(b_i^\alpha)}{Y_i(b_i^\alpha)} \right] - \frac{1}{\beta_j} \log \left[\frac{\beta_j + Y_j(b_j^\alpha)}{Y_j(b_j^\alpha)} \right] \end{aligned} \tag{B.5}$$

For $\alpha = 0$, the above equation becomes

$$\begin{aligned} 0 &= \frac{1}{\beta_i}\log\left[\frac{\beta_i + Y_i(b_i^\alpha)}{Y_i(b_i^\alpha)}\right] - \frac{1}{\beta_j}\log\left[\frac{\beta_j + Y_j(b_j^\alpha)}{Y_j(b_j^\alpha)}\right] \\ \frac{1}{\beta_i}\log\left[\frac{\beta_i + Y_i(b_i^\alpha)}{Y_i(b_i^\alpha)}\right] &= \frac{1}{\beta_j}\log\left[\frac{\beta_j + Y_j(b_j^\alpha)}{Y_j(b_j^\alpha)}\right] \\ \frac{\beta_i}{\beta_j} &= \log\left[\frac{\beta_i + Y_i(b_i^\alpha)}{Y_i(b_i^\alpha)} - \frac{\beta_j + Y_j(b_j^\alpha)}{Y_j(b_j^\alpha)}\right] \end{aligned} \tag{B.6}$$

Moreover, for 2 auctions z_1 and z_2, we observe different lower bounds $\underline{b}(z_1)$ and $\underline{b}(z_2)$ but the same equality in $(B.6)$ such that

$$\frac{\beta_i}{\beta_j} = \log\left[\frac{\beta_i + Y_i(z_1)}{Y_i(z_1)} - \frac{\beta_j + Y_j(z_1)}{Y_j(z_1)}\right]$$

and

$$\frac{\beta_i}{\beta_j} = \log\left[\frac{\beta_i + Y_i(z_2)}{Y_i(z_2)} - \frac{\beta_j + Y_j(z_2)}{Y_j(z_2)}\right]$$

where $Y_i(z_\ell)$ stands for the function $Y(\cdot)$ evaluated at $b_i^\alpha(z_\ell)$, where $\alpha = 0$, for $i = 1, \cdots, I$ and $\ell = 1, \cdots, L$.

It implies that

$$\log\left[\frac{\beta_i + Y_i(z_1)}{Y_i(z_1)} - \frac{\beta_j + Y_j(z_1)}{Y_j(z_1)}\right] = \log\left[\frac{\beta_i + Y_i(z_2)}{Y_i(z_2)} - \frac{\beta_j + Y_j(z_2)}{Y_j(z_2)}\right].$$

Since $Y_i(z) = g_i(\underline{b}|z) \equiv g_i(z)$ and logs appear on both sides, the latter equation simplifies into

$$\frac{\beta_i + g_j(z_1)}{g_j(z_1)} - \frac{\beta_j + g_i(z_1)}{g_i(z_1)} = \frac{\beta_i + g_j(z_2)}{g_j(z_2)} - \frac{\beta_j + g_1(z_2)}{g_1(z_2)},$$

which, after some manipulations, becomes

$$\frac{g_j(z_2)\left[\beta_i + g_j(z_1)\right] - g_j(z_1)\left[\beta_i + g_j(z_2)\right]}{g_j(z_1)g_j(z_2)} = \frac{g_i(z_2)\left[\beta_j + g_i(z_1)\right] - g_i(z_1)\left[\beta_j + g_i(z_2)\right]}{g_i(z_1)g_i(z_2)},$$

or

$$\begin{aligned}
\frac{[g_j(z_2) - g_j(z_1)]}{g_j(z_1)g_j(z_2)}\beta_i &= \frac{[g_i(z_2) - g_i(z_1)]}{g_i(z_1)g_i(z_2)}\beta_j \\
\beta_i &= \frac{g_j(z_1)g_j(z_2)}{[g_j(z_2) - g_j(z_1)]}\frac{[g_i(z_2) - g_i(z_1)]}{g_i(z_1)g_i(z_2)}\beta_j.
\end{aligned}$$

Replace this last formula into equation $(B.5)$, for any given $\alpha \neq 0$. The expression becomes an equation in one unknown, β_j. The CARA model is identified.

Proof of Lemma 3:

For illustrative purpose, I detail here the proof for the CRRA case, where the equation to be estimated is linear. The same reasoning may be extended to a non-linear equation.

Consistency: Suppose $Y = X\theta_0$, but one can only observe $\hat{X} = X - e_X$ and $\hat{Y} = Y - e_Y$, where $\hat{X}$ and $\hat{Y}$ are X and Y respective predicted values. The residuals e_X and e_Y satisfy $E(e_X|\hat{X}) = 0$ and $E(e_Y|\hat{Y}) = 0$.

The observed relation between X and Y is

$$\hat{Y} = \hat{X}\theta_0 + \epsilon. \tag{B.7}$$

where $\epsilon = e_X\theta_0 - e_Y$. Then the estimates of the parameters θ_0 appears as the solution of the least squares minimization program: $\min_\beta(\hat{Y} - \hat{X}\theta)'(\hat{Y} - \hat{X}\theta)$.

One may want to use instrumental variables W to reduce the variance of the estimator and/or to solve for the possible correlation between $\hat{X}$ and e_Y. These instruments should satisfy $E(P_W|\hat{X}) \neq 0$ and $E(P_We) = 0$. The matrix P_W defines the projection matrix that projects any vector in the sample space onto the space spanned by the vector W, $\delta(W)$.

Equation $(B.7)$ becomes

$$P_W\hat{Y} = P_W\hat{X}\theta_0 + P_W(e_X\theta_0 - e_Y) \tag{B.8}$$

and the estimator $\hat{\theta}$ of θ_0 can be defined as

$$\begin{aligned}
\hat{\theta} &= (\hat{X}'P_W\hat{X})^{-1}\hat{X}'P_W\hat{Y} \\
&= (\hat{X}'P_W\hat{X})^{-1}\hat{X}'P_W(X\theta_0 - e_Y) \\
&= (\hat{X}'P_W\hat{X})^{-1}\hat{X}'P_W X\theta_0 - (\hat{X}'P_W\hat{X})^{-1}\hat{X}'P_W e_Y
\end{aligned}$$

since $Y = X\theta_0 = \hat{Y} + e_Y$.
For N observations, we also have $\hat{Y} = Y + O(N^{-1/2})$, and $\hat{X} = X + O(N^{-1/2})$, such that

$$\begin{aligned}
1/N(\hat{X}'P_W\hat{X}) &= 1/N((X + O(N^{-1/2}))'P_W(X + O(N^{-1/2}))) \\
&= 1/N(X'P_W X) + O(N^{-1/2})
\end{aligned}$$

and

$$\begin{aligned}
1/N(\hat{X}'P_W X\theta_0) &= 1/N(X + O(N^{-1/2})P_W X\theta_0) \\
&= 1/N(X'P_W X\theta_0) + O(N^{-1/2})
\end{aligned}$$

It implies that, for $N \to \infty$, and the appropriate choice of instruments W

$$\text{plim}_{N\to\infty}(1/N(\hat{X}'P_W\hat{X}))^{-1}1/N(\hat{X}'P_W e_Y) = 0,$$

so that

$$\text{plim}_{N\to\infty}(\frac{1}{N}(\hat{X}'P_W\hat{X}))^{-1}\frac{1}{N}(\hat{X}'P_W X\theta_0) = \text{plim}_{N\to\infty}\hat{\theta} = \theta_0.$$

The estimator $\hat{\theta}$ of θ_0 is consistent.

Asymptotic distribution:
The identifying equality the regression equation

$$b_i^\alpha(z_\ell, \hat{\beta}) - b_j^\alpha(z_\ell, \hat{\beta}) = \frac{\theta_i}{Y_i(z_\ell, \alpha, \hat{\beta})} + \frac{\theta_j}{Y_j(z_\ell, \alpha, \hat{\beta})} + e_{ij\ell}$$

The equality holds for all auctions $\ell = 1, \cdots, L_{ij}$, and $i \neq j = 1, \cdots, I$. Residuals $e_{ij\ell}$ are the result of the estimation of the left and right-hand side variables.
Let $\Psi_{ij}(\theta, \hat{\beta}, Z) = P_W' e_{ij}$ define the set of orthogonality conditions such that $E\left(\Psi_{ij}(\theta, \hat{\beta}, Z)|Z\right) = 0$.

For $L \to +\infty$ (if agents i and j, $i \neq j = 1, \cdots, I$, participate to $L_{ij} = L$ auctions), we have

$$\text{plim}_{L\to\infty} \frac{1}{I} \sum_{i} \frac{1}{J_{i\ell}} \sum_{j\neq i} \frac{1}{L} \sum_{\ell} \Psi_{ij}(\theta_0, \hat{\beta}, z_\ell) = 0,$$

where $J_{i\ell}^{+}$ denotes both the set of bidders $j \neq i, j > i$ who compete with bidder i in auction ℓ and its cardinality. Let's drop for the moment the subscript ij. The Taylor expansion of $\Psi(\cdots)$ around the true value of the parameters $\theta = \theta_0$ yields

$$\begin{aligned}
0 &\simeq \frac{1}{L} \sum_{\ell=1}^{L} \Psi(\hat{\theta}, \hat{\beta}, z_\ell) \\
0 &\simeq \underbrace{\frac{1}{L} \sum_{\ell=1}^{L} \Psi(\theta_0, \hat{\beta}, z_\ell)}_{A} + \underbrace{\frac{1}{L} \sum_{\ell=1}^{L} \frac{\partial \Psi(\theta_0, \hat{\beta}, z_\ell)}{\partial \theta} (\hat{\theta} - \theta_0)}_{B}
\end{aligned}$$

Let's study separately A and B. Their Taylor expansion around β_0 yields

$$A \simeq \frac{1}{L} \sum_{\ell=1}^{L} \left[\Psi(\theta_0, \beta_0, z_\ell) + \frac{\partial \Psi(\theta_0, \beta_0, z_\ell)}{\partial \beta'} (\hat{\beta} - \beta_0) \right]$$

and

$$B \simeq \frac{1}{L} \sum_{\ell=1}^{L} \left[\frac{\partial \Psi(\theta_0, \beta_0, z_\ell)}{\partial \theta} + \frac{\partial^2 \Psi(\theta_0, \beta_0, z_\ell)}{\partial \theta \partial \beta'} (\hat{\beta} - \beta_0) \right]$$

respectively.

Notice that, for every observation z_ℓ, $\ell = 1, \cdots, L$,

$$\Psi(\theta_0, \beta_0, z_\ell) = 0$$

$A + B$ becomes

$$\begin{aligned}
A + B &\simeq \left[\frac{1}{L} \sum_{\ell=1}^{L} \frac{\partial \Psi(\theta_0, \beta_0, z_\ell)}{\partial \beta'} (\hat{\beta} - \beta_0) \right] \\
&\quad + \left[\frac{1}{L} \sum_{\ell=1}^{L} \frac{\partial \Psi(\theta_0, \beta_0, z_\ell)}{\partial \theta} + \frac{1}{L} \sum_{\ell=1}^{L} \frac{\partial^2 \Psi(\theta_0, \beta_0, z_\ell)}{\partial \theta \partial \beta'} (\hat{\beta} - \beta_0) \right] (\hat{\theta} - \theta_0)
\end{aligned}$$

Assume

$$\text{plim}_{L\to\infty}\frac{1}{L}\sum_{\ell=1}^{L}\frac{\partial\Psi(\theta_0,\beta_0,z_\ell)}{\partial\theta\partial\beta'}=O(L),$$

then $\sqrt{L}(\hat{\theta}-\theta_0)$ satisfies

$$\sqrt{L}(\hat{\theta}-\theta_0)=\left[-\frac{1}{L}\sum_{\ell=1}^{L}\frac{\partial\Psi(\theta_0,\beta_0,z_\ell)}{\partial\theta}\right]^{-1}\left[\frac{1}{L}\sum_{\ell=1}^{L}\frac{\partial\Psi(\theta_0,\beta_0,z_\ell)}{\partial\beta}\right]\sqrt{L}(\hat{\beta}-\beta_0) \quad \text{(B.9)}$$

If we use Q quantiles in the estimation of θ, the empirical moment condition becomes

$$\frac{1}{Q}\sum_{\alpha}\frac{1}{I}\sum_{i}\frac{1}{L}\sum_{\ell=1}^{L}\frac{1}{J_{i\ell}^{+}}\sum_{j\in J_{i\ell}^{+}}\Psi_{ij}(\theta,\hat{\beta},\alpha,z_\ell)\equiv\frac{1}{Q}\sum_{\alpha}\Psi(\theta,\hat{\beta},\alpha)$$

where Q is the number of quantile or α values used in the estimation of θ. Equation $(B.9)$ becomes

$$\sqrt{L}(\hat{\theta}-\theta_0)=\left[-\frac{1}{Q}\sum_{\alpha}\frac{\partial\Psi(\theta_0,\beta_0,\alpha)}{\partial\theta}\right]^{-1}\left[\frac{1}{Q}\sum_{\alpha}\frac{\partial\Psi(\theta_0,\beta_0,\alpha,z_\ell)}{\partial\beta}\right]\sqrt{L}(\hat{\beta}-\beta_0). \text{(B.10)}$$

With Q, one gets a faster convergence of the $K_1$and K_2 matrices, higher efficiency, but the $\hat{\theta}$ asymptotics are still restricted by the behavior of $\hat{\beta}$ at the limit.

For either $L\to+\infty$ or $Q\to+\infty$,

$$\text{plim}\frac{1}{Q}\sum_{\alpha}\frac{\partial\Psi(\theta_0,\beta_0,\alpha)}{\partial\theta}=E\left[\frac{\partial\Psi(\theta_0,\beta_0,z)}{\partial\theta}\right]\equiv K_1.$$

while

$$\text{plim}\frac{1}{Q}\sum_{\alpha}\frac{\partial\Psi(\theta_0,\beta_0,\alpha,z_\ell)}{\partial\beta}=E\left[\frac{\partial^2\Psi(\theta_0,\beta_0,z)}{\partial\beta}\right]\equiv K_2$$

If $L\to+\infty$, $\sqrt{L}(\hat{\beta}-\beta_0)$ follows the asymptotic normal distribution $\mathcal{N}(0,\Sigma_\beta)$, $\sqrt{L}(\hat{\theta}-\theta_0)$ is asymptotically normal $\mathcal{N}(0,\Sigma_\theta)$ with

$$\Sigma_\theta=K_1^{-1}K_2\Sigma_\beta K_2K_1^{-1}.$$

As such, the asymptotics of $\hat{\theta}$ rely on the properties of $\hat{\beta}$.

What happens if we relax the assumption that a bidder participates only once to every auction?

Suppose that bidder i bids P_i times, and participates to L_i auctions, $i = 1, \cdots, I$. One type may bid more than once in every auction: P_i may not be equal to L_i . I had assumed $P_i = L_i = L$ until now. The estimates of β satisfy $\sqrt{P_i}(\hat{\beta}_i - \beta_{i0}) \sim \mathcal{N}(0, \sigma_i)$, where $(\beta_{i0}, \sigma_{io})$ are the true values of the parameter β_i and σ_i, the respective mean and variance of bidder i's bid distribution.

Consider two cases:
1. Either $L_i = L$ or $P_i = P$ for all $i = 1, \cdots, I$. The asymptotics are straightforward. The estimator $\hat{\theta}$ is defined by: $\sqrt{L}(\hat{\theta} - \theta_0)$ is asymptotically normal $\mathcal{N}(0, \Sigma_\theta)$ where

$$\Sigma_\theta = K_1^{-1} K_2 \Sigma_\beta K_2 K_1^{-1}.$$

2. $\lim_{L \to +\infty} P_i/P_j = 1$, for $j \neq i = 1, \cdots, I$, and $\lim_{L \to +\infty} P_i = P$, $\forall i$.
Then $\sqrt{P}(\hat{\theta} - \theta_0)$ follows asymptotically a normal distribution of mean 0 and variance

$$\Sigma_\theta = K_1^{-1} K_2 \Sigma_\beta K_2 K_1^{-1}.$$

where $\sqrt{P}(\hat{\beta} - \beta_0)$ asymptotic distribution is a $\mathcal{N}(0, \Sigma_\beta)$.

Up to this point, I have considered cases where the β coefficients were computed for separate pools of observations: β_i which characterizes the agent i's bid distribution i and help reconstruct b_i^α and the $Y(\cdot)$ functions were estimated over the number of times P_i that agent i participated in the auctions, for $i = 1, \cdots, I$. Suppose that the bid distribution is defined as $G(\cdot|Z, W_i, X_i; \beta)$, where X_i, W_i are agent i's characteristics, and all the sample observations may be pooled (for example, a normal distribution with mean $\mu_i = \beta_0 + \beta_1 Z + \beta_2 X_i + \beta_3 W_i$). The rate of convergence improves from $\sqrt{L}$ ($\sqrt{P}$) to $\sqrt{N}$, the total number of bids in the sample.