

Illegal Sports Bookmakers
 Koleman S. Strumpf
 UNC Chapel Hill

Sports Betting in US (**Table 1**)

Type	Annual Bet Volume	Location
Legal	\$2b	Nevada
Internet	\$20-\$30b	Internet/offshore
Illegal	\$80-\$380b	Ubiquitous

Fiscal Year	Number of Locations	Bet Volume	Winnings	% Hold
1969-70	12	\$395,763	NA	NA
1974-75	9	\$13,083,910	\$960,359	7.34%
1979-80	30	\$289,630,191	\$12,106,542	4.18%
1984-85	61	\$930,810,122	\$22,897,929	2.46%
1989-90	67	\$1,482,362,000	\$48,325,000	3.26%
1994-95	109	\$2,250,252,000	\$89,335,000	3.97%
1998-99	145	\$2,301,837,000	\$98,979,000	4.30%
1999-2000	157	\$2,560,893,000	\$117,545,000	4.59%
2000-01	156	\$2,106,579,000	\$112,702,000	5.35%

Legal Sports Wagering in Nevada

Year	Bet Volume	Winnings
1974	\$2,500,000,000	\$220,000,000
1982	\$15,628,000,000	\$1,329,300,000
1985	\$20,087,000,000	\$1,714,102,000
1989	\$29,506,901,093	\$2,517,938,875
1995	\$84,000,000,000	NA
1999	\$80-\$380,000,000,000	NA

Illegal Sports Wagering Estimates

Year	Number On-Line Sites	Bet Volume	Winnings	% Hold
1995	0	0	0	-
1996	2	\$60,000,000	NA	NA
1997	NA	\$600,000,000	NA	NA
1998	53	NA	\$260,000,000	NA
1999	139	\$14,300,000,000	\$640,000,000	4.5%
2000	204	NA	\$1,000,000,000	NA

Internet Sports Wagering

Table 1: Sports Wagering Trends (excludes horse racing and greyhounds)

Motivation

- Interesting environment
 - large illegal with legal fringe
 - similar: music (P2P), prostitution
- Key features
 - all segments provide same product (bets, not amenities)
 - yet behave/organized in different ways
- *Goal is to explain source of these differences*
 - focus on illegal vs internet
 - both unfettered markets
 - but different legal status in home jurisdiction
- Legal environment
 - internets not concerned with arrest
 - illegals can be arrested (pr=5%/yr) → asset confiscation but no jail
 - NB: bettors not arrested but limited access to illegals

This paper

1. Explain Industrial Organization illegal market
 - e.g. firm size, behavior, contracts, ...
 - illegal status \uparrow info issues and \downarrow competitive pressures
 - *qualitative evidence in the spirit of the law & econ lit*
2. Economic incentives are central
 - not obvious: little formal education (no college let alone MBA)
 - not brainwashed into using standard structure and conduct
 - inconsistent CW for many practices
 - quantitative evidence here (prices, fn. diversification)*
3. Provides new support for economic theory of crime (Becker, 1968)
 - lit focused on labor issues
 - (entry decision: wages/UE, deterrence)
 - new labor evidence here
 - (low eqbm income \rightarrow consistent with RL prefs)
 - additional evidence on IO
 - incentives shape criminal enterprise

Data Used in Analysis – Illegal Bookies

- 6 New York city bookies arrested 1995-2000
[videos of arrests: www.unc.edu/~cigar]
 - wide range sizes:
 - * \$5-\$150 million/yr bet volume
 - * 15-300 bettors
 - * 0-80 hired employees
 - long-lived (min=2 yrs; average=5 yrs)
- Very detailed internal records (**Table 2**):
 - observation period: 1 week - 3 months
 - individual bets
 - individual bettors
 - employee records
 - other:
 - PC backup, court docs, audio tapes,
 - address book, bank statements, . . .

	Location	Observation Period	Number of Bettors	Number of Bets	Bet Volume	Available Records
Bookie 0	Brooklyn, Manhattan, Queens, Staten Island, NY	7/1/95-8/4/95 (22 days with bets)	280 active	10,252	\$11,459,310	<ul style="list-style-type: none"> • bettor records • figure sheets • PC backup • court documents
Bookie 1	Staten Island, NY	12/16/96-1/12/97 (17 days with bets)	54	1,332	\$314,810	<ul style="list-style-type: none"> • bettor records • figure sheets • audio tapes • address book • financial records • court documents
Bookie 2	Newark, NJ	8/31/92-9/30/92 (31 days with bets)	≈ 500	8,400	≈ \$9,700,000	court documents
Bookie 3	Brooklyn, NY	1/17/00-1/23/00 (7 days with bets)	112	1,308	\$479,433	bettor records
Bookie 4	Brooklyn, NY	1/17/00-1/23/00 (7 days with bets)	15	134	\$220,880	<ul style="list-style-type: none"> • bettor records • desk contents • audio tapes
Bookie 5	Secaucus, NJ	12/30/96-3/14/97 (77 days with bets)	249	—	\$4,732,879	<ul style="list-style-type: none"> • figure sheets • audio tapes • financial records
Bookie 6	Philadelphia, PA	11/1/97	—	—	—	game lines

Table 2: Data Overview

Data Used in Analysis – Internet Bookies

- Internal docs + discussions w/ CEOs of two books (public, private)
 - large: \$1⁺ billion/yr bet volume
 - mainly US bettors
 - started in 1995; 1998
 - financial records for lifetime of firm
- Financial reports from ten public books
 - Sportingbet, CanBet, BetInternet, . . .
 - larger ones operate > 1 internet site
 - 1-5 years of annual reports
 - \$0.1-\$1 billion/yr bet volume

Data Used in Analysis – Internet Bookies (CONT)

- Press releases from three large books
 - 2 private (WSEX, Intertops)
 - 1 public (WWTS–Betcorp)
 - \$1+ billion/yr bet volume
- Real-time price/odds data (DBC):
 - Feb 1999 - Feb 2000
 - all price changes for 6600 major sport games
 - data for 15 books (11 Nevada; 4 internet)

Sample Selection with Illegal Books

- Non-random data generation.
 - arrests by Kings County DA
 - cases subsequently resolved
- Might bias sample:
 - less successful bookies?
 - counter: bettors “dropping a dime”
 - more risk-loving?
 - geographic concentration
 - counter: consistency with non-NYC bookies (lit)
- Helps case for rationality: over-sample mistake prone bookies!
- **Argument against SS**
 - consistency with non-arrested bookies (lit)

1. Background and Qualitative Results

STYLIZED FACTS – Firm Structure I (details in paper/next)

Issue	Legal	Internet	Illegal
Separation of Owner/Executive	Y	Y/N	N
Closely Held/Partnership	N	Y/N	Y
Customer Interaction	Direct	Direct + Indirect(?)	Direct + Indirect
Customer Recruiters	N	Y/N	Y
Incentive Contracts	N	N	Y
Bet on Credit	N	N	Y

STYLIZED FACTS – Firm Structure II (details in paper/next)

Issue	Legal	Internet	Illegal
Bet Size	Small	Medium	Large
Market Power	N	N	Y
Firm Size	Medium	Medium-Large	Small-Medium
High Growth (Mergers)	N	Y	Y/N
Horizontal Integration (Casino betting)	Y	Y/N	N
Innovative	N	Y (exchange)	N

Key to above:

- info asymmetry → agency (monitor/trust workers and bettors)
- enforceability of contracts

Descriptive Stats: Table 3

- Large bookie: 0
 - \$500K/day bets, 280 bettors, 80 employees
 - same volume as large Vegas bookie: Caesars
- Moderate-sized bookie: 3, 5
 - \$60K/day bets, 100⁺ bettors, 10-20 employees
- Small bookie: 1, 4
 - \$25K/day bets, 0-10 employees. few bettors exit.
- General characteristics:
 - average bet \$250 - \$1000
 - bettors wager 1⁺ /day
 - majority bettors lose \$
 - volatile daily net revs (more later)

VARIABLE	Bookmaker 0	Bookmaker 1	Bookmaker 3	Bookmaker 4	Bookmaker 5
Bet Size					
Average	\$1,117.8	\$236.3	\$366.5	\$1,648.4	—
Maximum	\$23,820	\$3,000	\$5,000	\$4,000	—
Bets per Bettor-Day					
Average	3.58	1.45	4.44	1.28	—
Maximum	43	18	36	25	—
Bettors (%)					
Bet every day	7.1%	3.7%	9.9%	0%	2.0%
Exit	23.9%	11.1%	5.4%	6.7%	39.0%
Winings ≥ 0	49.3%	38.9%	22.5%	53.3%	48.4%
Bet Frequency					
Bets/Minute	1.23	—	—	—	—
Bet Types (%)					
Exotic Wagers	16.5%	26.3%	19.3%	0%	—
Football	2.5%	81.7%	21.5%	0%	—
Basketball	0%	16.2%	71.2%	98.5%	—
Baseball	97.5%	0%	0%	0%	—
Daily Bet Volume					
Average	\$520,877.7	\$18,518.2	\$68,490.4	\$31,554.3	\$61,466.0
Daily Net Revenue					
Average	\$5,892.4	\$1,554.9	\$9,228.4	-\$1,310.0	\$9,055.8
Standard Deviation	\$87,418.8	\$3,501.0	\$13,913.5	\$6,205.0	\$17,916.3
Maximum	\$163,232	\$8,461	\$37,429	\$7,480	\$47,960
Minimum	-\$103,891	-\$4,700	-\$3,190	-\$12,400	-\$61,205
Employees (number)					
Salariated pay	4	1	2	0	6
Commission pay	75	7	10	0	13

Table 3: Descriptive Statistics

Basic Facts – Contrast to Internet Bookies

- Internet bet size smaller (\$100-\$300)
 - reflects adverse selection
 - illegals screen bettors
- Internet book size is much larger
 - annual bet volume = \$0.1b-\$2b
 - number bettors = 20k-1000k
 - organic growth + mergers
 - illegals require trust (below) → limits expansion/mergers
- Horizontal Integration
 - many internet books provide casino, lottery games
 - high margins; leverage customer base (spread out FC)
 - illegals only sports → avoid bettors interacting (↑ arrest prob)

Market Power

- Approach 1: firm-level data (**Appendix B**).
 - Lerner index \equiv markup prices to MC
 - elasticity-adjusted Lerner index = 0.35
 - e.g. 3 firms under Cournot
- Approach 2: market concentration.
 - big bookie 0 is < 5% NYC market
 - C4 < 35% for NYC
 - suggests perfect competition

Market Power (CONT)

- Reconciliation:
 - bet market is neighborhood
 - local oligopoly, e.g. neighborhood bookie
 - most bookies only serve one market
- Barriers to entry:
 - \$ for K reserves
 - outsider's difficulty in gaining trust
 - access financial diversification services
- Internet bookies (contrast)
 - no local markets
 - less concentrated (no book > 5% US market)

Organization and Operation

- Textbook firm structure:
 - min transaction costs
- Bookmaker (one or two guys):
 - owner: residual claimant on profits/losses
 - executive/policy-setter: prices, new bettors, employees, layoffs
 - multiple prior arrests: rarely served prison sentence
 - but asset forfeiture common
- Wireroom (regular hours):
 - writers/clerks
 - collector/bag-man/runner
 - flat wages: \$200-\$500/week $\approx 2 \times$ Min Wage
 - mainly close friends/relatives of bookie

Organization and Operation (CONT)

- Sheetholders:
 - recruit and service customers
 - wages based on bettor losses (next)
 - hi salary: \$1500/week for bookie 0
 - most are prior acquaintance of bookie
- redundancy:
 - management deal with bettors *and* with sheetholders
- return to sheetholders below

Organization and Operation (CONT)

- Common practices:
 - low tech (paper records)
 - bet commissions: identical to Vegas
- Betting on credit:
 - each bettor given credit limit
 - weekly settle-up
 - ST credit involves a lot \$:
 - \$3400/week for bettor with bookie 0
 - some allow LT extensions: up to 3 months
 - bettors unable to pay debt allowed to work it off
 - NB: credit requires trust

Organization and Operation – Internet Bookies

- Difference 1: impersonal relationship with bettors
 - implications: no credit, no/few sheetholders
 - geographic separation hinders establishing personal relationship
- Difference 2: innovative (bet exchanges)
 - exchanges require thick market with diverse opinions
 - illegals have small, homogeneous mkts (neighborhood)
 - exchanges require continuous transmission of info
 - illegals have costly info transmission
 - innovations due to competitive forces
 - illegals have barriers to entry and less need to innovate
- Difference 3: competition on commissions
 - easy entry
 - illegals – trust limits entry into neighborhood

Organization and Operation – Internet Bookies (CONT)

- Similarity 1: little separation ownership/management
 - {less true for public internet books}
 - lack of separation for different reasons in each sector
 - internet: economize on costs
 - illegal: info asymmetry (risk of manager stealing), no enforceable contracts (promise to pay manager) discourage separation
- Similarity 2: closely held ownership
 - {less true for public internet books}
 - both overcome classic problems in partnerships
 - free-riding 1: effort is private cost; π is PG
 - free-riding 2: monitor other partners is PG
 - mitigated – few partners, daily interaction, long-time relationship
 - still limits ability of illegals to grow
 - how outside investors monitor; adv selection in seeking financing

Revenues, Costs, Profits: **Table 4**

- Net revenues:
 - -4% to 15% hold rate
 - no clear relation to bookie size
 - no clear relation legal mkt hold rate
- Costs:
 - mainly sheetholder commissions
- Profits:
 - \$250,000 - \$1m per year (<1% bet volume)
 - π rate \approx non-arrested bookie
 - $\pi >$ leader's wages for drug-selling gang (Levitt and Venkatesh, 2000)
 - π attractive?
 - * bookie has limited outside opportunities
 - * but must invest substantial K (next!)

VARIABLE	Bookmaker 0	Bookmaker 1	Bookmaker 3	Bookmaker 4	Bookmaker 5
Obs. Period (days)	22	17	7	7	77
REVENUES					
Bet Volume	\$11,459,310	\$314,810	\$479,433	\$220,880	\$4,732,879
(Net) Revenue	\$129,632	\$26,434	\$64,599	-\$9170	\$697,294
% Hold	1.31%	8.40%	13.47%	-4.15%	14.73%
COSTS					
Sheetholder	\$99,189	\$8,610	\$36,840	\$0	\$453,350
Commissions					
Unpaid/Delayed	\$11,486	\$720	NA	\$50	\$40,462
Debt Payment					
Salaries to Writers and Collectors	\$2,520	\$1,350	\$500	\$0	\$19,800
Rent	\$3,100	\$1,900	\$600	\$250	\$11,000
Line Service	\$200	\$550	\$200	\$100	\$1,350
Subscription					
Utilities (phone, electric, pagers)	\$497	\$611	\$701	\$478	\$3,567
Total Costs	\$116,992	\$13,741	\$38,841	\$878	\$529,529
PROFIT					
Profit	\$12,640	\$12,693	\$25,758	-\$9,648	\$167,765
Profit annualized	≈ \$210,000	≈ \$220,000	≈ \$1,300,000	≈ -\$400,000	≈ \$800,000

Table 4: Bookmaker Accounting Statement

Revenues, Costs, Profits – Internet Bookies

- Costs:
 - lower (no sheetholders)
 - higher (marketing, customer service)
 - costs are 3-7% bet volume (vs 1-11% for illegals)
- π rate:
 - -20% - 2% bet volume
 - \uparrow with age
 - illegals: -4% - 5% bet volume
- higher opportunity cost K?

Qualitative Result: Role of Trust

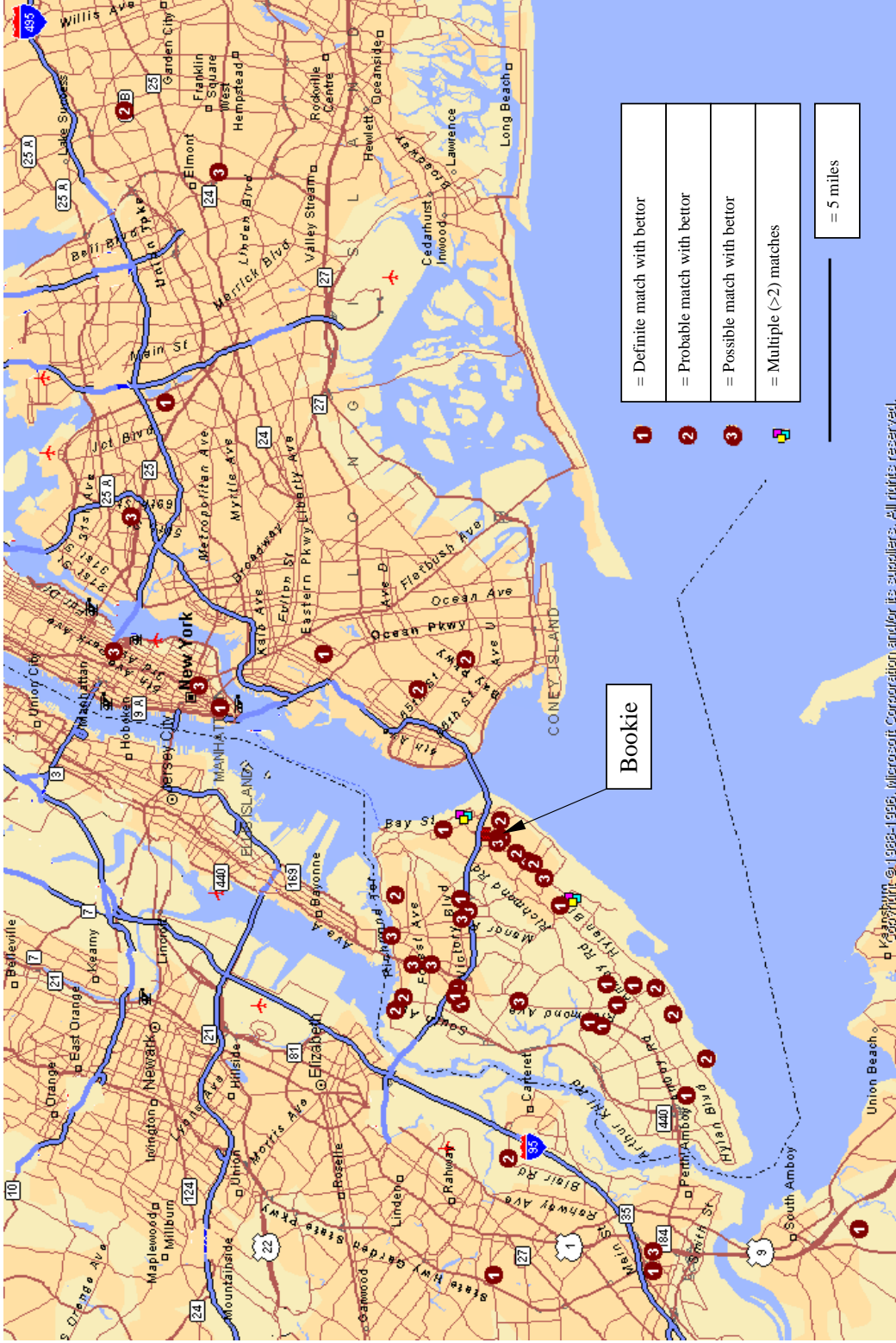
- Bookie must trust bettor (e.g. not to inform police)
 - Legally binding contracts not possible
1. Small bookmaker (< 100 bettors)– **Appendix C**
 - Bookie knows/personally interacts with bettors
 - Stable client base (**Table 3**)
 - Map listings in address book (**Figure 5**)
 - Same neighborhood + similar ethnic background
(match Fig. 5 address to census tract)

∴ trust develops
 2. Big bookmaker (> 100 bettors)
 - more significant problem: little personal \cap
 - solution: sheetholding arrangement (below)

Qualitative Result: Role of Trust (CONT)

Importance: allow betting on credit

- Bookie trusts bettors to repay him
- *Adv 1:* ↓ \$ exchanges → ↓ theft, arrest
- *Adv 2:* ↑ number bettors
illiquid, TI indiv. start addictive behavior
- *Adv 3:* induce more betting | bettor
end of week “catch up”



Staten Island Bookie

Figure 5: Bookmaker 1's Place of Business and His Bettors' Place of Residence

Sheetholding Arrangement

- Sheetholder recruit customers (average: 8.5)
 - acquaintances
 - exclusive relationship
 - bettors from same neighborhood
 - similar betting patterns
- Multiple layers possible (**Figure 1**)
- Will see like *limited liability franchise*

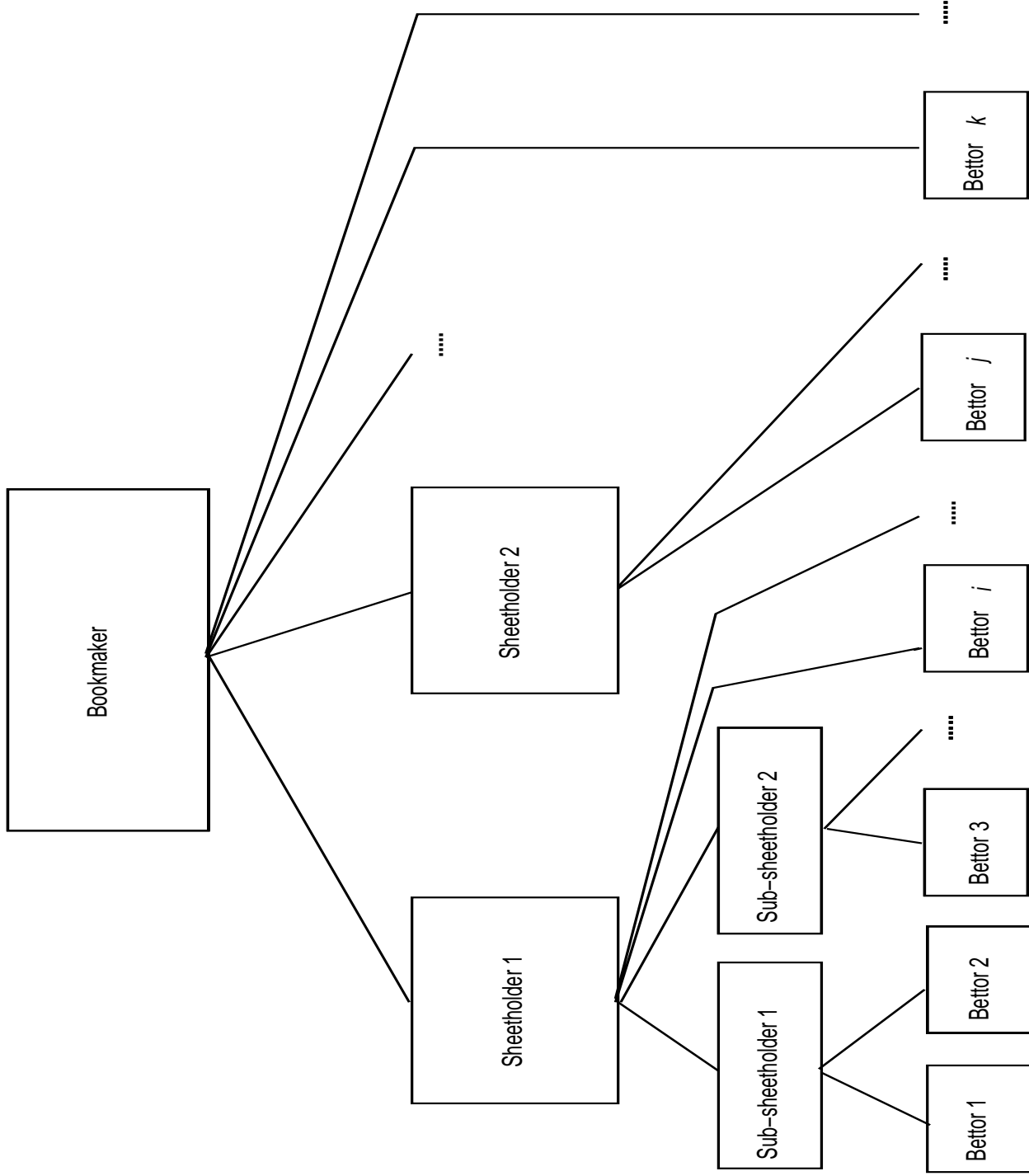


Figure 1: Sheetholding Arrangement for Large Bookmaking Operation

Sheetholding Arrangement (CONT)

- *Red figure system*
 - weekly salary =
sheet's rate \times net weekly position
 - net weekly position =
 $\max(\text{bettors' current losses} - \text{red fig}, 0)$
 - red fig = cumulative bettor wins ...
 - example from text ...
 - NB: only paid if bettors repay debts
- Advantage to sheetholder:
 - cannot lose \$
 - not need access to capital

Sheetholding Arrangement (CONT)

- Why bookie use red figure?
 1. sheetholders credit constrained (above)
 2. sheetholders superior info on bettors:
 - high power incentives along with insurance sheetholder shares π but limited liability
 - incentive to bring “high quality” bettors . . .
 - aligns more informed agents’ prefs with principal
 - more on aligning sheetholder preferences.
 - * tenure premium in wages (below)
 - * like implicit investment in firm
 - * gives incentive to ensure firm longevity
 - not have classical problems w/ high power incentives
 - * output/effort (bettor losses, payments) accurately measured
 - * risk neutral agent \rightarrow not need insurance

Sheetholding Arrangement (CONT)

- DAs for bookie:
 1. high red fig \rightarrow “sheetholder flight”
solution: reputation + tenure premium (empirics below)
 2. \Downarrow E(net inc) AND little effect on Var(net inc)
 3. implicit sheetholder debt repayed with lag.
bookie 0: hi within-sheet payoff correlation
 \rightarrow pay commissions even if net rev = 0.
- Still nash among feasible incentive contracts
(if info asymmetry, sheeth. credit constrained)

Sheetholders: Evidence on Link to Information Asymmetry

- Small bookies have no sheetholders → no info problems
- Higher growth rate for big bookies
- Internet books typically not use → trust less important
no bet on credit; not need to worry about arrest
- Redundancy in structure (Radner, 1993)
 - larger organization has more elaborate arrangement
 - management deals with both bettors and sheetholders (Fig 1)
 - hierarchy shapes management ability to process information
 - bettor behavior provides real time check on sheetholder behavior
 - not true in “balanced tree”

1. Wage rates: cols (1) - (2)

$$\textit{Sheetholding Rate}_{it} = \alpha \times X_{i,t-1} + \nu_i + \epsilon_{it}$$

- (1): productivity premium
- (2): reputation/tenure wage premium

2. Sheetholder entry/exit: col (3)

$$\Pr(\textit{Employed}_{it}) = \lambda_0(t) \exp(X_{i,t-1}\beta)$$

- {(instrumented) wage rate ↓ exits}
- no red figure “sheetholder flight”

3. Sheetholder “effort” intensity: cols (4) - (5)

$$\textit{Sheetholder Intensity}_{it} = \gamma \times X_{i,t-1} + \mu_i + \delta_t + u_{it}$$

- effort = num bettors, bet volume
- red figure has no effect
- e.g. not channel portion bettors away

DEPENDENT VAR.	<i>Sheet. Rate</i>	<i>Pr(Employed)</i>	<i>Bet Volume</i> ($\times 10^{-4}$)	<i>Num. Bettors</i>
COVARIATE (lagged)	(1)	(2)	(3)	(5)
constant	0.220 (0.014)	0.333 (0.109)		0.214 (0.021)
Net Revenue from Bettors ($\times 10^{-3}$)	0.090 (0.020)	0.071 (0.045)	-0.030 (0.056)	6.204 (0.456)
Bet Volume ($\times 10^{-4}$)	0.012 (0.003)	0.009 (0.007)	0.911 (0.292)	
Number Bettors	0.003 (0.005)	0.004 (0.009)		
Previous Years of Service		0.031 (0.016)		
Previous Year's Net Rev. ($\times 10^{-4}$)		0.061 (0.009)		
Commission Rate			0.886 (0.422)	0.446 (0.084)
Red Figure ($\times 10^{-3}$)			0.006 (0.011)	0.011 (0.078)
Sheetholder FE?	Yes	No	No	Yes
Time FE?	No	No	baseline hazard	Yes
Frequency	Week	Week	Week	Day
R^2	0.41	0.19		0.63
$\log L$			-159.23	
N	121	121	121	825

Table 5: Sheetholder Incentive System Evaluation: Bookmaker 5.

Trust/Sheetholders – Internet Bookies

- Hard to develop trust
 - no personal/physical interaction
 - in different countries
 - no property rights
- Trust less important anyway
 - not need to worry about arrest
- Implications:
 - (typically) not use sheetholders
 - no bet on credit (cash accounts only)
 - faster growth (easier to add bettors)

Trust/Sheetholders – Internet Bookies (CONT)

- WWTS uses “agents” (bettor recruiters)
 - likely illegal bookies/sheetholders in US
 - pay proportionate to losses of referred bettors
 - huge expense: 9% net revs in 2003
 - www.betwwts.com: “You can earn up to 40% back on your referred customers’ losses”
 - only a few recruiters
 - top agent=40% agent revs; top eight agents=80% agent revs
 - bring in over a quarter of bet volume
- Formal alliances with illegal bookies:
 - used to extend credit
 - overcomes inability to enforce contracts (illegal bookies collect)
 - source: NYC arrests; CEO discussions

2. Prices and Financial Risk/Profits

STYLIZED FACTS – Firm Behavior

Issue	Legal	Internet	Illegal
Pricing	Vegas	Vegas	Vegas + Home Team + Discrimination
Competition on Commissions	N	Y	N
Financial Risk Management	Balanced Book	little hedging + small risk	little hedging + large risk

2a. Prices

Prices – Overview

- “Price” fixed at time of wager
 - F, BB, H: price = –spread
 - BS: price = odds (from (8))
- {No competition on commissions (discuss later)}

Prices – Objectives

1. Compare with legal price:
 - easy for bookies to offer legal price if want
 - real-time Vegas prices readily available
 - bets only on games that day
2. Check for price discrimination:
 - harder for bookie to do
 - difficulty 1: competition from other bookies (not too important)
 - difficulty 2: bettor picks side he wagers on
 - bad price on $A \rightarrow$ good price on $\sim A$
e.g. spread
 - {NB: never set separate lines for each side of game in data}
 - BUT bookie has betting history of each bettor:
can use this to infer bettor prefs and price discriminate

Theory: **Appendix D.2**

Large Bookmaker (Bookie 0) Prices: Preliminaries

- Records exact minute wager placed (mainly BS):
 $price_t$
- Match with real time Las Vegas price:
 $price_{vegas,t}$

Bookie 0 Price Empirics: Regressions (Table 7)

$$price_{i,t} = \alpha + \beta \times price_{vegas,t} + \gamma_1 X_{i,t} + \gamma_2 X_i + \nu_i + \nu_t + \epsilon_{i,t} \quad (4)$$

- (1): prices largely follow legal line
 - $\alpha \approx 0$, $\beta \approx 1 \rightarrow$ BUT can reject
 - $\alpha > 0 \rightarrow$ unfavorable bettor price.
matches raw data (Table 6)
 - NB: not linear shift legal price (game \times team FE)
- (2): deviations due to bettor-specific factors
 - $R^2 \rightarrow 1$
 - 10% bettors FE $\gg 0 \rightarrow$ unfavorable price
 - $< 1\%$ bettors FE $\ll 0$

Bookie 0 Price Regressions: **Table 7 (CONT)**

$$price_{i,t} = \alpha + \beta \times price_{vegas,t} + \gamma_1 X_{i,t} + \gamma_2 X_i + \nu_i + \nu_t + \epsilon_{i,t} \quad (4)$$

- (3): bettor-specific factors help explain prices
 - small effect:
Bookie Position, Time from Game, Debt
 - big effect:
Herfindahl = bet pattern persistence
 - ex: perfect team loyalty
Herf_{FA=1} \rightarrow *price* \uparrow 0.024 (allow bookie arbitrage)
 - interaction:
loyalty effect \uparrow bet frequency

NY Yankees games

- sentimental favorite in much of NYC
- (4): R^2 only 0.8
- (5): pro-NYY bets have constant $\gg 0$
 - anti-NYY bets have constant ≈ 0 (*omitted*)
 - \therefore not linear price shift
- Better FE: (*omitted*)
 - $R^2 \rightarrow 1$. Herf key to explain better FE.

Bookie 0 Price Regressions: NY Yankees games (CONT)

- Implication:
 - Yankees loyalists ($\frac{1}{4}$ sample)
 - $WTP = -(\Pr(Y) \times \frac{1-price}{price} - (1 - \Pr(Y)) \times 1)$
 - Yankees loyalists $WTP = 4.2\phi$ / dollar bet
→ 2x BS commission
- State-dependent utility function: $U(Y, SOW)$
 - VN prefs:
 - insure against bad SOW → bet against Yankees
 - SD utility:
 - ↑ MU income in happier SOW → bet on Yankees
 - prior evidence – extreme SOW (death) or health outcomes
 - here – may apply in financial context as well
 - not insure against loss by “sentimental favorite”
 - ex: election outcome

Bookie 0 Price Regressions: NY Yankees games (CONT)

- Small effect on π :
 - unfavorable wagers involve small stakes
 - \uparrow net revs per bet by 0.3%

VARIABLE	Definition	Mean	St. Dev.	Min	Max
SAMPLE SIZE					
N	number of wagers	11797	—	—	—
N_i	number of bettors	263	—	—	—
WAGER-LEVEL					
$price_{bookmaker}$	bookmaker price on wager, see (8)	0.522	0.080	0.278	0.737
$price_{vegas}$	Las Vegas price at the exact minute of the bookmaker wager	0.519	0.008	0.286	0.737
Bookie Position	\$ bookmaker must pay if wagered team wins game	11734.78	13490.36	-59566	61847
Time from Game	time between bet and games (minutes)	140.90	160.35	0	1465
Bet Amount	\$ on each part of wager presuming the favorite team wins (this assumption is needed since under a money line the size of the payoff depends on which side wins)	920.73	1293.95	0	10400
BETTOR-LEVEL					
Prior Debt	\$ which bettor owes the bookmaker at the start of the betting day (it is reset each week and equals zero if the bettor has cumulative winnings)	2410.56	5488.34	0	45750
Sunday Debt	$PriorDebt \times \mathcal{I}_{Sunday}$	550.058	3120.01	0	39040
Number Bets	number of bets bettor places over the sample period	37.91	55.90	1	533
Herfindahl _T	$\frac{\sum_{t \in teams} Number\ Bets \times p_t^2 - 1}{Number\ Bets - 1}$ where p_t is the proportion of the bettor's wagers on team t	0.086	0.166	0	1
Herfindahl _{FA}	$\sum_{t \in teams} 2p_t(p_{tF}^2 + p_{tA}^2) - 1$ where p_{tF} (p_{tA}) is the proportion of wagers involving team t which are for (against) team t	0.624	0.273	0.036	1

Table 6: Definitions and Descriptive Statistics: Bookmaker 0 Prices and Betting Patterns

DEPENDENT VAR. SAMPLE COVARIATE	<i>price</i> _{bookmaker,t}				
	All Games (1)	(2)	(3)	NY (4)	pro-NY (5)
constant	0.013 (0.001)	0.001 (0.001)	0.006 (0.001)	0.016 (0.003)	0.024 (0.005)
<i>price</i> _{vegas,t}	0.979 (0.002)	0.996 (0.002)	0.989 (0.001)	0.970 (0.005)	0.974 (0.009)
Bookie Position ($\times 10^{-6}$)			0.167 (0.010)		
Time from Game ($\times 10^{-5}$)			-0.194 (0.077)		
Bet Amount ($\times 10^{-5}$)			-0.199 (0.010)		
Prior Debt ($\times 10^{-7}$)			0.547 (0.272)		
Sunday Debt ($\times 10^{-6}$)			0.180 (0.072)		
Number Bets ($\times 10^{-4}$)			0.135 (0.018)		
Herfindahl _T ($\times 10^{-2}$)			0.104 (0.077)		
Herfindahl _{FA}			0.013 (0.003)		
Herfindahl _{FA} × Number Bets ($\times 10^{-3}$)			0.230 (0.061)		
Day of the week indicators?	No	No	Yes	No	No
Bettor FE?	No	Yes	No	No	No
R^2	0.91	0.98	0.98	0.81	0.85
N	11797	11797	11786	1223	811

Table 7: Bookmaker 0 Prices. OLS Regression Analysis

Price Discrimination Interp

1. Most bookie 0 bettors wager at Vegas prices
2. Price discriminate against bettors with team loyalty
 - e.g. always bet *for* some set of teams
 - $\frac{1}{10}$ bettors
 - consistently wager at unfavorable price
 - NB: bookie must know bettor will pick unfavorably priced side (use betting history)
3. Bettors wagering at favorable price
 - few bettors \rightarrow heavy, infrequent gambles
 - arbitageurs?

Yankees games: patterns accentuated

Supplemental Evidence for Price Discrimination

1. Simultaneous wagers on same game
 - $> 50\%$ at different price
 - only bettors with strong team loyalty consistently wager at hi price
 - evidence of bettor-specific prices
2. Convert to price for favorite team
 - controls for bettor's choice of side
 - persistence vars no longer significant
 - consistent with price discrim:
sentimental team not always favorite
3. Only wagers with big price diff from Vegas
 - i.e. allow arbitrage
 - 8% allow bookie arbitrage
→ mainly with persistent bettors
 - $< 1\%$ allow bettor arbitrage

Price Discrimination: Qualitative Evidence

voice 1: Was that Norm40? Did he play the Knicks again?

voice 2: Yeh. Picked them for a nickel. [a \$500 bet]

voice 1: What a *****! Don't he realize the boss is charging him four points extra? We might as well just pick his pocket...Andy50 never gets a soft line, he always gets Vegas. A real wise guy [a well informed bettor]. [Re]member when he busted us on Jack[sonville]? He can always get his boys in Vegas to lay off the other way.

excerpted conversation between wire room operators employed by Bookmaker 5

• **Old Table 7:**

- most bettors receive price near Las Vegas
- few get mainly good lines ($\Delta price < 0$)
very few get (and bet on) **Arbitrage**
- $\approx 10\%$ get mainly bad lines.
 $\approx 10\%$ get \sim **Arbitrage**
- bettor characteristics
- **Herfindahl**: bet pattern persistence
- consistent with price discrimination . . .

• **Old Table 8:** Just Yankees games

- earlier patterns accentuated
- loyalists (“mainly bad”).
many; $\frac{3}{4}$ prices satisfy \sim **Arbitrage**
- not just uniform line shift . . .

VARIABLE	Bettor-Level (conditional on price category)					
	Overall	Mainly Good	Many Good Arb.	Mainly Bad	Many Bad Arb.	
<i>price</i>	0.530 (0.041)	0.525 (0.035)	0.534 (-)	0.556 (0.037)	0.584 (0.027)	
$\Delta price$	0.0021 (0.0078)	-0.0093 (0.0041)	-0.0206 (-)	0.0131 (0.0045)	0.0145 (0.0034)	
Arbitrage	0.007 (0.012)	0.009 (0.015)	0.417 (-)	0.001 (0.003)	0.000 (0.001)	
\sim Arbitrage	0.084 (0.116)	0 (-)	0 (-)	0.214 (0.161)	0.349 (0.082)	
Bet Amount per wager	1052.66 (1290.84)	1513.81 (1330.56)	2837.08 (-)	264.89 (372.94)	472.95 (772.83)	
Prior Debt at start of day	1321.90 (2897.96)	865.79 (1628.67)	1514.58 (-)	1483.54 (664.38)	1373.5038 (1542.35)	
Number Bets in sample	37.91 (55.90)	7.75 (8.01)	4.00 (-)	33.08 (38.22)	31.73 (35.13)	
Herfindahl _T	0.086 (0.166)	0.140 (0.435)	0 (-)	0.258 (0.355)	0.179 (0.106)	
Herfindahl _{FA}	0.624 (0.273)	0.678 (0.234)	0.500 (-)	0.878 (0.234)	0.877 (0.229)	
Herfindahl _{BT}	0.878 (0.213)	0.977 (.078)	1 (-)	0.878 (0.203)	0.880 (0.228)	
<i>N</i>	263	12	1	39	26	

OLD Table 7: Cross Tabulations of Bookmaker 0 Prices and Betting Patterns

VARIABLE	For and Against Yankees Wagers			For Yankees Wagers only		
	Overall	Mainly Good	Mainly Bad	Overall	Mainly Good	Mainly Bad
<i>price</i>	0.532 (0.060)	0.516 (0.055)	0.559 (0.067)	0.572 (0.106)	0.562 (0.052)	0.568 (0.067)
$\Delta price$	0.0079 (0.0112)	-0.0049 (0.0046)	0.0219 (0.0086)	0.0151 (0.0113)	-0.0035 (0.0037)	0.0242 (0.0087)
Arbitrage	0.003 (0.012)	0.004 (0.014)	0 (-)	0.000 (.007)	0 (-)	0 (-)
\sim Arbitrage	0.300 (0.257)	0.005 (0.030)	0.772 (0.367)	0.449 (0.331)	0 (-)	0.797 (0.400)
Bet Amount per wager	1137.01 (1457.59)	1839.20 (1920.27)	552.51 (821.86)	1044.13 (1426.11)	1801.50 (1959.47)	745.87 (1116.42)
Prior Debt at start of day	1465.213 (3402.66)	1167.96 (1887.23)	1749.117 (3699.49)	1576.222 (4301.28)	1508.28 (2900.31)	2115.80 (4886.22)
Number Bets in sample	5.91 (7.74)	2.81 (2.03)	8.36 (4.00)	7.665 (4.023)	1.722 (1.301)	6.620 (3.073)
Herfindahl _{FA}	0.774 (0.323)	0.605 (0.323)	0.909 (0.280)	0.897 (0.248)	0.702 (0.235)	0.945 (0.209)
Herfindahl _{BT}	0.917 (0.193)	0.966 (0.127)	0.939 (0.160)	0.946 (-)	0.988 (0.070)	0.946 (0.162)
<i>N</i>	207	29	45	179	12	71

OLD Table 8: Bookmaker 0 Prices and Betting Patterns: Bets on New York Yankees games

Smaller Bookmakers Prices (**Table 8**)

- Bookie 1 (very small)
 - (1): legal prices explain 99.9% variation.
 - (2)-(3): 1 pt uniform line shift against NY teams
 - (1)-(3): bettor FE, other covariates insign.
- Bookie 3 (moderate size):
 - (4): mainly legal prices
 - (5): but some bettor-specific prices
 - (6): high price linked to persistent betting
 - e.g. price discrimination.
BUT only $\frac{1}{16}$ bettors (\ll bookie 0)
- Bookie 4:
 - contemporaneous with bookie 3
 - Corr(prices)=0.98
 - e.g. little illegal mkt. price dispersion
 - contrast: 1970s had dispersion (role tech Δ)

SAMPLE DEP. VAR	Bookmaker 1			Bookmaker 3		
	full $price_{i,t}$ (1)	pro-NY bets $price_{i,t}$ (2)	anti-NY bets $price_{i,t}$ (3)	full $price_{i,t}$ (4)	full $price_{i,t}$ (5)	full FE from (5) (6)
COVARIATE						
constant	0.018 (0.029)	1.212 (0.087)	-1.226 (0.063)	0.146 (0.056)	-0.025 (0.014)	-0.121 (0.009)
$price_{vegast}$	1.001 (0.001)	1.007 (0.003)	1.005 (0.002)	0.987 (0.002)	0.998 (0.001)	
Herfindahl _T						0.530 (0.135)
Herfindahl _{FA}						1.172 (0.070)
Bettor FE?	No	No	No	No	Yes	No
R^2	0.999	0.999	1.000	0.948	0.999	0.51
N	1893	58	101	1532	1532	112

Table 8: Bookmakers 1, 3 Prices. OLS Regression Analysis

Prices: Conclusion

- Bookie price almost lockstep with legal
- . . . except home-town price unfavorable
- . . . except price discrimination by moderate, big books
→ more evidence of mkt power

Prices – Internet Books Differences

1. Competition on commissions
 - illegal/legal: standardized commissions (4.5% for F, BB)
 - Tradesports:
 - 8¢ per lot → 2% commission
 - BOS:
 - loyalty rewards; initial deposit bonus
 - WWTS:
 - Friday night reduced commissions; initial deposit bonus
 - explanation:
 - role of competition (price shopping difficult in illegal sector)

Prices – Internet Books Differences (CONT)

2. Legal prices offered

- initial lines from Vegas / Don Best
- only small price shifts when bet imbalance
- little price dispersion (across internet book and with Vegas)
0.001% have > 1 pt differential in football
- evidence:
 - real-time DBC data (6600 games; $N = 10^6$ prices for day of event); www.oddschecker.com; CEO discussion
- explanation:
 - role of competition (easy to arbitrage on internet)
- note:
 - illegal books have less “representative” bettors (local mkt)
→ ↑ need to shift prices

- 3. Little price discrimination
 - explanation:
greater churning of bettors; role of competition
 - NB: technically feasible to price discriminate
→ individ-specific prices when log-in
 - evidence:
CEO discussion + internal docs

Overall

- legal status is crucial
- influences info about bettor, need for price competition
- but constrained by bettors' ability to choose

2b. Financial Risk/Profits

Financial Risk Taking

- CW: “balanced book” \rightarrow Std Dev(net rev)=0
- How obtain balanced book:
 - layoff
 - adjust price
- Costs to each form of hedging:
 - layoff: pay commissions
 - adjust price: “middling;” bet cascades
- \therefore CW hard to understand:
 - hedging is costly
 - balanced book requires extreme RA
- **Goal:**
 - Calc extent of financial risk in practice

Calculating Net Revs Distribution

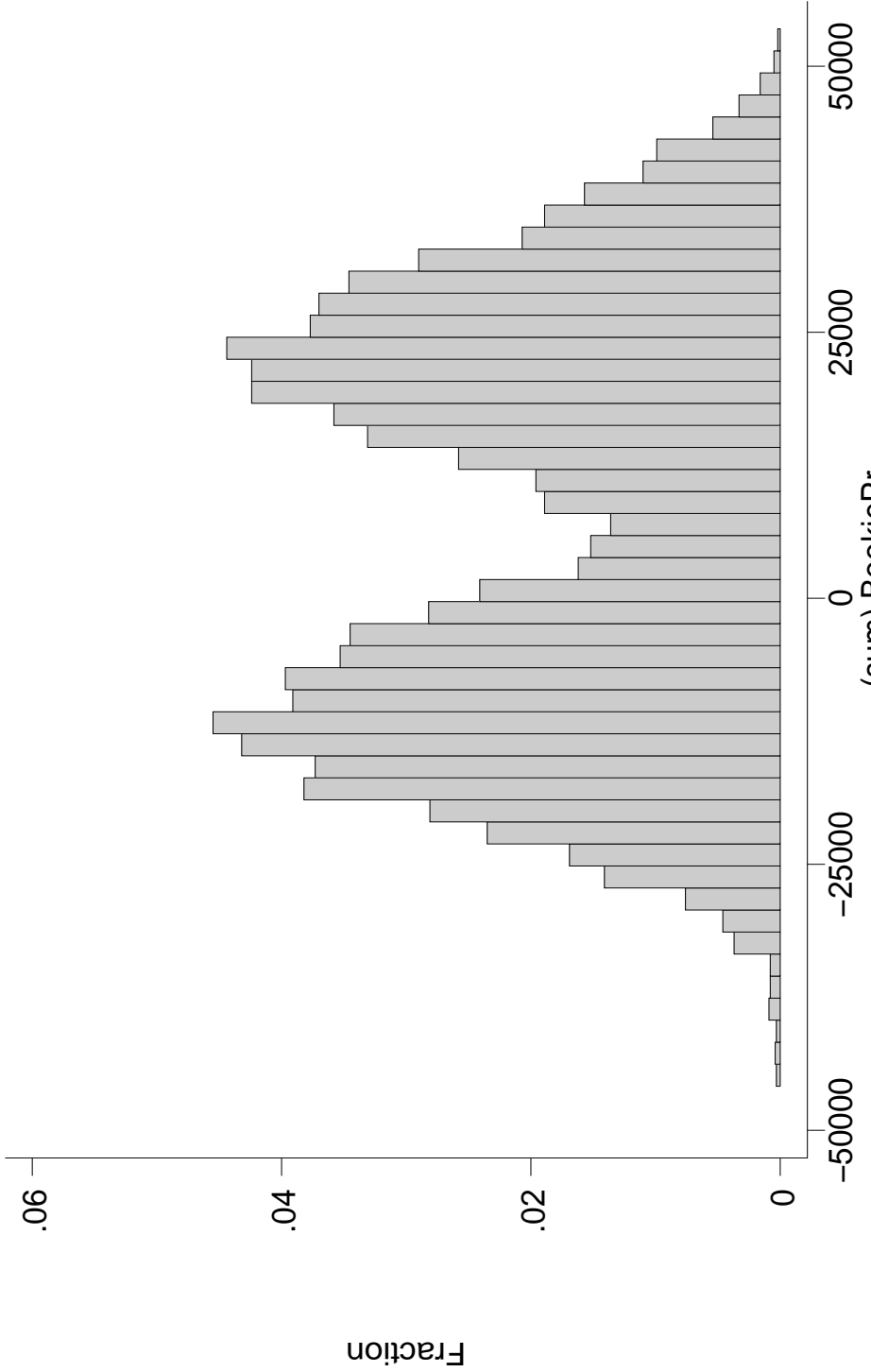
Overview

- distribution \equiv prob-wt. sum all possible rev outcomes
- prob: use prices to infer distribution of game results
- rev outcome: calc bookie net rev for given outcome (vector)
- **Figure:** unbalanced bets on one event

Calculating Net Revs Distribution

Details (Appendix E.1)

- *game/event*:
 - prob of outcome \rightarrow use mkts
 - * BS (money line): odds via (8)
 - * F, BB, H: outcome $\sim \mathcal{N}(\text{spread}_V, \sigma_{\text{season}}^2)$
 - revenue \rightarrow net revs given outcome and bets which were placed
- *daily* net rev distribution:
 - BS: calc exact distrib ($2^N \approx 16\text{K}$ outcomes)
 - F, BB, H: draw from distribution vector
 - NB: presume game outcomes independent
- caveat:
 - calc compromised if a central authority pools risk \rightarrow e.g. O.C.
 - evidence against:
 - competition, layoffs, observed losses

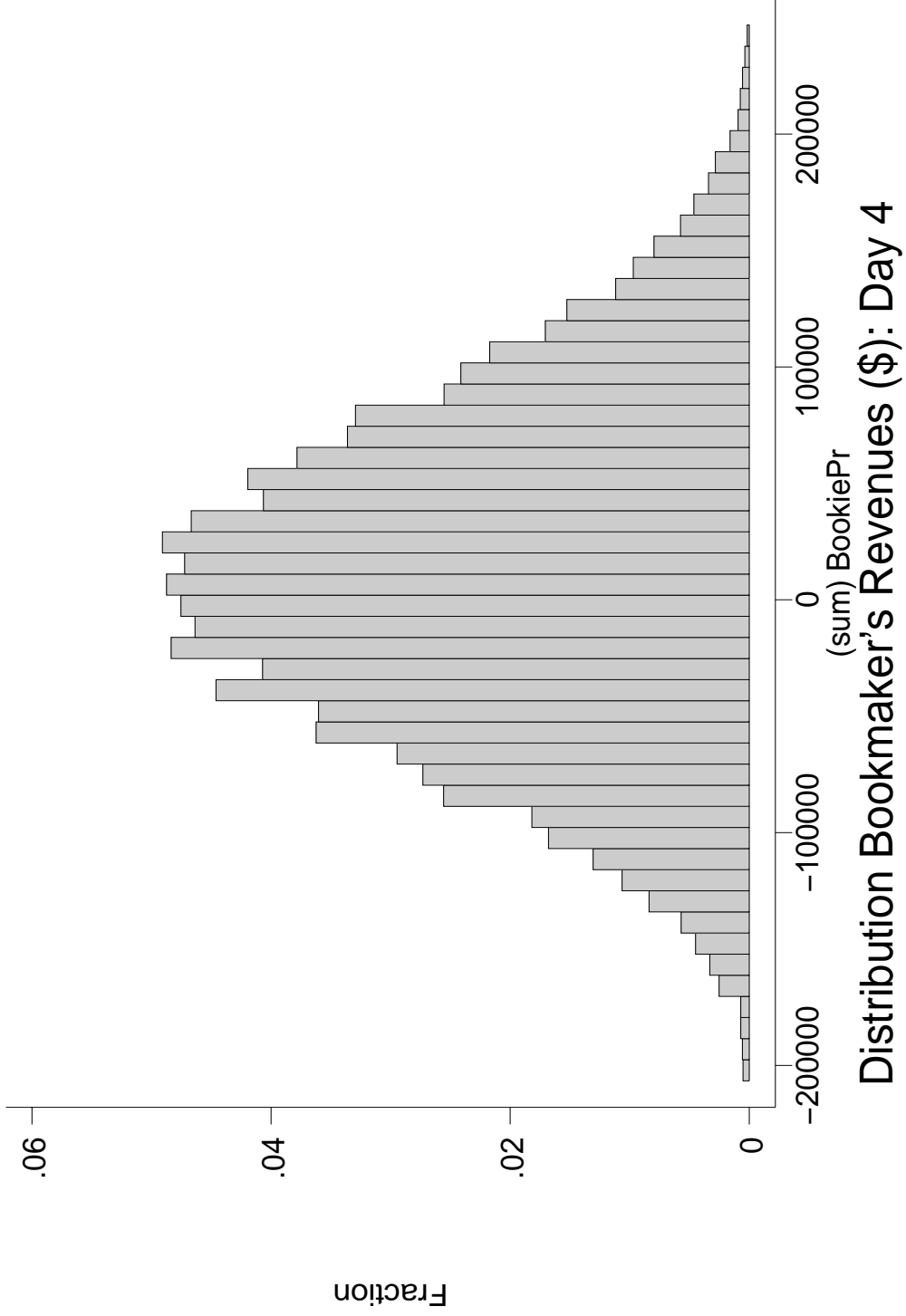


Distribution Bookmaker 3's [Simulated] Revenues (\$): Day 7

Risk Taking: Large Bookmaker (Bookie 0)

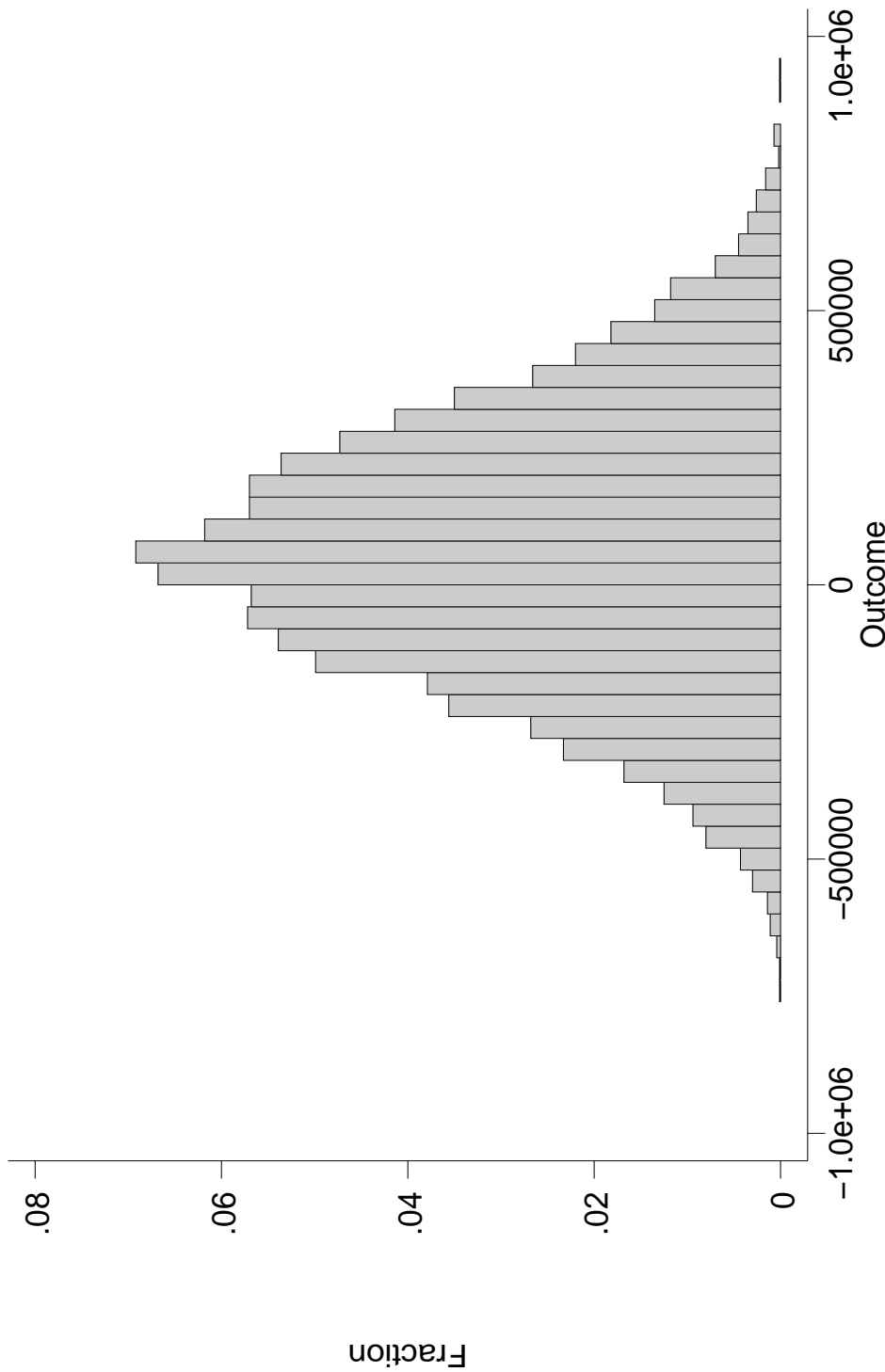
- Takes very large positions (and never lays off)
e.g. payout depend on which team wins
- Daily/weekly distribution net revs, \tilde{R} .
weekly \rightarrow since settle-up period.
Monte Carlo simulated
- Net revs quite variable (**Figs 2,3**):
 - **Fig 2**: mean=\$20K, std dev=\$70K
 - **Fig 3**: mean=\$60K, std dev=\$240K
 - Actual: weekly net revs $\in [-\$400k, \$300k]$
 - $\tilde{R} = \sum_{g \in G} \tilde{R}_g \xrightarrow{CLT} \text{Std Dev}(\tilde{R}) \propto \sqrt{G}$
 - intuition:
 - (i) idiosyncratic risk not wash out \rightarrow games independent
 - (ii) systematic risk too (home-town teams)

Figure 2: Daily Revenue Distribution: Bookmaker 0



Distribution Bookmaker's Revenues (\$): Day 4

Figure 3: [Simulated] Weekly Revenue Distribution: Bookmaker 0



Distribution Bookmaker's [Simulated] Weekly Revenues: Week 2

- Min K : **Table 9** (top panel)
 - negative outcomes necessitate \$ reserve
 - deal with worst weekly loss
- $$K_T^* \equiv -\min R^* : \Pr(\tilde{R} < R^*) = T$$
- $K_{0.001}^* \approx \$750,000$
 - NB: close to available \$ when arrested!

VARIABLE	Average Daily Return	Normal Distribution	Monte Carlo Simulation
MINIMUM CAPITAL (\$)			
K_0^*	1,219,132	1,023,564	934,262
$K_{0.001}^*$	1,095,576	669,090	766,527
$K_{0.01}^*$	889,619	539,607	630,615

Bookmaker 0

VARIABLE	Average Daily Return	Normal Distribution	Monte Carlo Simulation
MINIMUM CAPITAL (\$)			
K_0^*	47,852	47,639	27,295
$K_{0.001}^*$	33,768	17,804	17,765
$K_{0.01}^*$	27,690	10,837	11,474

Bookmaker 1

VARIABLE	Average Daily Return	Normal Distribution	Monte Carlo Simulation
MINIMUM CAPITAL (\$)			
K_0^*	143,280	154,503	85,663
$K_{0.001}^*$	103,542	77,182	73,307
$K_{0.01}^*$	76,665	59,125	51,813

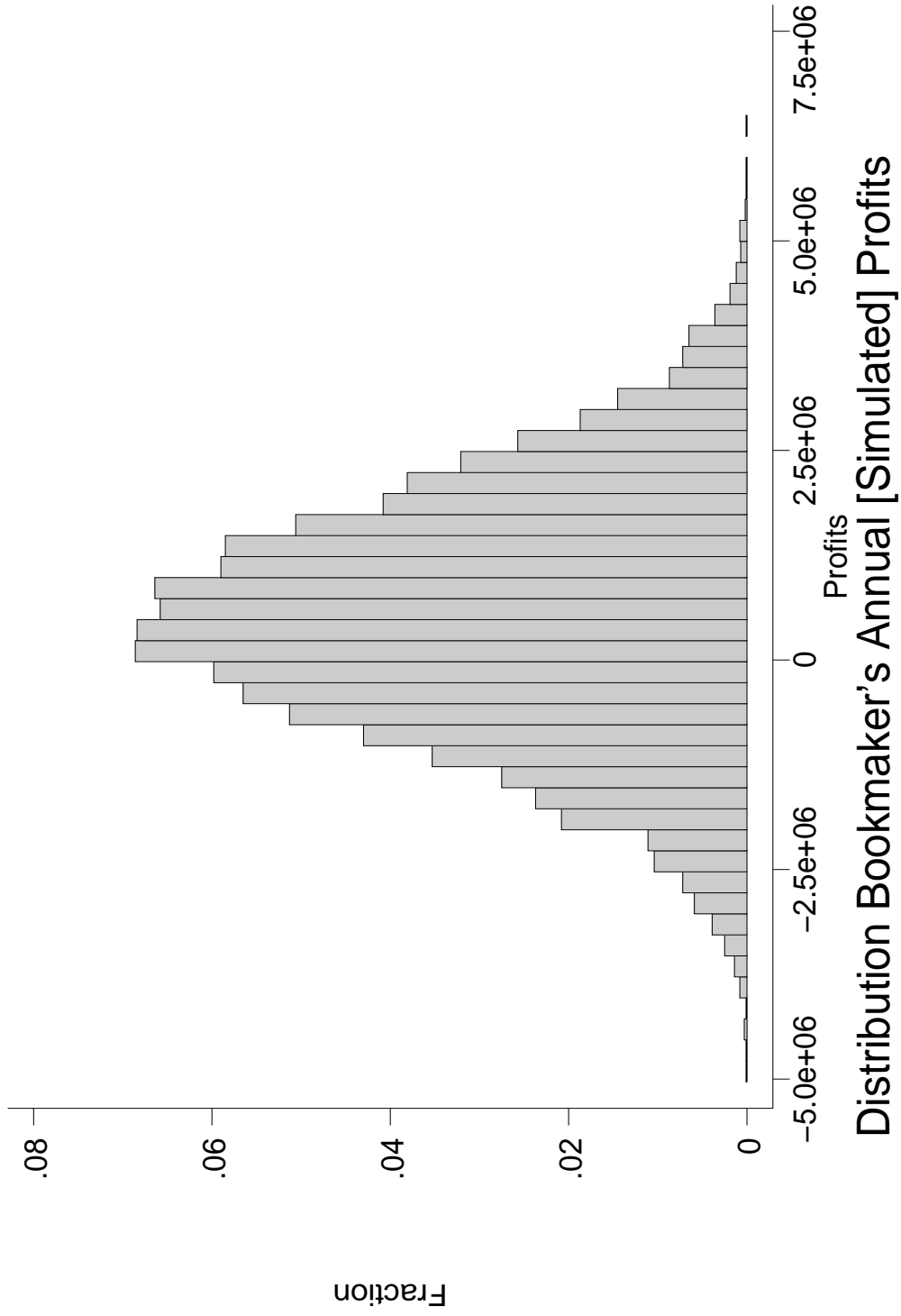
Bookmaker 3

Table 9: Minimum Capital Reserve

Annual Returns Bookie 0

- Algorithm:
 - Monte Carlo simul from weekly distribution
 - Instead: calibrated model for realism
 - details: **Appendix E.2**, below
- $Net\ Revs \sim \mathcal{N}(\$6.7m, \$2.0m^2)$
 - $\Pr(NetRevs < 0) \approx 0$
- $Profits \sim \mathcal{N}(\$0.9m, \$2m^2)$
 - **old Fig 5**: mean=\$500k, std dev=\$1.5m
 - $\Pr(Profits < 0) = \frac{1}{3}$
 - NB: $\pi < 0$
 - ... 1 of 5 yrs with data
 - ... 8% S&P 500 firms
 - ... 50% public internet books

OLD Figure 5: [Simulated] Annual Profits Distribution: Bookmaker 0



Distribution Bookmaker's Annual [Simulated] Profits

Annual Returns Bookie 0 (CONT)

- Annual ror on K :
 - $E(ror) = 90\%$, $Std\ Dev(ror) = 200\%$
 - below one std dev confidence band of legal fin mkt efficient frontier: SEE (5)

Framework

- bet payoff: $(0.5, -M) \oplus (0.5, (1 + v)M)$
- s offsetting wagers, $N - 2s$ unmatched
- G independent events

Distributions (via CLT)

$$\text{Net Revs} \sim \frac{M}{2} \mathcal{N}(NGv, (N - 2s)^2 G(2 + v)^2) \quad (19)$$

$$\text{Profits} = \text{Net Revs}(v = \tilde{v}) - DFC - IFC \quad (20)$$

Advantage: realism

- bookie commission rate (v)
- sheetholder commission ($v \rightarrow \tilde{v}$)
- costs (DFC, IFC)
- see text for param values
- NB: \approx if just Monte Carlo from weekly dist

1. Not invest K in legal mkt.
get higher risk-adjusted ror in legal mkt
2. Not invest K in safe asset.
 - intuition: annual π highly uncertain
 - details: **Appendix E.3**, next sheet
3. Not hedge/layoff
 - \downarrow std dev \$1 \rightarrow \downarrow mean \$0.8.
comparable to legal fin. mkt
 - hedging inexpensive and yet not use
 \rightarrow not strongly dislike risk
 - details: **Appendix E.3**, next sheet

Aside: bias to find RA (since ignore non-fin risk).

Explanation for results: Becker (1968) eqbm entry model of crime

Risk Pref Calc Details (Appendix E.3)

Framework

- **A1** CARA utility, $u(x) = -\exp(-\gamma x)$
- **A2** Annual profits, $x \sim \mathcal{N}(\mu, \sigma^2)$

2. Not invest K in safe asset

$$\begin{aligned} CE &\geq \text{safe} \\ CE &= \mu - \frac{1}{2}\gamma\sigma^2 \\ &\rightarrow \gamma \leq 0 \end{aligned}$$

3. Not hedge/layoff

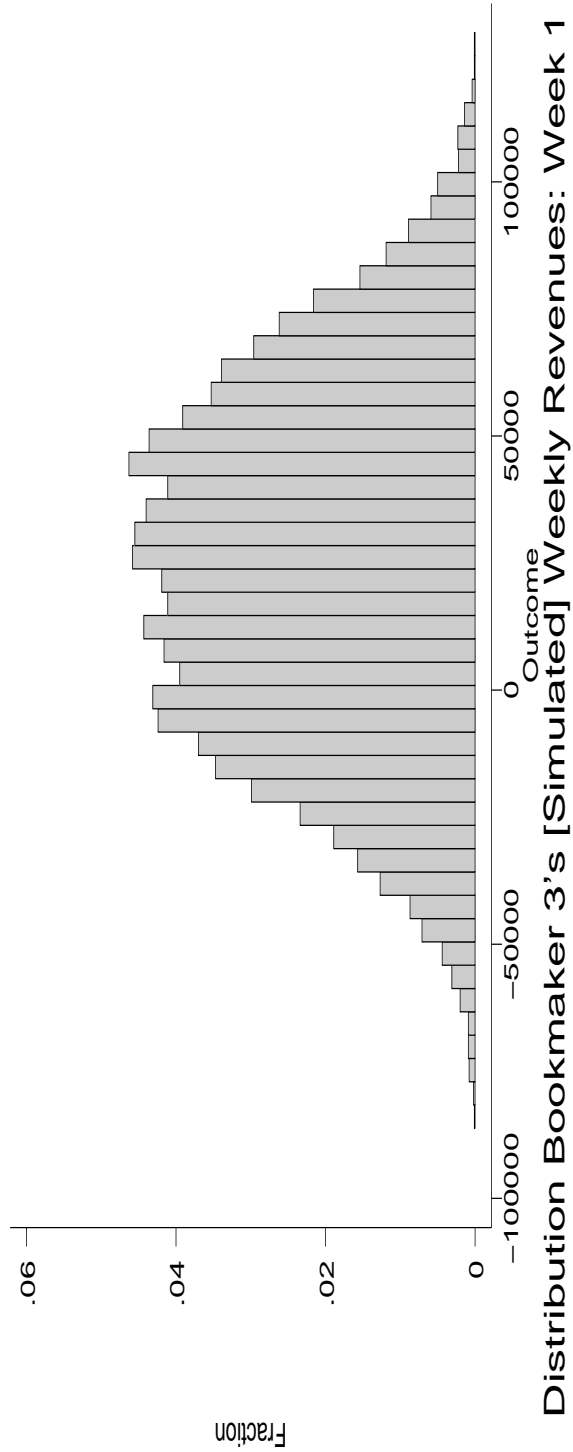
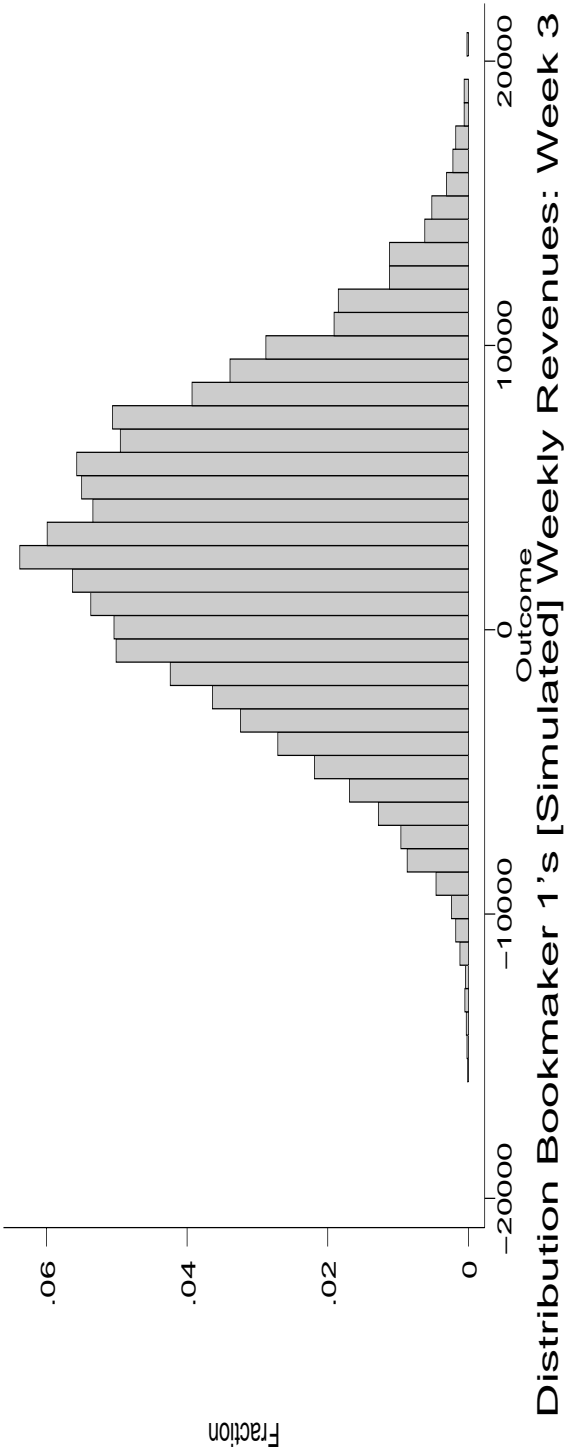
$$\begin{aligned} \max_{b \geq 0} CE &= \mu(b) - \frac{1}{2}\gamma\sigma(b)^2 \\ \text{s.t.} \quad \mu'/\sigma' &= 0.8 \\ b^* &= 0 \quad \rightarrow \quad \gamma \leq 0 \end{aligned}$$

NB: $CRRRA \equiv \gamma \times \text{income} \leq 0$

Risk Taking: Smaller Bookmakers (Bookies 1, 3)

- Some hedging (layoffs) but still unbalanced books
- Weekly net rev dist (**Fig 4**)
 - Bookie 1: \$5k
 - Bookie 3: \$28k
 - (volume-scaled) dispersion \uparrow bookie size
- Min K : **Table 9** (bottom panels)
 - $K_{bookie\ 1}^* \approx \$15K$
 - $K_{bookie\ 3}^* \approx \$75K$
 - scale by bet volume:
 $K^* \uparrow$ bookie size
- $\Pr(\pi \leq 0) \ll$ bookie 0

Figure 4: [Simulated] Weekly Revenue Distribution: Bookmakers 1, 3



Risk Taking: Smaller Bookmakers (CONT)

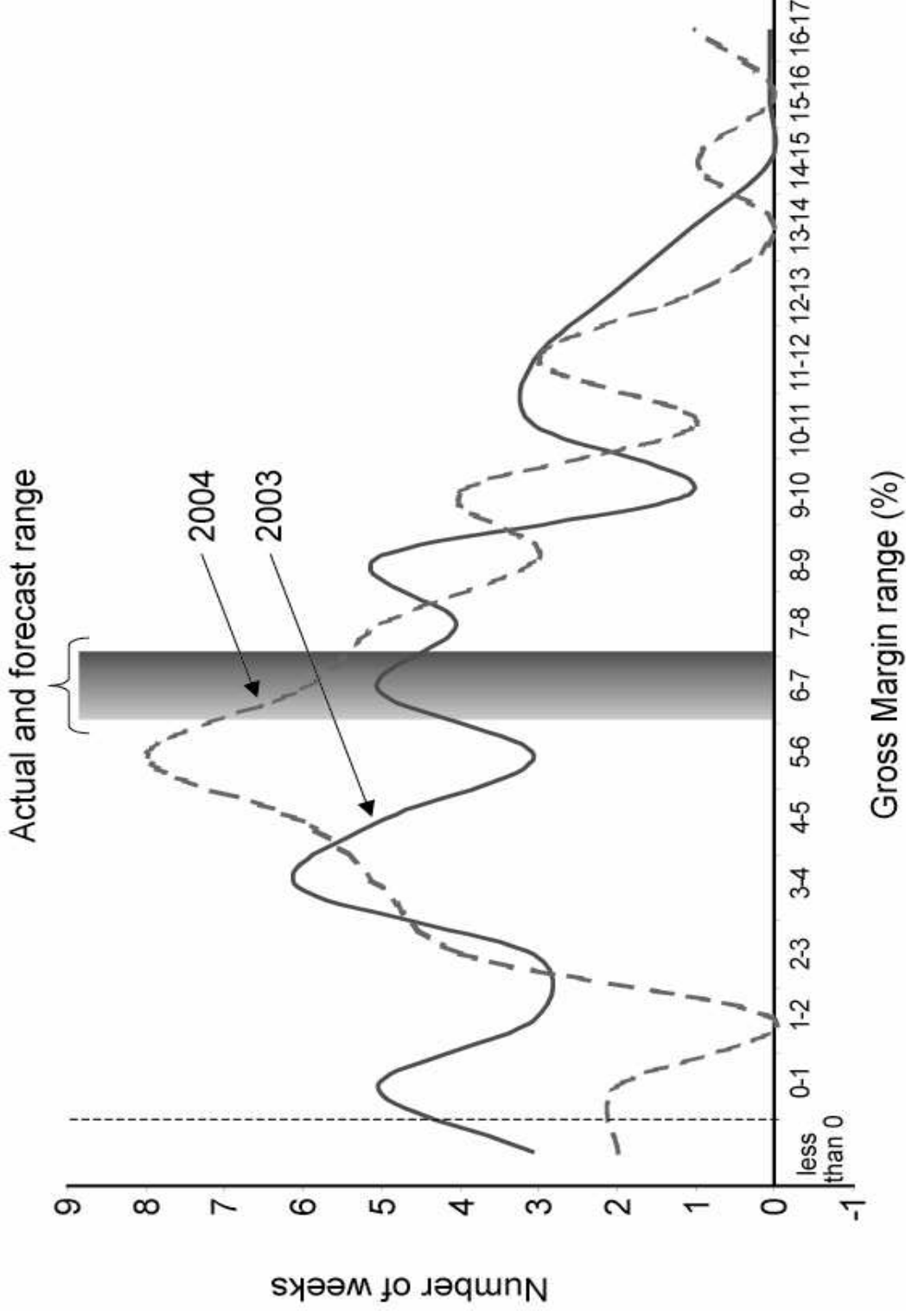
- Risk prefs:
 - must be RA (since layoff)
 - but \approx RN (since net rev dispersion)
 - risk tolerance \uparrow bookie size

Risk Taking: Conclusion

- all bookies take on financial risk
- only small ones even partially hedge.
consistent with π max
- balanced book is poor approximation

Risk Taking – Internet Books Differences

- Hedging: virtually never layoff (CEO; internal docs)
- Net revs less volatile
 - Annual.
Internet: $\text{StdDev}(\text{Hold}) = 0.6\% - 1.0\%$ (firms w/ ≥ 3 yrs data)
Bookie 0: $\text{StdDev}(\text{Hold}) = 1.3\%$
 - Weekly.
Internet: $\text{Pr}(\text{Weekly Net Revs} < 0) = 0.05 \rightarrow$ **Figure**
Bookie 0: $\text{Pr}(\text{Weekly Net Revs} < 0) = 0.32$
- {Profits
 - annual $\pi < 0$ for 50% public internet books ($N = 51$)
 - somewhat volatile (above) }
- Data sources: Internal docs; CEO discussion; annual reports



BetOnSports: Weekly gross margin distribution

Source: Company data

Role of legality

1. Illegal
 - mkt size = neighborhood and/or city (trust ...)
 - means systematic differences in demand (\neq bettors in Vegas)
 - hard to attract bettors on both sides since homogeneous prefs
 - hard to cheaply diversify
 - what about across-city betting?
 - could arbitrage (given home town bias documented here)
 - but hard to do
 - difficult to attract bettors in unfamiliar area
 - layoffs increase chance of arrest (other bookie)
 - layoffs expensive (pay commission)
 - RICO (federal prosecution)

Role of legality (CONT)

2. Internet Books

- not have such imbalanced demand
- since have worldwide pool of bettors (less systematic risk)
- reduces financial risk from charging Vegas prices

Conclusion

- Self-interested behavior powerful
 - explains expensive incentive contract
 - highlights weakness CW on risk
 - why prices track legal mkt
- Outstanding issues:
 - identical commissions as legal mkt
 - bookie's precise obj function (π max?)
 - role legal fringe
- Do results extend to other organized illegal markets with legal quasi-substitutes?
 - ex: file sharing on internet; prostitution
 - how does trust develop?

Conclusion: Policy Implications

1. All sports gambling move to internet/off shore?
 - not completely
 - bettors not have tech sophistication
 - no financial credit:
 - unlikely in future since no trust (below)
 - bettors \$ constrained and unlikely to have access to legit credit mkts
 - hard to develop trust
 - no personal/physical interaction
 - in different countries
 - no property rights
 - alternative: specialization
 - off shore contract with on shore to service accounts
 - common in recent NYC arrests

Conclusion: Policy Implications (CONT)

2. Can prohibitions against internet bookies work?
 - unlikely
 - internet is a-geographic and porous; overcome trust issue
 - little success at eliminating on-shore version
3. Changes if legalized:
 - ↓ role trust
 - ↓ credit (?): might ↓ gambling addiction
 - no price discrimination
 - bookie size grow (size, geographic area)
 - no sheetholders
 - illegal fringe persist.
horses currently

EXTRA SLIDES: betting primer

- Spread:
 - bet team A for \$1
 - spotted P points ($P \lesseqgtr 0$)
 - win if actual points $> P$
 - if win: get \$1
 - if lose: pay $\$(1 + vig)$ where $vig = 0.1$
- Money line:
 - bet team at money line l for \$1
 - win bet if team wins (no spread)
 - if lose: pay \$1
 - if win and $l \geq 100$ (underdog): get $l/100$
 - if win and $l < -100$ (favorite): get $100/|l|$
 - bookie commission:
 - gap in favorite-underdog line

- **Table 10** for more advanced stuff

Bet Type	Definition	(FB = football, BB = basketball, B = baseball, H = Hockey) Payoff	Commission Rate
Straight	Bet on A	$S(A) \equiv (I(\text{win } A) - (1 - I(\text{win } A))(1 + \text{vig})) \text{Bet}_A$ $S(A) \equiv (I(\text{win } A)c_A - (1 - I(\text{win } A)) \text{Bet}_A$	FB, BB: 4.8% B: $\leq 2.3\%$
Exotics			
If	If A then B	$\mathcal{I}(A, B) \equiv S(A) + I(\text{win } A)S(B)$	FB, BB: 7.0% B: $\leq 2.3\%$
Reverse	If A then B , AND If B then A	$\mathcal{R}(A, B) \equiv \mathcal{I}(A, B) + \mathcal{I}(B, A)$	FB, BB: 7.0% B: $\leq 2.3\%$
Reverse Box	$\binom{N}{2}$ Reverses	$\sum_{i,j \in \text{Bets}} \mathcal{R}(i, j)$	FB, BB: 7.0% B: $\leq 2.3\%$
Parlay	Joint bet on multiple events	$\mathcal{P}(\text{Bets}) \equiv \left(\prod_{i \in \text{Bets}} I(\text{win } i) \times 2^{\#\text{Bets}} - 1 - \#\text{Bets} \times \text{vig} \right) \text{Bet}$	FB, BB: 12.5% B: $\leq 3.2\%$
N team Round Robin	$\binom{N}{2}$ two team Parlays	$\mathcal{P}(\text{Bets}) \equiv \left(\prod_{i \in \text{Bets}} I(\text{win } i) \times (1 + c_i) - 1 \right) \text{Bet}$	FB, BB: 12.5% B: $\leq 3.2\%$

Table 10: Types of Bets