

Six anomalies looking for a model.
A consumption based explanation of
international finance puzzles

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A long list of international finance puzzles:

- Why is there an apparent disconnect between exchange rates and consumption?
- Why do high interest rate currencies appreciate?
- Why is risk-sharing high with prices and low with quantities?
- Can we account for the relative smoothness of exchange rate movements and stock markets' fluctuations?
- What is driving the high correlation of international prices, if fundamentals are almost uncorrelated?
- ...

Can we explain the disconnect between international prices and quantities?



Can we explain the disconnect between international prices and quantities?



This paper:

- 1 two slowly moving predictive components of consumption growth
- 2 risk-sensitive preferences
- 3 correlation of long-lasting and temporary shocks to consumption growth
- 4 stochastic volatility

Relation to the literature



- Bansal and Yaron (2004): long-run risks can explain equity premium puzzle
- Colacito and Croce (2007): highly cross-country correlated long-run risks can explain
 - volatility of real exchange rate movements
 - degree of int'l correlation of stock markets
- Colacito and Croce (2008): general equilibrium analysis with risk sensitive preferences
- Colacito, Croce, and Ghysels (in progress): estimation of long-run risks model

Plan of the talk



- The puzzles that we want to explain

Plan of the talk



- The puzzles that we want to explain
- A look at the data

Plan of the talk



- The puzzles that we want to explain
- A look at the data
- The economy

Plan of the talk



- The puzzles that we want to explain
- A look at the data
- The economy
- Inside the black box

Plan of the talk



- The puzzles that we want to explain
- A look at the data
- The economy
- Inside the black box
- Quantitative findings

Things we would like to explain



$$m_{t+1}^* - m_{t+1} = \Delta e_{t+1}$$

Things we would like to explain



$$m_{t+1}^* - m_{t+1} = \Delta e_{t+1}$$

Time Additive preferences

Things we would like to explain

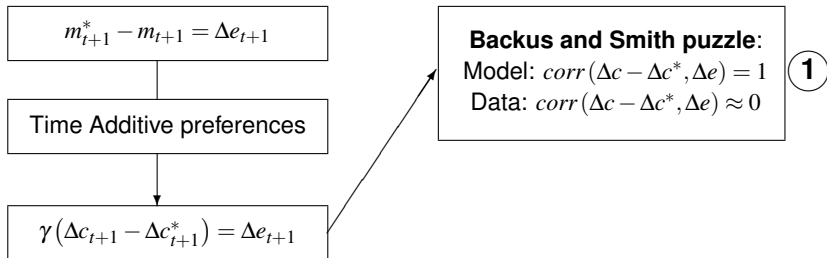


$$m_{t+1}^* - m_{t+1} = \Delta e_{t+1}$$

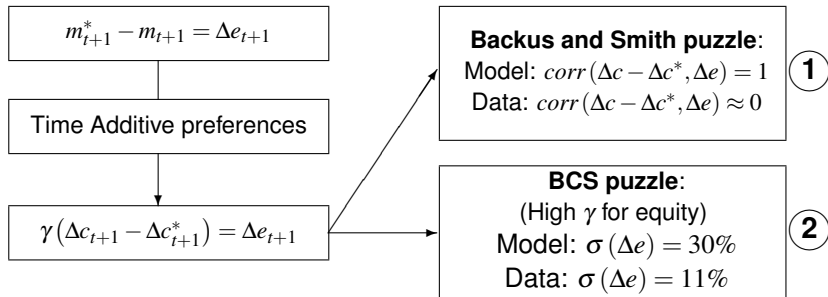
Time Additive preferences

$$\gamma(\Delta c_{t+1} - \Delta c_{t+1}^*) = \Delta e_{t+1}$$

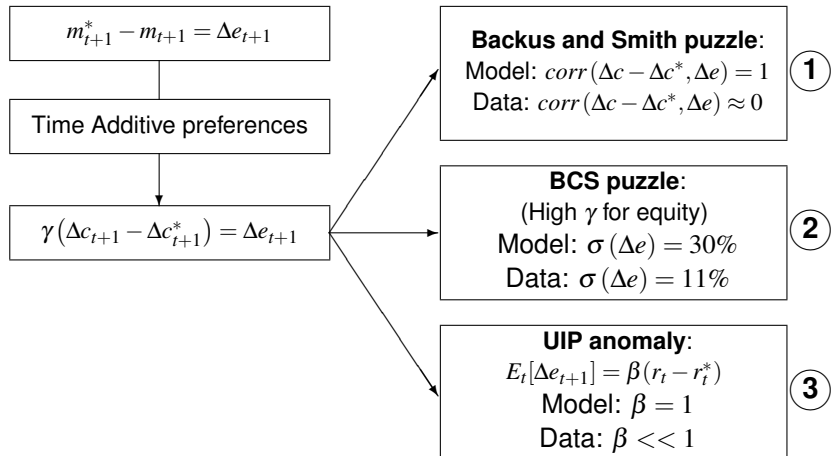
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Things we would like to explain



Things we would like to explain

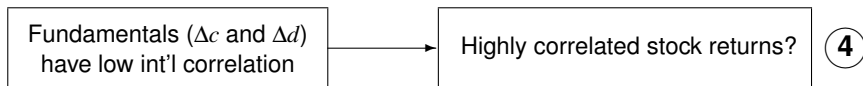


Things we would like to explain (cont'd)

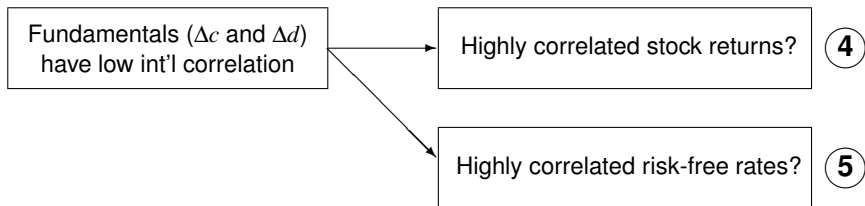


Fundamentals (Δc and Δd)
have low int'l correlation

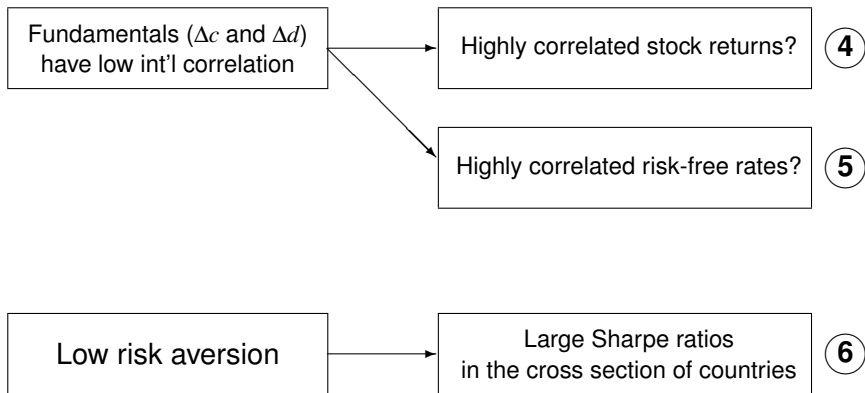
Things we would like to explain (cont'd)



Things we would like to explain (cont'd)



Things we would like to explain (cont'd)





The Backus and Smith anomaly

$$\text{corr}(\Delta c - \Delta c^*, \Delta e)$$

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	-0.223	-0.013	0.166	-0.146	-0.007	-0.037	-0.043
CAN	-0.223	-	0.098	0.185	0.007	0.105	0.153	0.054
JPN	-0.013	0.098	-	0.220	-0.053	0.251	0.069	0.095
GER	0.166	0.185	0.220	-	0.108	0.123	0.157	0.160
UK	-0.146	0.007	-0.053	0.108	-	0.042	0.224	0.030
FRA	-0.007	0.105	0.251	0.123	0.042	-	0.085	0.100
ITA	-0.037	0.153	0.069	0.157	0.224	0.085	-	0.108
Average								0.072



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Average								0.072

Brandt, Cochrane, and Santa-Clara puzzle



Volatility of real exchange rate growth

	US	CAN	JPN	UK	GER	FRA	ITA	Average
US	-	4.506	13.131	10.965	12.479	11.629	11.620	10.722
CAN	4.506	-	13.615	11.435	13.127	12.422	12.365	12.593
JPN	13.131	13.615	-	12.585	11.597	11.470	13.038	12.461
UK	10.965	11.435	12.585	-	9.784	8.967	9.540	10.462
GER	12.479	13.127	11.597	9.784	-	4.631	8.804	9.588
FRA	11.629	12.422	11.470	8.967	4.631	-	7.003	8.898
ITA	11.620	12.365	13.038	9.540	8.804	7.003	-	10.150
Average								10.696

Brandt, Cochrane, and Santa-Clara puzzle



Volatility of real exchange rate growth

	US	CAN	JPN	UK	GER	FRA	ITA	Average
US	-	4.506	13.131	10.965	12.479	11.629	11.620	10.722
CAN	4.506	-	13.615	11.435	13.127	12.422	12.365	12.593
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UK	10.965	11.435	12.585	-	9.784	8.967	9.540	10.462
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FRA	11.629	12.422	11.470	8.967	4.631	-	7.003	8.898
ITA	11.620	12.365	13.038	9.540	8.804	7.003	-	10.150
Average								10.696

Markets' Sharpe Ratios

	US	CAN	JPN	GER	UK	FRA	ITA	Average
Sharpe ratio	58.694	31.278	22.038	43.169	46.286	40.904	22.099	37.781



Correlations of fundamentals

$$\text{corr}(\Delta c, \Delta c^*)$$

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.356	0.077	0.242	0.214	0.266	-0.018	0.228
CAN	0.356	-	-0.074	0.094	0.259	0.165	0.099	0.150
JPN	0.077	-0.074	-	0.067	0.054	0.151	0.126	0.067
GER	0.242	0.094	0.067	-	0.115	-0.066	-0.068	0.064
UK	0.214	0.259	0.054	0.115	-	0.311	-0.001	0.158
FRA	0.266	0.165	0.151	-0.066	0.311	-	0.134	0.160
ITA	-0.018	0.099	0.126	-0.068	-0.001	0.134	-	0.045
Average								0.125



Correlations of fundamentals

$$\text{corr}(\Delta c, \Delta c^*)$$

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.356	0.077	0.242	0.214	0.266	-0.018	0.228
CAN	0.356	-	-0.074	0.094	0.259	0.165	0.099	0.150
JPN	0.077	-0.074	-	0.067	0.054	0.151	0.126	0.067
GER	0.242	0.094	0.067	-	0.115	-0.066	-0.068	0.064
UK	0.214	0.259	0.054	0.115	-	0.311	-0.001	0.158
FRA	0.266	0.165	0.151	-0.066	0.311	-	0.134	0.160
ITA	-0.018	0.099	0.126	-0.068	-0.001	0.134	-	0.045
Average								0.125

$$\text{corr}(\Delta d, \Delta d^*)$$

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.151	0.014	-0.020	-0.053	0.019	0.045	0.026
CAN	0.151	-	0.057	0.173	0.255	-0.035	-0.110	0.082
JPN	0.014	0.057	-	-0.002	0.076	0.062	0.239	0.074
GER	-0.020	0.173	-0.002	-	0.203	0.202	0.183	0.123
UK	-0.053	0.255	0.076	0.203	-	0.219	0.244	0.157
FRA	0.019	-0.035	0.062	0.202	0.219	-	0.343	0.135
ITA	0.045	-0.110	0.239	0.183	0.244	0.343	-	0.157
Average								0.108



Correlations of returns

Correlations of stock markets' returns

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.821	0.481	0.487	0.637	0.588	0.433	0.574
CAN	0.821	-	0.392	0.428	0.540	0.486	0.407	0.512
JPN	0.481	0.392	-	0.294	0.422	0.359	0.389	0.389
GER	0.487	0.428	0.294	-	0.478	0.630	0.498	0.469
UK	0.637	0.540	0.422	0.478	-	0.541	0.376	0.499
FRA	0.588	0.486	0.359	0.630	0.541	-	0.587	0.532
ITA	0.433	0.407	0.389	0.498	0.376	0.587	-	0.448
Average								0.489



Correlations of returns

Correlations of stock markets' returns

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.821	0.481	0.487	0.637	0.588	0.433	0.574
CAN	0.821	-	0.392	0.428	0.540	0.486	0.407	0.512
JPN	0.481	0.392	-	0.294	0.422	0.359	0.389	0.389
GER	0.487	0.428	0.294	-	0.478	0.630	0.498	0.469
UK	0.637	0.540	0.422	0.478	-	0.541	0.376	0.499
FRA	0.588	0.486	0.359	0.630	0.541	-	0.587	0.532
ITA	0.433	0.407	0.389	0.498	0.376	0.587	-	0.448
Average								0.489

Correlations of risk-free rate

	US	CAN	JPN	GER	UK	FRA	ITA	Average
US	-	0.695	0.664	0.502	0.709	0.520	0.625	0.619
CAN	0.695	-	0.758	0.686	0.754	0.719	0.671	0.714
JPN	0.664	0.758	-	0.609	0.659	0.648	0.784	0.687
GER	0.502	0.686	0.609	-	0.630	0.701	0.563	0.615
UK	0.709	0.754	0.659	0.630	-	0.761	0.737	0.708
FRA	0.520	0.719	0.648	0.701	0.761	-	0.837	0.698
ITA	0.625	0.671	0.784	0.563	0.737	0.837	-	0.703
Average								0.678

Setup of the economy



- Complete markets



Setup of the economy

- Complete markets
- Agents have risk-sensitive preferences:

$$U_t = (1 - \delta) \log C_t + \frac{\delta}{1 - \gamma} \log E_t \exp \{ (1 - \gamma) U_{t+1} \}$$

$$U_t^* = (1 - \delta) \log C_t^* + \frac{\delta}{1 - \gamma} \log E_t \exp \{ (1 - \gamma) U_{t+1}^* \}$$



Setup of the economy

- Complete markets
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$$U_t^* = (1 - \delta) \log C_t^* + \frac{\delta}{1 - \gamma} \log E_t \exp \{ (1 - \gamma) U_{t+1}^* \}$$

- Long-run risks in consumption

$$\Delta c_{t+1} = \mu_c + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + \sigma_c \varepsilon_{t+1}$$

$$\Delta c_{t+1}^* = \mu_c^* + \lambda_2 z_{1,t} + \lambda_1 z_{2,t} + \sigma_c^* \varepsilon_{t+1}^*$$

$$z_{1,t} = \rho_1 z_{1,t-1} + \sigma_1 \varepsilon_{1,t+1}$$

$$z_{2,t} = \rho_2 z_{2,t-1} + \sigma_2 \varepsilon_{2,t+1}$$

Calibration



$$U_t = (1 - \delta) \log C_t + \frac{\delta}{1 - \gamma} \log E_t \exp \{ (1 - \gamma) U_{t+1} \}$$

$$\Delta c_{t+1} = \mu_c + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + \sigma_c \varepsilon_{t+1}$$

$$\Delta c_{t+1}^* = \mu_c^* + \lambda_2 z_{1,t} + \lambda_1 z_{2,t} + \sigma_c^* \varepsilon_{t+1}^*$$

$$z_{1,t} = \rho_1 z_{1,t-1} + \sigma_1 \varepsilon_{1,t+1}$$

$$z_{2,t} = \rho_2 z_{2,t-1} + \sigma_2 \varepsilon_{2,t+1}$$

Calibration



$$U_t = (1 - .991) \log C_t + \frac{.991}{1 - 8} \log E_t \exp \{ (1 - 8) U_{t+1} \}$$

$$\Delta c_{t+1} = \mu_c + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + \sigma_c \varepsilon_{t+1}$$

$$\Delta c_{t+1}^* = \mu_c^* + \lambda_2 z_{1,t} + \lambda_1 z_{2,t} + \sigma_c^* \varepsilon_{t+1}^*$$

$$z_{1,t} = \rho_1 z_{1,t-1} + \sigma_1 \varepsilon_{1,t+1}$$

$$z_{2,t} = \rho_2 z_{2,t-1} + \sigma_2 \varepsilon_{2,t+1}$$

Calibration



$$U_t = (1 - .991) \log C_t + \frac{.991}{1 - 8} \log E_t \exp \{ (1 - 8) U_{t+1} \}$$

$$\Delta c_{t+1} = .001 + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + .006 \varepsilon_{t+1}$$

$$\Delta c_{t+1}^* = .001 + \lambda_2 z_{1,t} + \lambda_1 z_{2,t} + .006 \varepsilon_{t+1}^*$$

$$z_{1,t} = .987 \cdot z_{1,t-1} + .00005 \varepsilon_{1,t+1}$$

$$z_{2,t} = .987 \cdot z_{2,t-1} + .00005 \varepsilon_{2,t+1}$$

Calibration



$$U_t = (1 - .991) \log C_t + \frac{.991}{1 - 8} \log E_t \exp \{ (1 - 8) U_{t+1} \}$$

$$\Delta c_{t+1} = .001 + 5.5 \cdot z_{1,t} + 1 \cdot z_{2,t} + .006 \epsilon_{t+1}$$

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$$z_{2,t} = .987 \cdot z_{2,t-1} + .00005 \varepsilon_{2,t+1}$$

$$\text{corr}(\varepsilon_{t+1}, \varepsilon_{t+1}^*) = 0.1$$

Calibration



$$U_t = (1 - .991) \log C_t + \frac{.991}{1 - 8} \log E_t \exp \{ (1 - 8) U_{t+1} \}$$

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$$z_{2,t} = .987 \cdot z_{2,t-1} + .00005 \varepsilon_{2,t+1}$$

$$\text{corr}(\varepsilon_{t+1}, \varepsilon_{t+1}^*) = 0.1, \quad \text{corr}(\varepsilon_{1,t+1}, \varepsilon_{2,t+1}) = 0.6$$

Calibration



$$U_t = (1 - .991) \log C_t + \frac{.991}{1 - 8} \log E_t \exp \{ (1 - 8) U_{t+1} \}$$

$$\Delta c_{t+1} = .001 + 5.5 \cdot z_{1,t} + z_{2,t} + .006 \varepsilon_{t+1}$$

$$\Delta c_{t+1}^* = .001 + z_{1,t} + 5.5 \cdot z_{2,t} + .006 \varepsilon_{t+1}^*$$

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$$\text{corr}(\varepsilon_{t+1}, \varepsilon_{t+1}^*) = 0.1, \quad \text{corr}(\varepsilon_{1,t+1}, \varepsilon_{2,t+1}) = 0.6$$

$$\text{corr}(\varepsilon_{t+1}, \varepsilon_{1,t+1}) = 0.06, \quad \text{corr}(\varepsilon_{t+1}^*, \varepsilon_{2,t+1}) = 0.06$$

Calibration



$$U_t = (1 - .991) \log C_t + \frac{.991}{1 - 8} \log E_t \exp \{ (1 - 8) U_{t+1} \}$$

$$\Delta c_{t+1} = .001 + 5.5 \cdot z_{1,t} + z_{2,t} + .006 \varepsilon_{t+1}$$

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$$z_{2,t} = .987 \cdot z_{2,t-1} + .00005 \varepsilon_{2,t+1}$$

$$\text{corr}(\varepsilon_{t+1}, \varepsilon_{t+1}^*) = 0.1, \quad \text{corr}(\varepsilon_{1,t+1}, \varepsilon_{2,t+1}) = 0.6$$

$$\text{corr}(\varepsilon_{t+1}, \varepsilon_{1,t+1}) = 0.06, \quad \text{corr}(\varepsilon_{t+1}^*, \varepsilon_{2,t+1}) = 0.06$$

$$\text{corr}(\varepsilon_{t+1}, \varepsilon_{2,t+1}) = 0.60, \quad \text{corr}(\varepsilon_{t+1}^*, \varepsilon_{1,t+1}) = 0.60$$

Explaining the Backus and Smith anomaly



Explaining the Backus and Smith anomaly



$$\Delta e_t - E_{t-1}[\Delta e_t] = (\varepsilon_{c,t} - \varepsilon_{c^*,t})$$

$$(\Delta c_t - \Delta c_t^*) - E_{t-1}(\Delta c_t - \Delta c_t^*) = (\varepsilon_{c,t} - \varepsilon_{c^*,t})$$

Explaining the Backus and Smith anomaly



$$\Delta e_t - E_{t-1}[\Delta e_t] = (\varepsilon_{c,t} - \varepsilon_{c^*,t}) + \left[(1 - \gamma) \frac{\delta(\lambda_2 - \lambda_1)}{1 - \delta\rho} \right] (\varepsilon_{1,t} - \varepsilon_{2,t})$$

$$(\Delta c_t - \Delta c_t^*) - E_{t-1}(\Delta c_t - \Delta c_t^*) = (\varepsilon_{c,t} - \varepsilon_{c^*,t})$$

Explaining the Backus and Smith anomaly



$$\Delta e_t - E_{t-1}[\Delta e_t] = (\varepsilon_{c,t} - \varepsilon_{c^*,t}) + \underbrace{\left[(1-8) \frac{\delta(1-5)}{1-\delta\rho} \right]}_{>0} (\varepsilon_{1,t} - \varepsilon_{2,t})$$

$$(\Delta c_t - \Delta c_t^*) - E_{t-1}(\Delta c_t - \Delta c_t^*) = (\varepsilon_{c,t} - \varepsilon_{c^*,t})$$

Explaining the Backus and Smith anomaly



$$\Delta e_t - E_{t-1}[\Delta e_t] = (\varepsilon_{c,t} - \varepsilon_{c^*,t}) + \underbrace{\left[(1-\delta) \frac{\delta(1-\psi)}{1-\delta\rho} \right]}_{>0} (\varepsilon_{1,t} - \varepsilon_{2,t})$$

$$(\Delta c_t - \Delta c_t^*) - E_{t-1}(\Delta c_t - \Delta c_t^*) = (\varepsilon_{c,t} - \varepsilon_{c^*,t})$$

Can we engineer negative correlation between $(\varepsilon_{c,t} - \varepsilon_{c^*,t})$ and $(\varepsilon_{1,t} - \varepsilon_{2,t})$?

Explaining the Backus and Smith anomaly



$$\Delta e_t - E_{t-1}[\Delta e_t] = (\varepsilon_{c,t} - \varepsilon_{c^*,t}) + \underbrace{\left[(1-\delta) \frac{\delta(1-\delta)}{1-\delta\rho} \right]}_{>0} (\varepsilon_{1,t} - \varepsilon_{2,t})$$

$$(\Delta c_t - \Delta c_t^*) - E_{t-1}(\Delta c_t - \Delta c_t^*) = (\varepsilon_{c,t} - \varepsilon_{c^*,t})$$

Can we engineer negative correlation between $(\varepsilon_{c,t} - \varepsilon_{c^*,t})$ and $(\varepsilon_{1,t} - \varepsilon_{2,t})$?

Yes, by setting:

- high correlation b/w $\varepsilon_{c,t}$ and $\varepsilon_{2,t}$ and b/w $\varepsilon_{c^*,t}$ and $\varepsilon_{1,t}$

Explaining the Backus and Smith anomaly



$$\Delta e_t - E_{t-1}[\Delta e_t] = (\varepsilon_{c,t} - \varepsilon_{c^*,t}) + \underbrace{\left[(1-\delta) \frac{\delta(1-5)}{1-\delta\rho} \right]}_{>0} (\varepsilon_{1,t} - \varepsilon_{2,t})$$

$$(\Delta c_t - \Delta c_t^*) - E_{t-1}(\Delta c_t - \Delta c_t^*) = (\varepsilon_{c,t} - \varepsilon_{c^*,t})$$

Can we engineer negative correlation between $(\varepsilon_{c,t} - \varepsilon_{c^*,t})$ and $(\varepsilon_{1,t} - \varepsilon_{2,t})$?

Yes, by setting:

- high correlation b/w $\varepsilon_{c,t}$ and $\varepsilon_{2,t}$ and b/w $\varepsilon_{c^*,t}$ and $\varepsilon_{1,t}$
- low correlation b/w $\varepsilon_{c,t}$ and $\varepsilon_{1,t}$ and b/w $\varepsilon_{c^*,t}$ and $\varepsilon_{2,t}$

Explaining the Brandt, Cochrane, and Santa-Clara anomaly (and its side-effects!)



Low real exchange rate volatility

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Drawback: are risk-free rates too correlated?

$$\begin{aligned} r_t^f - \bar{r}^f &= \lambda_1 z_{1,t} + \lambda_2 z_{2,t} \\ r_t^{f^*} - \bar{r}^{f^*} &= \lambda_2 z_{1,t} + \lambda_1 z_{2,t} \end{aligned}$$

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- Reduce $\text{corr}(\varepsilon_{1,t}, \varepsilon_{2,t})$
- Turn on $\text{corr}(\varepsilon_{c,t}, \varepsilon_{2,t}) = \text{corr}(\varepsilon_{c^*,t}, \varepsilon_{1,t}) = 0.6$



Remainder of the model

- Stochastic volatility:

$$\Delta c_{t+1} = \mu_c + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + \lambda_t^{1/2} \varepsilon_{t+1}$$

$$\Delta c_{t+1}^* = \mu_c + \lambda_1^* z_{1,t} + \lambda_2^* z_{2,t} + \lambda_t^{*1/2} \varepsilon_{t+1}^*$$

$$z_{1,t} = \rho_1 z_{1,t-1} + \lambda_{t-1}^{1/2} \varepsilon_{1,t+1}$$

$$z_{2,t} = \rho_2 z_{2,t-1} + \lambda_{t-1}^{*1/2} \varepsilon_{2,t+1}$$

$$\lambda_t = \sigma(1 - \rho_\lambda) + \rho_\lambda \lambda_{t-1} + \varphi_\lambda \lambda_{t-1} \varepsilon_{\lambda,t}$$

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- Cash flows of redundant assets:

$$\Delta d_{t+1} = \mu_d + \lambda_{d_1} z_{1,t} + \lambda_{d_2} z_{2,t} + \lambda_t^{1/2} \varepsilon_{d,t+1}$$

$$\Delta d_{t+1}^* = \mu_d + \lambda_{d_1}^* z_{1,t} + \lambda_{d_2}^* z_{2,t} + \lambda_t^{*1/2} \varepsilon_{d,t+1}^*$$

Further results



- Well known results:
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Without long-run risks

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With long-run risks

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$$\text{Low Correlation} \begin{cases} \Delta d_t - E_{t-1} [\Delta d_t] = \varepsilon_{d,t} \\ \Delta d_t^* - E_{t-1} [\Delta d_t^*] = \varepsilon_{d,t}^* \end{cases}$$



Results

	Data	Model	95% CI
Volatility of exchange rate growth	10.696	10.929	(9.105,12.954)
UIP regression slope	-0.136	-0.062	(-0.609,0.458)
Sharpe ratio	37.781	40.564	(2.783,76.447)
Backus and Smith anomaly	0.072	-0.056	(-0.279,0.165)
Correlation of risk-free rates	0.678	0.793	(0.189,0.954)
Correlation of excess returns	0.489	0.422	(0.232,0.578)



Results

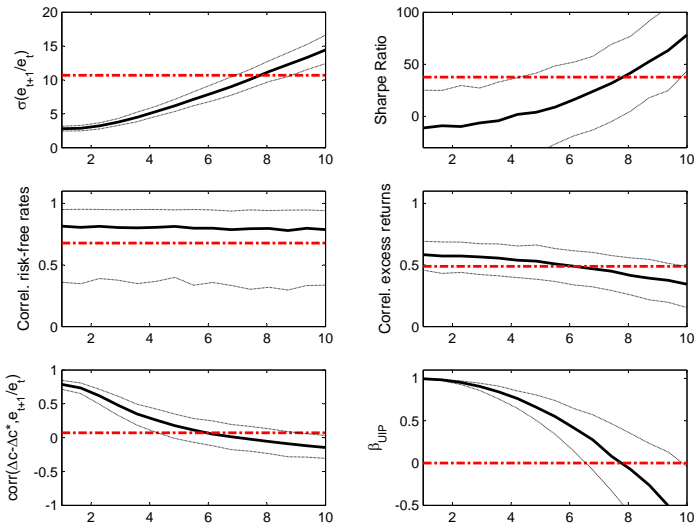
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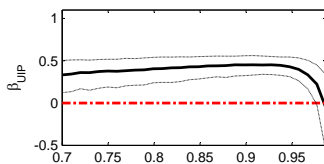
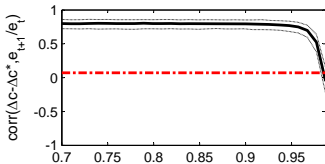
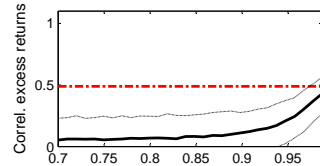
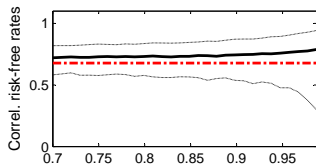
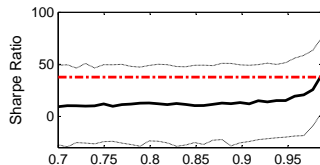
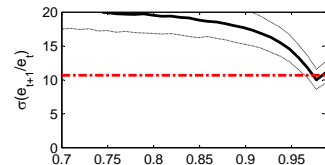
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Volatility of dividend growth	9.329	9.845	(8.115,12.045)
AC(1)	0.158	0.109	(-0.142,0.384)
AC(2)	0.093	0.088	(-0.161,0.376)
AC(3)	0.086	0.087	(-0.161,0.358)
AC(4)	0.182	0.075	(-0.164,0.356)
Correlation of dividend growths	0.108	0.135	(-0.113,0.380)

The case for risk-sensitive preferences

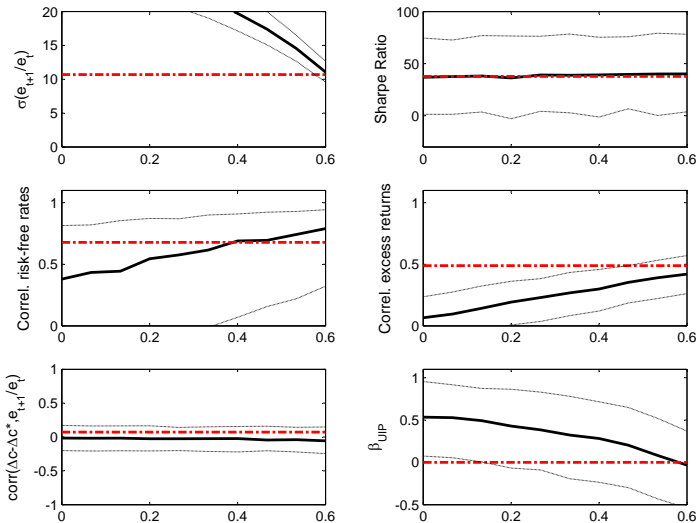


The case for long-run risks



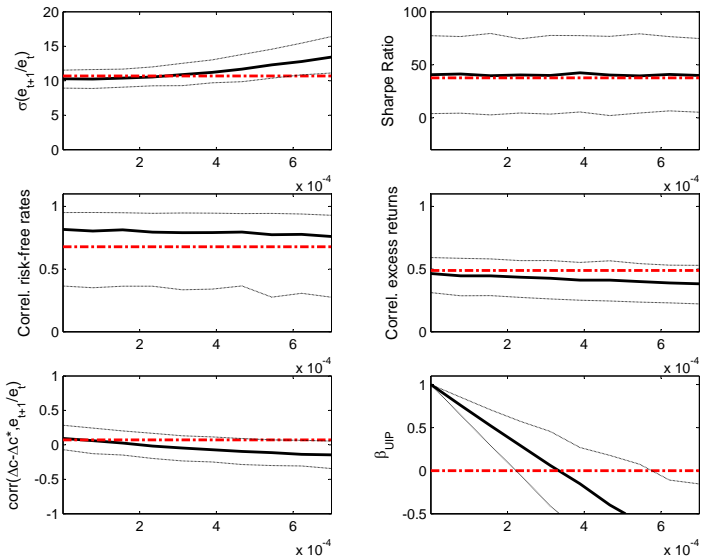


The case for two factors





The case for stochastic volatility





Concluding remarks

- A simple two-country long-run risks model can explain
 - ① Backus-Smith anomaly
 - ② Brandt, Cochrane, and Santa-Clara puzzle
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 - ④ High correlation of stock markets
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 - ⑥ Large Sharpe ratios in the cross-section of countries



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 - Identification of low frequency components of consumption growth: Colacito, Croce, and Ghysels (2008)