Common Risk Factors in Currency Markets

Discussion by Ric Colacito

AFA Meetings, Atlanta, 1/2010
Contributions

1 Empirical: the difference in returns between the low and high interest rate currencies explains a large share of the cross-section of currency returns.

- Sizeable spread in the cross-sectional variation of currency returns: Lustig and Verdehlan (2007)

→ The carry trade risk factor (or $HML_{FX}$)
Contributions

1. **Empirical**: the difference in returns between the low and high interest rate currencies explains a large share of the cross-section of currency returns.
   
   - Sizeable spread in the cross-sectional variation of currency returns: Lustig and Verdehlan (2007)
   
   → The carry trade risk factor (or $HML_{FX}$)

2. **Theoretical**: refines the set of restrictions on the joint conditional distribution of international stochastic discount factors.
   

   → SDF's: heterogenous exposure to a common risk factor
Empirical contribution in a nutshell

The cross-section of carry trade returns

<table>
<thead>
<tr>
<th></th>
<th>ALL COUNTRIES</th>
<th>DEVELOPED COUNTRIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-minus-Low:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_{x_j} - r_{x_1} ) (without b-a)</td>
<td>2.95 4.33 6.59 6.46 8.83</td>
<td>2.75 5.35 4.47 6.29</td>
</tr>
<tr>
<td>Mean</td>
<td>2.95</td>
<td>2.75</td>
</tr>
<tr>
<td>Std</td>
<td>5.36</td>
<td>6.42</td>
</tr>
<tr>
<td>SR</td>
<td>0.55</td>
<td>0.43</td>
</tr>
<tr>
<td>High-minus-Low:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_{x_{net,j}} - r_{x_{net,1}} ) (with b-a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.75</td>
<td>0.57</td>
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<tr>
<td>Std</td>
<td>5.36</td>
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<tr>
<td>SR</td>
<td>0.14</td>
<td>0.09</td>
</tr>
</tbody>
</table>
Empirical contribution in a nutshell

1. The cross-section of carry trade returns
2. Explanatory power of $HML_{FX}$ factor

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{HML_{FX}}$</th>
<th>$\lambda_{RX}$</th>
<th>$b_{HML_{FX}}$</th>
<th>$b_{RX}$</th>
<th>$R^2$</th>
<th>RMSE</th>
<th>$\chi^2$</th>
</tr>
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<tbody>
<tr>
<td>$GMM_1$</td>
<td>5.46</td>
<td>1.35</td>
<td>0.59</td>
<td>0.26</td>
<td>69.28</td>
<td>0.95</td>
<td>13.83</td>
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<tr>
<td></td>
<td>[2.34]</td>
<td>[1.68]</td>
<td>[0.25]</td>
<td>[0.32]</td>
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<td>$GMM_2$</td>
<td>4.88</td>
<td>0.58</td>
<td>0.52</td>
<td>0.12</td>
<td>47.89</td>
<td>1.24</td>
<td>15.42</td>
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<td>[2.23]</td>
<td>[1.63]</td>
<td>[0.24]</td>
<td>[0.31]</td>
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<td>5.46</td>
<td>1.35</td>
<td>0.58</td>
<td>0.26</td>
<td>69.28</td>
<td>0.95</td>
<td>13.02</td>
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<tr>
<td></td>
<td>[1.82]</td>
<td>[1.34]</td>
<td>[0.19]</td>
<td>[0.25]</td>
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<tr>
<td></td>
<td>(1.83)</td>
<td>(1.34)</td>
<td>(0.20)</td>
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Empirical contribution in a nutshell

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<tr>
<th>Portfolio</th>
<th>$\alpha_0^j$</th>
<th>$\beta_{HML_{FX}}^j$</th>
<th>$\beta_{RX}^j$</th>
<th>$R^2$</th>
<th>$\chi^2(\alpha)$</th>
<th>$p-value$</th>
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<tbody>
<tr>
<td>1</td>
<td>-0.56</td>
<td>-0.39</td>
<td>1.06</td>
<td>91.36</td>
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<tr>
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<td>[0.02]</td>
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<tr>
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<td>-1.21</td>
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<td>0.97</td>
<td>78.54</td>
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<tr>
<td>3</td>
<td>-0.13</td>
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All

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0.12
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Theoretical contribution in a nutshell

- Candidate SDF’s

\[ m_{t+1}^i - E_t \left[ m_{t+1}^i \right] = -\sqrt{\gamma^i z_t^i u_t^i} - \sqrt{\delta^i z_t^w u_t^w} \]
Theoretical contribution in a nutshell

- Candidate SDF’s

\[ m_{t+1}^i - E_t [m_{t+1}^i] = -\sqrt{\gamma^i z_t^i u_{t+1}^i} - \sqrt{\delta^i z_t^w w_{t+1}^w} \]

- Currency risk premium

\[ E_t [RX_{t+1}^i] = \sqrt{\delta^{us}} \left( \sqrt{\delta^{us}} - \sqrt{\delta^i} \right) z_t^w + \gamma^{us} z_t \]
Theoretical contribution in a nutshell

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- Heterogeneity in \( \delta^i \)'s explains cross-section of currency returns.
a) Complete markets’ assumption, b) cross-section of currency risk premia, and c) volatility of exchange rates’ fluctuations impose restrictions on

i) volatility of domestic SDF
ii) volatility of foreign SDF
iii) correlation between domestic and foreign SDF’s
1. a) Complete markets’ assumption, b) cross-section of currency risk premia, and c) volatility of exchange rates’ fluctuations impose restrictions on
   i) volatility of domestic SDF
   ii) volatility of foreign SDF
   iii) correlation between domestic and foreign SDF’s

2. Where else are the heterogeneous loadings on the world factor going to show up?
   - The cross section of currencies’ volatilities
   - The cross section of stock market returns’ correlations
Comment 1: existence region

- Stochastic discount factors are \( m \) and \( m^* \).
- Volatilities are \( \sigma_m \) and \( \sigma_{m^*} \); correlation is \( \rho_{m,m^*} \).
Comment 1: existence region

- Stochastic discount factors are $m$ and $m^*$.
- Volatilities are $\sigma_m$ and $\sigma_{m^*}$; correlation is $\rho_{m,m^*}$.
- Markets are complete.
Comment 1: existence region

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- Volatilities are $\sigma_m$ and $\sigma_{m^*}$; correlation is $\rho_{m,m^*}$.
- Markets are complete.
- Exchange rate volatility is

$$\sigma_t [\Delta e_{t+1}] = \sigma_m^2 + \sigma_{m^*}^2 - 2\rho_{m,m^*} \sigma_m \sigma_{m^*}$$
Comment 1: existence region

- Stochastic discount factors are $m$ and $m^*$. 
- Volatilities are $\sigma_m$ and $\sigma_{m^*}$; correlation is $\rho_{m,m^*}$. 
- Markets are complete. 
- Exchange rate volatility is 
  \[ \sigma_t[\Delta e_{t+1}] = \sigma_m^2 + \sigma_{m^*}^2 - 2\rho_{m,m^*}\sigma_m\sigma_{m^*} \]
- Carry trade risk premium is 
  \[ E_t[RX_{t+1}] = \sigma_m^2 - \rho_{m,m^*}\sigma_m\sigma_{m^*} \]
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- Volatilities are $\sigma_m$ and $\sigma_{m^*}$; correlation is $\rho_{m,m^*}$.
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- Carry trade risk premium is
  \[ E_t [RX_{t+1}] = \sigma_m^2 - \rho_{m,m^*}\sigma_m\sigma_{m^*} \]
- Exchange rate volatility and carry trade premia impose restrictions on the joint dynamics of $\sigma_m$, $\sigma_{m^*}$, and $\rho_{m,m^*}$. 
SDF’s volatilities and correlations

\[ \rho_{m,m} \]

\[ \sigma_m = 0.25, 0.3, 0.35 \]

\[ \sigma_{\Delta e} = 14\% \]
SDF’s volatilities and correlations

- Correlations and Volatilities

\[ \sigma_m \]
\[ \sigma_m^* \]
\[ \rho_{m,m}^* \]
\[ E[R_X] = 1\% \]
\[ \sigma[\Delta e] = 14\% \]
SDF’s volatilities and correlations
SDF’s volatilities and correlations

Motivation

In a nutshell

Existence Region

Correlations and Volatilities

E[RX]=5%

σ[Δe]=31%
Comment 1: questions

1. How much cross-sectional dispersion of FX volatilities is needed to account for the cross-section of currency returns?
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2. Would forming portfolios according to the expected FX volatility deliver the same results?
Comment 1: questions

1. How much cross-sectional dispersion of FX volatilities is needed to account for the cross-section of currency returns?

2. Would forming portfolios according to the expected FX volatility deliver the same results?

3. Would $HML_{FX}$ preserve its explanatory power?
Comment 2: heterogeneous loadings on the world factor ($\delta^i$)

- Proposed stochastic discount factor

$$m_{t+1}^i = -\lambda^i z_t^i - \tau^i z_w^i - \xi^i_{t+1} - \sqrt{\delta^i z_w^i u_{t+1}^w}$$
Comment 2: heterogeneous loadings on the world factor ($\delta^i$)

- Proposed stochastic discount factor
  \[
  m^i_{t+1} = -\lambda^i z_t^i - \tau^i z_t^w - \xi_{t+1} - \sqrt{\delta^i z_t^w u_{t+1}}
  \]

- Specify a dividend process
  \[
  \Delta d^i_{t+1} = \lambda^i z_t^i + \tau^i z_t^w + \xi_{t+1}
  \]
Comment 2: heterogeneous loadings on the world factor ($\delta^i$)

- Proposed stochastic discount factor

$$m^i_{t+1} = -\lambda^i z_t^i - \tau^i z^w_t - \xi^i_{t+1} - \sqrt{\delta^i z^w_t u^w_{t+1}}$$

- Specify a dividend process

$$\Delta d^i_{t+1} = \lambda^i z_t^i + \tau^i z^w_t + \xi^i_{t+1}$$

- Excess returns in country $i$

$$r^i_{t+1} - E_t [r^i_{t+1}] = \underbrace{\xi^i_{t+1}}_{\text{country specific}} + \underbrace{\frac{\kappa^i_1 \sigma^w}{2(1 - \kappa^i_1 \phi^w)} \delta^i \sqrt{z^w_t v^w_{t+1}}}_{\text{global}}$$
Motivation
In a nutshell
Existence Region

Correlations and volatilities

\[ r_{t+1}^i - E_t [r_{t+1}^i] = \xi_{t+1}^i + \beta \delta^i \sqrt{Z_t^w} \nu_{t+1}^w \]

Returns Correlations

\[ \delta \]

\[ 5 \quad 10 \quad 15 \quad 20 \]

\[ 0.44 \quad 0.46 \quad 0.48 \quad 0.5 \quad 0.52 \quad 0.54 \quad 0.56 \quad 0.58 \quad 0.6 \quad 0.62 \]
Correlations and volatilities

\[ r_{t+1} - E_t[r_{t+1}] = \xi_i + \beta \delta^i \sqrt{z_{t+1}^w} \sqrt{v_{t+1}^w} \]

\[ \Delta e_{t+1} - E_t[\Delta e_{t+1}] = (\xi_{us} - \xi_i) + (\sqrt{\delta_{us} - \sqrt{\delta_i}}) \sqrt{z_{t+1}^w} u_{t+1}^w \]
Correlations and volatilities

\[
\begin{align*}
    r_{t+1}^i - E_t [r_{t+1}^i] &= \xi_{t+1}^i + \beta \delta^i \sqrt{z_{t+1}^w} \nu_{t+1}^w \\
    \Delta e_{t+1}^i - E_t [\Delta e_{t+1}^i] &= (\xi_{t+1}^{us} - \xi_{t+1}^i) + \left(\sqrt{\delta_{t+1}^{us}} - \sqrt{\delta^i}\right) \sqrt{z_{t+1}^w} u_{t+1}^w
\end{align*}
\]

![Graphs showing the relationship between returns, correlations, and exchange rates volatilities.](image)